Optimal Distributed Generation and Capacitor placement in Power Distribution Networks for Power Loss Minimization

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Abstract—This paper presents a new combined technique for minimizing the power loss in distribution system by optimal Distributed Generation (DG) installation together with capacitor placement. Sensitivity analysis is used to identify the optimal candidate locations of DGs and capacitor placement. Bacterial Foraging Optimization Algorithm (BFOA) is applied to find the optimal size of DGs and capacitors. BFOA is a swarm intelligence technique which models the individual and group foraging policies of the E. coli bacteria as a distributed optimization process. The technical constraints of voltage and branch current carrying capacity are included in the assessment of the objective function. Different cases of DG and capacitor placement are considered to assess the performance of the proposed method. Proposed method has been tested on IEEE 33-bus radial distribution system and the results obtained are encouraging.

Keywords— Distributed Generation; Capacitor placement; Bacterial Foraging Optimization Algorithm; Power Loss Minimization; Distribution system.

Nomenclature

\[ P_k \] Real power load at bus k.
\[ Q_k \] Reactive power load at bus k.
\[ P_{k,k+1} \] Real power flowing in the line between buses k and k+1.
\[ Q_{k,k+1} \] Reactive power flowing in the line between buses k and k+1.
\[ I_k \] Equivalent current injected at node k.
\[ J_{k,k+1} \] Branch current in line section between buses k and k+1.
\[ J^{\text{max}}_{k,k+1} \] Maximum branch current limit of line section between buses k and k+1.
\[ V_k \] Voltage magnitude at bus k.
\[ \Delta V_{\text{max}} \] Maximum voltage drop limit between buses 1 and k.
\[ V_{\text{worst}} \] Worst voltage magnitude of the system.
\[ R_{k,k+1} \] Resistance of the line section between buses k and k+1.
\[ X_{k,k+1} \] Reactance of the line section between buses k and k+1.
\[ P_{T,\text{Loss}} \] Total power loss of the system without DGs and capacitors.
\[ P_{T,\text{Loss}}^\text{DG}, P_{T,\text{Loss}}^C \] Total power loss of the system with DGs and capacitors.
\[ P_k \] Real power supplied from DG at the bus k.
\[ Q_k \] Reactive power compensated by capacitor at bus k.
\[ P_{k+1,\text{eff}} \] Total effective real power supplied beyond the bus k+1.
\[ Q_{k+1,\text{eff}} \] Total effective reactive power supplied beyond the bus k+1.
\[ n \] Total number of buses.
\[ nb \] Total number of branches.
\[ nd \] Total number of DGs installed.
\[ nc \] Total number of capacitors placed.

I. INTRODUCTION

In distribution systems, the voltage at buses reduces when moved away from the substation, and the losses are high. Electric distribution systems are becoming large and complex leading to higher system losses and poor voltage regulation. Literature indicates that almost 13% of the total power generated is consumed as \( I^2R \) losses at the distribution level [1]. The most common method for improving power factor, voltage profile enhancement, and line loss reduction in power distribution systems is DG and capacitor installation. The location and size of DG and capacitor unit should be optimal to maximize the benefits and reduce their impact on the power system. Inappropriate placement in some situations can reduce benefits and even endanger the entire system operation [2]. Thus the optimal integration between these two problems becomes a significant and complex problem.

Many methods have been developed to solve the DG and capacitor placement problem in distribution systems to obtain maximum benefits [3-13]. Acharya et al. presented an analytical approach for finding the optimal location and size of DG units for reducing power loss along with methodologies for identifying the optimal location [3]. Satish et al. proposed a Particle Swarm Optimization (PSO) based technique to solve the optimal placement of different types of DGs to minimize the power loss [4]. Sathish and Prema, Srinivasa Rao et al. used loss sensitivity factor for finding the optimal location of
DG units and applied Simulated Annealing (SA) and Harmony Search Algorithm (HSA) to identify the optimal size of DGs for power loss minimization [5, 6]. Mistry and Ranjit used PSO to find optimal size and location of multiple DGs to cater the incremental load on the system and power loss minimization.

