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Multi-objective economic emission load dispatch problem with trust-region strategy

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ABSTRACT

In this paper, we present a trust region algorithm for solving multi-objective economic emission load dispatch problem (EELD). The trust region algorithm has proven to be a very successful globalization technique for solving a single objective constrained optimization problems. The proposed approach is suitable for multi-objective problem (EELD) such that its objective functions may be ill- defined or having a non convex pareto-optimal front. Also, we identify the weight values which reflect the degree of satisfaction of each objective. The proposed approach is carried out on the standard IEEE 30-bus 6-generator test systems to confirm the effectiveness of the algorithm used to solve the multi-objective problem (EELD). Our results with the proposed approach have been compared to those reported in the literature. The comparison demonstrates the superiority of the proposed approach and confirm its potential to solve the multi-objective problem (EELD).

A Matlab implementation of our algorithm was used in solving one case study and the results are reported.

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1. Introduction

The multi-objective problem (EELD) is of great interest to many researches and several local methods have been proposed to solve it. By local method we mean that the method is designed to converge to optimal solution from closest starting point whether it is local or global one. For a local method, there is no guarantee that it converges if it starts from remote.

The purpose of multi-objective problem (EELD) is to figure out the optimal amount of the generated power of the fossil-based generating units in the system by minimizing the fuel cost and emission level simultaneously, subject to various equality and inequality constraints including the security measures of the power transmission/distribution. Various optimization techniques have been proposed by many researchers to deal with this multi-objective nonlinear programming problem with varying degree of success. In [1,2] the problem has been reduced to a single objective problem by treating the emission as a constraint with a permissible limit. This formulation, however, has severe difficulty in getting the trade-off relation between cost and emission.

Goal programming method was also proposed for the multi-objective problem (EELD) (see [3]). In this method a target or a goal to be achieved for each objective is assigned and the objective function will then try to minimize the distance from the targets to the objectives. Although the method is computationally efficient, it will yield an inferior solution rather than a non-inferior one if the goal point is chosen in the feasible domain.

Heuristic algorithms such as genetic algorithms have been recently proposed for solving multi-objective problem (EELD) (see for example [4–6]). The results reported were promising and encouraging for further research. Moreover the studies on heuristic algorithms over the past few years, have shown that these methods can be efficiently used to eliminate most of difficulties of classical methods (see for example [7–11]). Further more, these methods cannot be used to find pareto-optimal solutions in problems having a non convex pareto-optimal front or ill defined problems.

In this paper, we will use a trust-region globalization strategy to solve the multi-objective problem (EELD). Globalizing strategy means modifying the local method in such a way that it is guaranteed to converge at all even if the starting point is far away from the solution. This approach is applied to solve multi-objective problem (EELD) with no limitation to the number of objective functions and is efficient for solving ill-defined systems and non-convex multi-objective optimization problems.

In this work, we convert the multi-objective problem (EELD) to a single-objective constrained optimization problem by using a weighting approach. The weighting approach is considered as one of the most useful algorithms in treating multi-objective

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optimization problems to generate a wide set of optimal solutions (pareto set) [12]. Also, the effect of changing the weights on cost and emission were studied to show the degree of satisfaction of each objective function.

In this paper, an active set strategy is used together with a multiplier method to convert the single-objective constrained optimization problem to unconstrained optimization problem.

The trust-region strategy for solving the single-objective constrained optimization problem and unconstrained optimization problem has proved to be very successful, both theoretically and practically (see for example [13–18]). Also the trust-region technique for the multi-objective problem has proved to be very successful (see for example [19–21]).

Here, we introduce some notations for subscripted functions denote function values at particular points; for example, $f_k = f(x_k)$, $\nabla_x f_k = \nabla_x f(x_k)$, $L_{k+1} = L(x_{k+1}, \mu_{k+1}, \nu_{k+1})$, $\nabla_x L_k = \nabla_x L(x_k, \mu_k, \nu_k)$, and so on. The matrix H_k denotes the Hessian of the objective function at the point (x_k) or an approximation to it. Finally, all norms are l_2-norms.

This paper is organized as follows: In Section 2 we introduce in details the description of economic emission load dispatch problem. Section 3 is devoted for the mathematical formulation of multi-objective problem (EELD). In Section 4 we give a detailed discussion of the trust region algorithm problem (EELD). Furthermore, we then discuss in detail the implementation of the proposed approach in Section 5. The results and discussions are presented in Section 6. Finally, the conclusion and future works are given in Section 7.

2. Economic emission load dispatch problem

The economic emission load dispatch involves the simultaneous optimization of fuel cost and emission objectives which are conflicting ones. The deterministic problem is formulated as follows:

2.1. Objective functions

There are two objective functions which are described in details as follows:

(1) **Fuel cost objective function**. The classical economic dispatch problem of finding the optimal combination of power generation, which minimize the total fuel cost while satisfying the total required demand can be mathematically stated in [22] as follows:

$$f_1 = \sum_{i=1}^{n} C_i(P_{Gi}) = \sum_{i=1}^{n} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \qquad (2.1)$$

where f_1 is the total fuel cost($/h), C_i is the fuel cost of generator i, P_{Gi} is the power generated by generator i, n is the number of generator, and a_i, b_i, c_i, are the fuel cost coefficients of generator i.

(2) **Emission objective function**. The emission function can be expressed as the sum of all types of emission considered as NO_2, SO_2, thermal emission, etc., with suitable pricing or weighting on each pollutant emitted. In the present study, only one type of emission NO_2 is given as a function of generator output, that is, the sum of a quadratic and exponential function:

$$f_2 = \sum_{i=1}^{n} [10^{-2}(\tilde{a}_i + \tilde{b}_i P_{Gi} + \tilde{c}_i P_{Gi}^2) + \xi_i e^{\lambda_i P_{Gi}}], \qquad (2.2)$$

where f_2 is the amount of NO_2 emission (ton/h) and $\tilde{a}_i, \tilde{b}_i, \tilde{c}_i, \xi_i$, and λ_i are the coefficients of the ith generator's NO_2 emission characteristic.

2.2. Constraints

The optimization problem is bounded by the following constraints:

(1) **Power balance constraint**. The total power generated must supply the total load demand and the transmission losses

$$\sum_{i=1}^{n} P_{Gi} - P_D - P_{Loss} = 0, \qquad (2.3)$$

where P_D is a total load demand, and P_{Loss} represents a transmission losses. The transmission losses are given by [23] as follows:

$$P_{Loss} = \sum_{i=1}^{n}\sum_{j=1}^{n} [A_{ij}(P_i P_j + Q_i Q_j) + B_{ij}(Q_i P_j - P_i Q_j)], \qquad (2.4)$$

where $P_i = P_{Gi} - P_{Di}$, $Q_i = Q_{Gi} - Q_{Di}$, $A_{ij} = (R_{ij}/V_i V_j)\cos(\delta_i - \delta_j)$, and $B_{ij} = (R_{ij}/V_i V_j)\sin(\delta_i - \delta_j)$ such that n is a number of buses, R_{ij} is a series resistance connecting buses i and j, P_i is a real power injection at bus i, Q_i a reactive power injection at bus i, V_i is a voltage magnitude at bus i, δ_i is a voltage angle at bus i.

2 **Maximum and minimum limits of power generation**. The power generated P_{Gi} by each generator is constrained between its minimum and maximum limits, i.e., $P_{Gi_{min}} \le P_{Gi} \le P_{Gi_{max}}$, $Q_{Gi_{min}} \le Q_{Gi} \le Q_{Gi_{max}}$, and $V_{i_{min}} \le V_i \le V_{i_{max}}$.