Baran and Wu were the first to solve the capacitor placement problem for power loss minimization [8]. Later on, several optimization algorithms have been developed for loss minimization and/or voltage profile improvement. Attia and El-Fergany had integrated Differential evolution and Pattern Search (DE-PS) as meta-heuristic optimization tools to solve optimal capacitor placement problem with the objective of operating cost minimization of distribution network operator [9]. Sneha and Roy presented a Teaching Learning Based Optimization (TLBO) approach to allocate static energy cost by optimal placement of capacitors in radial distribution systems [10]. Attia et al. proposed an approach based on artificial bee colony algorithm to allocate static capacitors along radial distribution networks [11]. Gopia et al. proposed an analytical methodology for optimal allocation of DG together with capacitor for power loss minimization [12].

All the above methods discussed have gained encouraging results in finding the optimal location and size of DGs, but they also have some shortcomings such as computation time in solving real large-scale systems, calculation efficiency and convergence.

The present work is aimed to develop a fast and novel computation methodology to find the optimal location and size of multiple DGs together with capacitor placement for power loss minimization in power distribution network. In this paper, sensitivity analysis is used to identify the candidate locations for DG and capacitor placement. The LSF at each bus are ranked in descending order and the top three sensitive buses are selected as candidate location. BFOA, a distributed optimization technique is employed to minimize the objective function by determining the optimal size of DGs and capacitors at candidate locations. The advantage of exempting BFOA from the determination of optimal candidate locations is to reduce the search space, computation time and to improve convergence characteristics. The novelty of this work lies in the methodology proposed to identify the optimal placement and sizing of multiple-DG units together with capacitors in order to minimize the power losses. The proposed method is tested on a standard IEEE 33-bus test system and the results obtained are compared with other classical methods available in the literature.

II. PROBLEM FORMULATION

A. Power Flow Equations

Power flow in a distribution system is calculated by the following set of recursive equations derived from the single line diagram shown in Fig. 1.

From Fig.1, the equivalent current injected at node k is calculated as

\[ I_k = \left( \frac{P_k + jQ_k}{V_k} \right)^* \]  (1)

\[ V_{k+1} = V_k - J_{k,k+1} \cdot (R_{k,k+1} + jX_{k,k+1}) \]  (4)

\[ [J] = [BIBC] \cdot [I] \]  (3)

\[ P_{Loss, k+1} = R_{k,k+1} \cdot \left( \frac{P_{Loss,k+1}^2 + Q_{Loss,k+1}^2}{|V_k|^2} \right) \]  (5)

\[ \sum_{k=1}^{nb} P_{Loss, k+1} \]  (6)

B. Loss Reduction using DG placement

DG units installation at optimal location will leads to line loss reduction, improve voltage stability, peak demand saving, improved reliability and security. Total power loss in the system after DG installation is calculated as

\[ P_T^\text{DG} = \sum_{k=1}^{nb} P_{Loss}^\text{DG}(k, k+1) \]  (7)

Where \( P_{Loss}^\text{DG}(k, k+1) \) is the power loss in the line section between k and k+1 bus after DG installation in the system.

Net power loss reduced by DG installation in the system \( \Delta P_L \) is given as

\[ \Delta P_L = P_T^\text{Loss} - P_T^\text{DG} \]  (8)

C. Loss reduction using Capacitor placement

Total power loss in the system after Capacitor placement is determined as

\[ P_T^C = \sum_{k=1}^{nb} P_{Loss}^C(k, k+1) \]  (9)

Where \( P_{Loss}^C(k, k+1) \) is the power loss in the line section
between k and k+1 bus after Capacitor placement in the system. 

Net power loss reduced by Capacitor installation in the system \( \Delta P_L^C \) is given as

\[
\Delta P_L^C = P_{T,\text{Loss}} - P_{T,\text{Loss}}^C \tag{10}
\]

D. Objective function of the problem

The proposed Objective Function (F) of the problem is formulated to maximize the net power loss reduced by DG and Capacitor placement in the distribution system, which is given by

Maximize \( F = \text{max.} (\Delta P_L^{DG} + \Delta P_L^C) \tag{11} \)

The objective function is maximized subject to various operational constraints to satisfy the electrical requirements of distribution network. These constraints are discussed as follows.