3 **Security constraints**. A mathematical formulation of the security constrained EELD problem would require a very large number of constraints to be considered. However, for typical systems the large proportion of lines has a rather small possibility of becoming overloaded. The EELD problem should consider only the small proportion of lines in violation, or near violation of their respective security limits which are identified as the critical lines. We consider only the critical lines that are binding in the optimal solution. The detection of the critical lines is assumed done by the experiences of the DM. An improvement in the security can be obtained by minimizing the following objective function,

$$S = \sum_{q=1}^{m} \left(\frac{(T_q(P_G))^2}{T_q^{max}} \right),$$

where $T_q(P_G)$ is the real power flow, T_q^{max} is the maximum limit of the real power flow of the qth line, and m is the number of monitored lines. The line flow of the qth line is expressed in terms of the control variables P_{Gs} by utilizing the generalized generation distribution factors (GGDF) in [24] and is given as follows

$$T_q(P_G) = \sum_{i=1}^{n} (D_{qi} P_{Gi}),$$

where D_{qi} is the generalized GGDF for line q due to generator i.

For secure operation, the transmission line loading S_l is restricted by its upper limit as

$$S_l \le S_{lmax}, \quad l = 1, \ldots, n_l,$$

where n_l is the number of transmission line.

In the following section we rewrite the EELD problem in mathematical formulation and using a weighting approach to transform it to a single objective optimization problem.

3. Multi-objective formulation of problem (EELD)

The mathematical formulation of the multi-objective problem (EELD) with n-buses and m-generator is the following multi-objective optimization problem:

$$\begin{aligned}
&\text{minimize} \quad f_1=\sum_{i=1}^{n}(a_i+b_iP_{Gi}+c_iP_{Gi}^2)\\
&\text{minimize} \quad f_2=\sum_{i=1}^{n}[10^{-2}(\tilde{a}_i+\tilde{b}_iP_{Gi}+\tilde{c}_iP_{Gi}^2)+\xi_i e^{\lambda_i P_{Gi}}]\\
&\text{subject to} \quad \sum_{i=1}^{n}P_{Gi}-P_D-P_{Loss}=0,\\
&\qquad P_{Gi_{min}}\le P_{Gi}\le P_{Gi_{max}},\\
&\qquad Q_{Gi_{min}}\le Q_{Gi}\le Q_{Gi_{max}},\\
&\qquad V_{i_{min}}\le V_i\le V_{i_{max}},\\
&\qquad S_l\le S_{lmax},
\end{aligned} \tag{3.1}$$

where $i=1, \ldots, n$ and $l=1, \ldots, n_l$.

Using a weighting approach to transform problem (3.1) to a single-objective optimization problem which has the following form:

$$\begin{aligned}
&\text{minimize} \quad f(x)=w_1f_1+w_2f_2\\
&\text{subject to} \quad \sum_{i=1}^{n}P_{Gi}-P_D-P_{Loss}=0,\\
&\qquad P_{Gi_{min}}\le P_{Gi}\le P_{Gi_{max}},\\
&\qquad Q_{Gi_{min}}\le Q_{Gi}\le Q_{Gi_{max}},\\
&\qquad V_{i_{min}}\le V_i\le V_{i_{max}},\\
&\qquad S_l\le S_{lmax},
\end{aligned} \tag{3.2}$$

where $x=[P_{G1}, \ldots, P_{Gn}, Q_{G1}, \ldots, Q_{Gn}, V_1, \ldots, V_n, S_1, \ldots, S_{n_l}]^T$, $w_1+w_2=1$, and $w_1, w_2 \ge 0$. The above problem can be written as follows:

$$\begin{aligned}
&\text{minimize} \quad f(x)\\
&\text{subject to} \quad h(x)=0,\\
&\qquad g(x)\le 0,
\end{aligned} \tag{3.3}$$

where $h(x)=\sum_{i=1}^{n}P_{Gi}-P_D-P_{Loss}$ and $g(x)=[P_{Gi_{min}}-P_{Gi}, P_{Gi}-P_{Gi_{max}}, Q_{Gi_{min}}-Q_{Gi}, Q_{Gi}-Q_{Gi_{max}}, V_{i_{min}}-V_i, V_i-V_{i_{max}}, S_l-S_{lmax}]^T$.

The functions $f(x):\mathfrak{R}^n\to\mathfrak{R}$, $h(x):\mathfrak{R}^n\to\mathfrak{R}$, and $g(x):\mathfrak{R}^{3n+n_l}\to\mathfrak{R}^{6n+n_l}$ are twice continuously differentiable.