Power balance constraints

\[
\sum_{k=1}^{n_d} P_k^{DG} \leq \sum_{k=2}^{n_c} P_k + \sum_{j=1}^{n_b} P_{\text{Loss},k,k+1} \tag{12}
\]

Thermal limits

\[
\sum_{k=1}^{n_c} Q_k^C \leq 0.8 * \sum_{k=2}^{n_c} Q_k \tag{13}
\]

Voltage drop limits

\[
\left| V_1 - V_2 \right| \leq \Delta V_{\text{max}} \tag{15}
\]

Distributed generation capacity limits

\[
P_{k,\text{Min}} \leq P_k^{DG} \leq P_{k,\text{Max}} \tag{16}
\]

Reactive compensation limits

\[
Q_{k,\text{Min}} \leq Q_k^C \leq Q_{k,\text{Max}} \tag{17}
\]

If any one of the above constraints is violated, the resultant solution will be rejected.

III. SENSITIVITY ANALYSIS FOR DG AND CAPACITOR PLACEMENT

Loss Sensitivity Factor (LSF) is computed to determine the candidate locations for the DG and Capacitor placement [14]. The estimation of these candidate buses initially helps in reduction of the search space for the optimization procedure.

![Fig. 2. Simple distribution line](image)

Consider a distribution line section with an impedance of \( R_{k,k+1} + jX_{k,k+1} \) and a load of \( P_{k+1,\text{eff}} + jQ_{k+1,\text{eff}} \) connected between k and k+1 buses as shown in Fig. 2.

From Fig. 2, the active power loss in the line section between buses k and k+1 is calculated by

\[
P_{\text{Loss},k,k+1} = R_{k,k+1} \left( P_{k+1,\text{eff}} + \frac{Q_{k+1,\text{eff}}^2}{V_{k+1}^2} \right) \tag{18}
\]

Now, the LSFs can be obtained with the help of following equation

\[
\frac{\partial P_{\text{Loss},k,k+1}}{\partial P_{k+1,\text{eff}}} = 2 \times \frac{P_{k+1,\text{eff}} \times R_{k,k+1}}{V_{k+1}^2} \tag{19}
\]

\[
\frac{\partial P_{\text{Loss},k,k+1}}{\partial Q_{k+1,\text{eff}}} = 2 \times \frac{Q_{k+1,\text{eff}} \times R_{k,k+1}}{V_{k+1}^2} \tag{20}
\]

Using the above two equations, LSFs are computed from the load flows and values are arranged in descending order for all the line section of the given system. Eq. (19) is used to compute the candidate bus for DG installation and Eq. (20) is used to compute the candidate bus for capacitor placement. LSFs decide the sequence in which buses are to be considered for DG and capacitor installation. The optimal size of DG and capacitor at candidate buses are calculated by using BFOA.

IV. OVERVIEW OF BFOA

BFOA is an efficient swarm intelligence based stochastic search technique developed recently by Passino [15]. In recent years, BFOA has been applied to solve numerous optimization problems in power systems [16, 17] due to its ability in searching the promising areas of the solution space. The idea of BFOA algorithm is based on the social foraging behavior of E. coli bacteria present in human intestines. During foraging of a bacterium, the locomotion is achieved by a set of tensile flagella. Flagella help an E. coli bacterium to tumble or swim, which are two basic operations (movements) performed by a bacterium at the time of foraging. When they rotate the flagella in the clockwise direction, each flagellum pulls on the cell. This results in the moving of flagella independently and finally the bacterium tumbles less frequently. Alternatively, in a harmful place it tumbles frequently to find a nutrient gradient. The four prime steps of this algorithm are chemotaxis, swarming, reproduction and elimination and dispersion. The detailed steps of the algorithm can be referred from [18].