The Lagrangian function associated with problem (3.3) is the function

$$L(x,\mu,\nu)=f(x)+\mu^T h(x)+\nu^T g(x), \tag{3.4}$$

where $\mu\in\mathfrak{R}$ and $\nu\in\mathfrak{R}^{6n+n_l}$ are the Lagrangian multiplier vectors associated with equality and inequality constraints, respectively.

Following [13], we define a 0–1 diagonal indicator matrix $U(x)\in\mathfrak{R}^{6n+n_l\times 6n+n_l}$, whose diagonal entries are

$$u_e(x)=\begin{cases}1 & \text{if } g_e(x)\ge 0,\\ 0 & \text{if } g_e(x)<0\end{cases} \tag{3.5}$$

Using the above matrix, we transform problem (3.3) to the following equality constrained optimization problem

$$\begin{aligned}
&\text{minimize} \quad f(x)\\
&\text{subject to} \quad h(x)=0,\\
&\qquad \frac{1}{2}g(x)^T U(x) g(x)=0.
\end{aligned} \tag{3.6}$$

Using a multiplier method, we transform the equality constrained optimization problem (3.6) to the following unconstrained optimization problem

$$\begin{aligned}
&\text{minimize} \quad \Phi(x,\mu,\nu;\rho;r)=L(x,\mu,\nu)+\frac{\rho}{2}\|U(x)g(x)\|_2^2\\
&\qquad +\frac{r}{2}\|h(x)\|_2^2,\\
&\text{subject to} \quad x\in\mathfrak{R}^{3n+n_l},
\end{aligned} \tag{3.7}$$

where ρ is the positive parameter and $r>0$ is a parameter usually called the penalty parameter. A detailed description of the main steps of the trust-region algorithm for solving the above problem and it's an algorithmic framework is presented in the following section.

4. Trust-region algorithm outline

This section presents in details the description of our trust-region algorithm for solving problem (3.7).

4.1. Computing a trial step

We compute the trial step z_k by solving the following trust-region subproblem

$$\begin{aligned}
&\text{minimize} \quad L_k+\nabla L_k^T z+\frac{1}{2}z^T H_k z+\frac{\rho_k}{2}\|U_k(g_k+\nabla g_k^T z)\|^2\\
&\qquad +\frac{r_k}{2}\|(h_k+\nabla h_k^T z)\|^2\\
&\text{subject to} \quad \|z\|\le\Delta_k,
\end{aligned} \tag{4.1}$$

where H_k is the Hessian matrix of the Lagrangian function $L(x_k, \mu_k, \nu_k)$ or an approximation to it. Since our convergence theory is based on the fraction of Cauchy decrease condition, therefore a generalized dogleg algorithm introduced by Steihaug [25] and Toint [26] can be used to compute the trial step.

4.2. Testing the step and updating Δ_k

Once the trial step is computed, it needs to be tested to determine whether it will be accepted. To test the step, estimates for the two Lagrangian multipliers μ_{k+1} and ν_{k+1} are needed. Our way of evaluating the two Lagrangian multipliers μ_{k+1} and ν_{k+1} is presented in Step 5 of Algorithm (4.1) below.

Let μ_{k+1} and ν_{k+1} be the estimation of the two Lagrangian multiplier vectors. We test whether the point $(x_k+z_k, \mu_{k+1}, \nu_{k+1})$ will be taken as a next iterate.

Table 1
Generator cost and emission coefficients.