The uniqueness of BFOA lies in the chemotaxis process in which the bacterium moves by an amount of \( c \Delta \) if the objective function value is reduced for new location. Otherwise, it remains in the same position. This chemotactic process of BFOA has an edge over the other classical optimization algorithms, especially in context to the convergence behavior of BFOA very near the global optima.

V. APPLICATION OF BFOA FOR POWER LOSS MINIMIZATION

The prime steps of BFOA applied to minimize the objective function are as follows.

**Step 1:** Initialization of BFOA parameters

- Let \( j \) be the index for the chemotactic step. Let \( k \) be the index for the reproduction step. Let \( l \) be the index for the chemotactic step.

- Let \( x_{j,k,l} \) be the decision variable for the chemotactic step. Let \( d_{j,k,l} \) be the decision variable for the reproduction step.

- The chemotactic step is performed by a bacterium that moves towards a nutrient source (solution) with an amount of \( c \Delta \) if the objective function value is reduced for new location. Otherwise, it remains in the same position.

- The reproduction step is performed by a bacterium that divides into two daughter cells, each with half of the nutrients.

- The elimination step is performed by a bacterium that disappears from the solution.

- The dispersion step is performed by a bacterium that moves randomly in the solution space.

The objective function is maximized subject to various operational constraints to satisfy the electrical requirements of distribution network. These constraints are discussed as follows.

Power balance constraints

\[
\sum_{k=1}^{n_d} P_k^{DG} \leq \sum_{k=2}^{n_c} P_k + \sum_{j=1}^{n_b} P_{\text{Loss},k,k+1} \tag{12}
\]

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\[
Q_{k,\text{Min}} \leq Q_k^C \leq Q_{k,\text{Max}} \tag{17}
\]
of the elimination-dispersal event. Let $q$ be the index for the iteration loop. Also

- $p$: Dimension of the search space, number of DGs and capacitors to be placed to minimize the objective function. The loss sensitivity factors are calculated from the load flows and sorted in descending orders. The buses that are more sensitive are selected to install DG and capacitor units. First part of the dimension signifies a DG size and the second part signifies the capacitor size at the selected candidate buses. ($p=3+3$)

$p1$, $p2$, $p3$: DGs size in kW at candidate buses.

$P4$, $p5$, $p6$: Capacitors size in kVAR at candidate buses.

- $S$: Total number of bacteria in the population (50).

- $N$: Number of iterations (50).

- $Nc$: The number of chemotactic steps (4).

- $Ns$: The swimming length (4).

- $Nre$: The number of reproduction steps (4).

- $Ned$: The number of elimination-dispersal events (2).

- $Ped$: Elimination-dispersal probability (0.5).

- $c(i)$: The size of the step taken in the random direction specified by the tumble ($0.05*\text{ones}(S,1)$).

- $\theta$: the initial random location of each bacterium.

The base power loss and voltages at all nodes without DGs and capacitors is calculated from the power flow.

**Step 2**: Iteration loop: $q=q+1$

**Step 3**: Elimination-dispersal loop: $l=l+1$

**Step 4**: Reproduction loop: $k=k+1$

**Step 5**: Chemotaxis loop: $j=j+1$

i. For $i=1, 2...S$ take a chemotactic step for bacterium $i$ as follows.

ii. Compute the objective function, $F(i, j, k, l)$.

Let $F(i, j, k, l) = F(i, j, k, l) + F_{cc}(\theta(i, j, k, l), P(j, k, l))$. $F_{cc}$ is computed using the equation given below.

$$F_{cc}(\theta, P(j, k, l)) = \sum_{i=1}^{s} F_{cc}(\theta, \theta^i(j, k, l))$$

$$= \sum_{i=1}^{s} \left[ -d_{\text{attract}} \text{exp}\left( -w_{\text{attract}} \sum_{m=1}^{p} (\theta_m - \theta^i)^2 \right) \right]$$

$$+ \sum_{i=1}^{s} \left[ -h_{\text{repellant}} \text{exp}\left( -w_{\text{repellant}} \sum_{m=1}^{p} (\theta_m - \theta^i)^2 \right) \right]$$

(21)

iii. Let $F_{\text{last}} = F(i, j, k, l)$ to save this value since we may find a better cost via a swim.

iv. Tumble: generate a random vector $\Delta(i)$ with each element $\Delta_m(i)$, $m = 1, 2..., p$, is a random number in the range of [-1, 1].

v. Move $\theta^i(j+1, k, l)$ using the below equation.

$$\theta^i(j+1, k, l) = \theta^i(j, k, l) + c(i) \frac{\Delta(i)}{\sqrt{\Delta^T(i)\Delta(i)}}$$

Where $\Delta$ indicates a vector in the random direction whose elements lie in [-1, 1]. This results in a step of size $c(i)$ in the direction of the tumble for bacterium i.

vi. Compute $F(i, j+1, k, l)$.

vii. Swim

a) Let $m=0$ (counter for swim length).

b) While $m < Ns$ (if have not climbed down too long).

- Let $m=m+1$.

- If $F(i, j+1, k, l) > F_{\text{last}}$ (if doing better), let $F_{\text{last}} = F(i, j+1, k, l)$ and move $\theta^i(j+1, k, l)$ using equation (22) and use this $\theta^i$ to compute the new $F(i, j+1, k, l)$ as did in (vi).

Else, let $m=Ns$. This is the end of the while statement.

viii. Go to next bacterium ($i+1$) if $i \neq S$ (i.e., go to (ii) to process the next bacterium).

**Step 6**: If $j < Nc$, go to step 5. In this case continue chemotaxis since the life of the bacteria is not over.

**Step 7**: Reproduction

- The $S/2$ bacteria with the lowest $F$ values die and the remaining $S/2$ bacteria with the best values split. This process is performed by the copies that are made and placed at the same location as their parent.

**Step 8**: If $k < Nre$, go to step 4.

**Step 9**: Elimination and dispersal: For $i=1, 2...S$ with probability $Ped$, eliminate and disperse each bacterium so that the number of bacterium in the population remains constant. The dispersed bacteria’s gives the brand new DG and capacitor sizes at candidate buses.

**Step 10**: If $l < Ned$, then go to step 3.
Step 11: If \( q < N \), then goto step 2, otherwise end. Evaluate the objective function for the final bacterium population. Determine the bacterium which gives the minimum \( F \) and this bacterium give the optimal sizes of DGs and capacitors at candidate buses. Run the load flows with the obtained DG and capacitor sizes installed at candidate buses and display the results.

These are the prime steps of BFOA implemented to minimize the objective function \( (F) \).

VI. RESULTS AND DISCUSSIONS

To demonstrate the effectiveness of the proposed method using LSFs and BFOA, it is applied to a standard IEEE test system consisting of 33 buses. The maximum number of DGs and capacitors installed for the given test system is limited to three since the rate of improvement of percentage loss reduction decreases when the candidate locations are more than three \([6, 7]\). However, the proposed technique can be implemented for any number of DGs. In the simulation of the test system, four different scenarios are considered to analyze the superiority and performance of the proposed method.

Case I: The system is without DGs and Capacitors. (Base Case).

Case II: Optimal size of capacitors placed at the candidate buses of the system.

Case III: Optimal size of DG units installed at the candidate buses of the system.

Case IV: System with simultaneous optimal DG and Capacitor placement.

An analytical software tool has been developed in MATLAB environment to carryout load flow, calculate voltages and power losses.

A. 33-Bus test system

![Fig. 3. Single line diagram of 33-bus radial distribution system.](image)

The line and load data of this test system are from \([19]\). It has 33 buses and 32 branches. The single line diagram 33-bus system is shown in Fig. 3. The total real and reactive power loads of the system are 3.72 MW and 2.3 MVAr, respectively. LSFs are computed to identify the candidate bus location for Cases II, III and IV. After computing LSF at all buses, they are sorted in descending order and ranked. Then the top three locations which are more sensitive are selected to install DGs and capacitors in the system. The candidate locations selected