	Parameter	G_1	G_2	G_3	G_4	G_5	G_6
Cost	a	10	10	20	10	20	10
	b	200	150	180	100	180	150
	c	100	120	40	60	40	100
Emission	$\tilde{a}$	4.091	2.543	4.258	5.426	4.258	6.131
	$\tilde{b}$	−5.554	−6.047	−5.094	−3.55	−5.094	−5.555
	$\tilde{c}$	6.49	4.638	4.586	3.38	4.586	5.151
	ξ	2.0E−4	5.0E−4	1.0E−6	2.0E−3	1.0E−6	1.0E−5
	λ	2.857	3.333	8.000	2.000	8.000	6.667

The actual reduction in the merit function is defined as

$$Ared_k = L(x_k, \mu_k, \nu_k) - L(x_{k+1}, \mu_k, \nu_k) - \Delta\mu_k^T h_{k+1} - \Delta\nu_k^T g_{k+1} + \frac{\rho_k}{2}[g_k^T U_k g_k - g_{k+1}^T U_{k+1} g_{k+1}] + \frac{r_k}{2}[\|h_k\|^2 - \|h_{k+1}\|^2], \quad (4.2)$$

where $\Delta\mu_k = (\mu_{k+1} - \mu_k)$ and $\Delta\nu_k = (\nu_{k+1} - \nu_k)$.

The predicted reduction in the merit function is defined to be

$$Pred_k = \hat{q}_k(0) - \hat{q}_k(z_k) - \Delta\mu_k^T(h_k + \nabla h_k^T z_k) - \Delta\nu_k^T U_k g_k + \frac{r_k}{2}[\|h_k\|^2 - \|h_k + \nabla h_k^T z_k\|^2], \quad (4.3)$$

where

$$\hat{q}_k(z) = L_k + \nabla_x L_k^T z + \frac{1}{2} z^T H_k z + \frac{\rho_k}{2}\|U_k(g_k + \nabla g_k^T z)\|^2. \quad (4.4)$$

After computing a trial step and updating the Lagrangian multipliers, the penalty parameter is updated to ensure that $Pred_k \geq 0$. To update r_k, we use a scheme that has the flavor of the scheme proposed by [14]. This scheme is described in Step 6 of Algorithm 4.1. After that, the step is tested to know whether it is accepted. This is done by comparing $Pred_k$ against $Ared_k$.

If $Ared_k/Pred_k < \eta_1$ where $\eta_1 \in (0, 1)$ is a small fixed constant, then the step is rejected. In this case, the radius of the trust region Δ_k is decreased by setting $\Delta_k = \alpha_1 \| z_k \|$, where $\alpha_1 \in (0, 1)$, and another trial step is computed using the new trust-region radius.

If $Ared_k/Pred_k \geq \eta_1$, then the step is accepted. Our theory requires that at the beginning of the next iteration, Δ_{k+1} must be greater than or equal to Δ_{min}, where Δ_{min} is a positive constant chosen at the beginning of the algorithm. That is, Δ_k can be reduced below Δ_{min} while finding an acceptable step. But, $\Delta_{k+1} \geq \Delta_{min}$ is required at the beginning of the next iteration after accepting the step z_k.

Our way of evaluating the trial steps and updating the trust-region radius is presented in Step 7 of Algorithm (4.1). After accepting the step, we update the parameter ρ_k and the Hessian matrix H_k. To update ρ_k, we use a scheme suggested by [27]. This scheme is described in Step 8 of Algorithm 4.1.

Finally, the algorithm is terminated when either $\|z_k\| \leq \varepsilon_1$ or $\| \nabla_x L_k \| + \| \nabla g_k U_k g_k \| + \| h_k \| \leq \varepsilon_2$, for some $\varepsilon_1, \varepsilon_2 > 0$.

4.3. Main algorithm

A formal description of our trust-region algorithm for solving problem (3.3) is presented in the following algorithm.

Algorithm 4.1. (The Main Algorithm)