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Base Case (Case 1)</td>
<td>210.98</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Only Capacitor Installation (Case 2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacitor Size in MVAr (Candidate Bus)</td>
<td>0.3496 (18)</td>
<td>0.8206 (30)</td>
<td>0.2773 (33)</td>
</tr>
<tr>
<td>( P_{Tot} ) (KW)</td>
<td>144.04</td>
<td>151.41</td>
<td>164.6</td>
</tr>
<tr>
<td>( V_{min} ) in p.u. (Bus No)</td>
<td>0.9361</td>
<td>0.92 (18)</td>
<td>0.9165 (18)</td>
</tr>
<tr>
<td>% Loss Reduction</td>
<td>31.72</td>
<td>28.23</td>
<td>21.98</td>
</tr>
<tr>
<td>Only DG Installation (Case 3)</td>
<td></td>
<td></td>
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<tr>
<td>DG Size in MW (Candidate Bus)</td>
<td>0.6335 (17)</td>
<td>0.0908 (18)</td>
<td>0.9470 (33)</td>
</tr>
<tr>
<td>( P_{Tot} ) (KW)</td>
<td>98.30</td>
<td>115.29</td>
<td>142.34</td>
</tr>
<tr>
<td>( V_{min} ) in p.u. (Bus No)</td>
<td>0.9645</td>
<td>0.9502 (18)</td>
<td>0.9311 (33)</td>
</tr>
<tr>
<td>% Loss Reduction</td>
<td>53.41</td>
<td>43.35</td>
<td>32.53</td>
</tr>
<tr>
<td>Simultaneous DG and Capacitor allocation (Case 4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DG Size in MW (Candidate Bus)</td>
<td>0.5424 (17)</td>
<td>0.1604 (18)</td>
<td>0.0955 (33)</td>
</tr>
<tr>
<td>( P_{Tot} ) (KW)</td>
<td>41.41</td>
<td>58.45</td>
<td>84.28</td>
</tr>
<tr>
<td>( V_{min} ) in p.u. (Bus No)</td>
<td>0.9783</td>
<td>0.9570 (18)</td>
<td>0.9611 (30)</td>
</tr>
<tr>
<td>% Loss Reduction</td>
<td>80.37</td>
<td>72.29</td>
<td>60.65</td>
</tr>
</tbody>
</table>

![Fig. 4 Comparison of voltage profile of the system in all the cases.](image)

![Fig. 5 Comparison of power loss at each line of the system in all the cases.](image)
for cases II, III and IV are given in Table I. The performance of the proposed method is presented in Table I.

From Table I, it is observed that the base case power loss (kW) in the system is 210.98, which is reduced to 144.04, 98.30 and 41.41 using cases II, III and IV respectively. However the proposed technique is very efficient in finding the optimal solution at all the scenarios; case IV is more effective in reducing total power loss and improving voltage profile compared to other scenarios. This proves that the simultaneous DG and capacitor placement (case IV) is better than the other cases in terms of quality of solutions. The voltage profiles of all four cases are compared in Fig. 4, respectively. From the figure, it is seen that the improvement in voltage profile of the system in case IV is better. It is also observed that the fall in minimum voltage magnitude is least in case IV.

The power loss in each line of all four cases is compared in Fig. 5, respectively. It is observed that the power loss at each line has been greatly reduced in case IV than others. In order to illustrate the performance of the proposed method, the performance of proposed method is compared with the results of other classical methods available in the literature and is shown in Table I. From the table, it is perceived that the performance of the BFOA is better compared to PSO [3] and analytical approach [12] in terms of power loss minimization.

VII. CONCLUSION

In this paper, a new approach has been proposed to install DG and capacitor simultaneously in power distribution system. In addition different cases of loss reduction are also simulated to validate the superiority of the proposed method. An efficient Meta heuristic BFOA is used in the optimization process of capacitor and DG installation. The proposed method is tested on standard IEEE 33-bus system. The computational results shows that case IV (simultaneous DG and capacitor placement) is found to be more effective in minimizing the power loss and improving the voltage profile. The simulated results are compared with the results of other methods available in the literature. The results showed that the performance of the BFOA is better than the other methods compared.

REFERENCES