Step 0. (**Initialization**)
Given $x_1 \in \Re^{3n+n_l}$. Compute U_1. Evaluate ν_1 and μ_1 (see Step 5 with $k = 0$ and $\mu_0 = (0, 0, \ldots, 0)^T$). Set $\rho_1 = 1$, $r_0 = 1$, $\sigma_1 = 1$, and $\beta = 0.1$. Choose $\varepsilon_1 = \varepsilon_2 = 10^{-8}$, $\alpha_1 = 0.05$, $\alpha_2 = 2$, $\eta_1 = 10^{-4}$, and $\eta_2 = 0.5$ such that $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $0 < \alpha_1 < 1 < \alpha_2$, and $0 < \eta_1 < \eta_2 < 1$. Set $\Delta_{min} = 10^{-3}$ and $\Delta_{max} = 10^5 \Delta_1$ such that $\Delta_{min} \leq \Delta_1 \leq \Delta_{max}$. Set $k = 1$.
Step 1. (**Test for convergence**)
If $\| \nabla_x L_k \| + \| \nabla g_k U_k g_k \| + \| h_k \| \leq \varepsilon_2$, then terminate the algorithm.
Step 2. (**Compute a trial step**)
a) Compute the step z_k by solving (4.1)
b) Set $x_{k+1} = x_k + z_k$.
Step 3. (**Test for termination**)
If $\|z_k\| \leq \varepsilon_1$, then terminate the algorithm.
Step 4. (**Update the active set**)
Compute U_{k+1}.
Step 5. (**Compute the Lagrangian multipliers** μ_{k+1} **and** ν_{k+1})
a) Compute ν_{k+1} by solving

$$\text{minimize} \quad \|(\nabla f_{k+1} + \nabla h_{k+1}\mu_k + \nabla g_{k+1} U_{k+1} \nu)\|^2 \quad (4.5)$$
$$\text{subject to} \quad U_{k+1}\nu \geq 0,$$

and set the rest of the components of ν_{k+1} to zero.
b) **If** $\| \nabla f_{k+1} + \nabla h_{k+1}\mu_k + \nabla g_{k+1} U_{k+1} \nu_{k+1} \| \leq \varepsilon$, then set $\mu_{k+1} = \mu_k$.
Else, compute μ_{k+1} by solving

$$\textit{minimize} \quad \|\nabla f_{k+1} + \nabla g_{k+1}\nu_{k+1} + \nabla h_{k+1}\mu\|^2. \quad (4.6)$$

End if.
Step 6. (**Update the penalty parameter** r_k)
a) Set $r_k = \max(r_{k-1}, \rho_k^2)$.
b) **If** $Pred_k \leq \frac{r_k}{4}[\|h_k\|^2 - \|h_k + \nabla h_k^T z_k\|^2]$, then set

$$r_k = \frac{4[\hat{q}_k(z_k) - \hat{q}_k(0) + \Delta\mu_k^T(h_k + \nabla h_k^T z_k) + \Delta\nu_k^T U_k g_k]}{\|h_k\|^2 - \|h_k + \nabla h_k^T z_k\|^2} + \beta.$$

End if.
Step 7. (**Test the step and update the trust-region radius**)
If $\frac{Ared_k}{Pred_k} < \eta_1$.
Reduce the trust-region radius by setting $\Delta_k = \alpha_1 \| z_k \|$ and go to step 2.
Else if $\eta_1 \leq \frac{Ared_k}{Pred_k} < \eta_2$, then
Accept the step: $x_{k+1} = x_k + z_k$.
Set the trust-region radius: $\Delta_{k+1} = \max(\Delta_k, \Delta_{min})$.
Else
Accept the step: $x_{k+1} = x_k + z_k$.
Set the trust-region radius: $\Delta_{k+1} = \min\{\Delta_{max}, \max\{\Delta_{min}, \alpha_2\Delta_k\}\}$.
End if.
Step 8. (**Update the parameters** ρ_k **and** σ_k)
a) Set $\rho_{k+1} = \rho_k$ and $\sigma_{k+1} = \sigma_k$.
b) **If**
$\hat{q}_k(0) - \hat{q}_k(z_k) - \Delta\mu_k^T(h_k + \nabla h_k^T z_k) - \Delta\nu_k^T U_k g_k \leq \sigma_k\|\nabla g_k U_k g_k\| \min\{\|\nabla g_k U_k g_k\|, \Delta_k\}$,
then set $\rho_{k+1} = 2\rho_k$ and $\sigma_{k+1} = \frac{1}{2}\sigma_k$.
End if.
Step 9. Set $k = k + 1$ and go to Step 1.

5. Implementation of the proposed approach

The proposed approach is applied to the standard IEEE 30-bus 6-generator test system to investigate the effectiveness of this approach. The values of fuel cost and emission coefficient are given in Table 1. The values of minimum fuel cost and minimum NO_2 emission are given in Tables 2 and 3 , respectively.

Table 2
Best fuel cost (cost unit).

P_{Gi}	NSGA	NPGA	SPEA	HMEA	Proposed
P_{G1}	0.1168	0.1245	0.1086	0.1737	0.1114
P_{G2}	0.3165	0.2792	0.3056	0.3568	0.3011
P_{G3}	0.5441	0.6284	0.5818	0.5411	0.5218
P_{G4}	0.9447	1.0264	0.9846	0.9890	1.0002
P_{G5}	0.5498	0.4693	0.5288	0.4529	0.5281
P_{G6}	0.3964	0.39993	0.3584	0.3705	0.3613
Best cost	608.245	608.147	607.807	606.012	602.55
Corresponding emission	0.21664	0.22364	0.22015	0.20080	0.22202

Table 3
Best NO_2 emission cost (emission unit).

P_{Gi}	NSGA	NPGA	SPEA	HMEA	Proposed
P_{G1}	0.4113	0.3923	0.4043	0.3675	0.4002
P_{G2}	0.4591	0.4700	0.4525	0.4904	0.4434
P_{G3}	0.5117	0.5565	0.5525	0.5177	0.5025
P_{G4}	0.3724	0.3695	0.4079	0.4512	0.4009
P_{G5}	0.5810	0.5599	0.5468	0.5215	0.5203
P_{G6}	0.5304	0.5163	0.5005	0.5304	0.5268
Best emission	0.19432	0.19424	0.19422	0.1880	0.2000
Corresponding cost	647.251	645.984	642.603	644.5346	643.0562

In this work, our program was written in MATLAB and run under MATLAB 7 with machine epsilon about 10^{-16}. For computing the two components of the trial steps, we used the dogleg algorithm. Successful termination with respect to our trust-region algorithm means that the termination condition of the algorithm is met with $\varepsilon_2 = 10^{-8}$. On the other hand, unsuccessful termination means that the number of iterations is greater than 300, the number of function evaluations is greater than 500, or the length of the trial step is less than $\varepsilon_1 = 10^{-8}$.

6. Results and discussions

We compare our obtained results with different previous works, which introduced different algorithms handle the same problem (the standard IEEE 30-bus 6-generator test system) to investigate the effectiveness of this approach. These approaches are (a) Non-dominated Strong Genetic Algorithm (NSGA) [7], (b) Niched Pareto Genetic Algorithm (NPGA) [8], (c) Strength Pareto Evolutionary Algorithm (SPEA) [9], and (d) Hybrid Multi-objective Evolutionary Algorithm (HMEA) [28]. For comparison purposes with the reported results, the system is considered as losses and the security constraints is released. The results declare the implementation of trust-region globalization strategy to solve the multi-objective problem (EELD)modifies the local method in such a way that it is guaranteed to converge at all even if the starting point is far away from the solution and improves the solution quality for the same approach. Also, our approach could discuss the effects of changing weights on fuel cost, as well as the emission. As one weight is changed linearly such as $\{w_1 = 0.0, 0.1, 0.2, \ldots, 1\}$ and $\{w_2 = 1.0, 0.9, 0.8, \ldots, 0.0\}$, then we can obtain the best range of the weights to help the decision maker to choose the most prefer weights for the objective functions. We do this idea as a some sort of parametric study to help the D.M. if he/she not experience enough in this field. That mean that the D.M. can choose the weights from the following ranges $0.4 \le w_1 \le 0.6$ and $0.4 \le w_2 \le 0.6$. where $w_1 + w_2 = 1$. These ranges guarantee that minimum of both fuel cost and NO_2 emission. Finally our algorithm the fuel cost function and keep the NO_2 emission function in its ranges comparable with all reported results. The values of minimum fuel cost and the minimum NO_2 emission are given in Tables 2 and 3, respectively (Fig. 1).

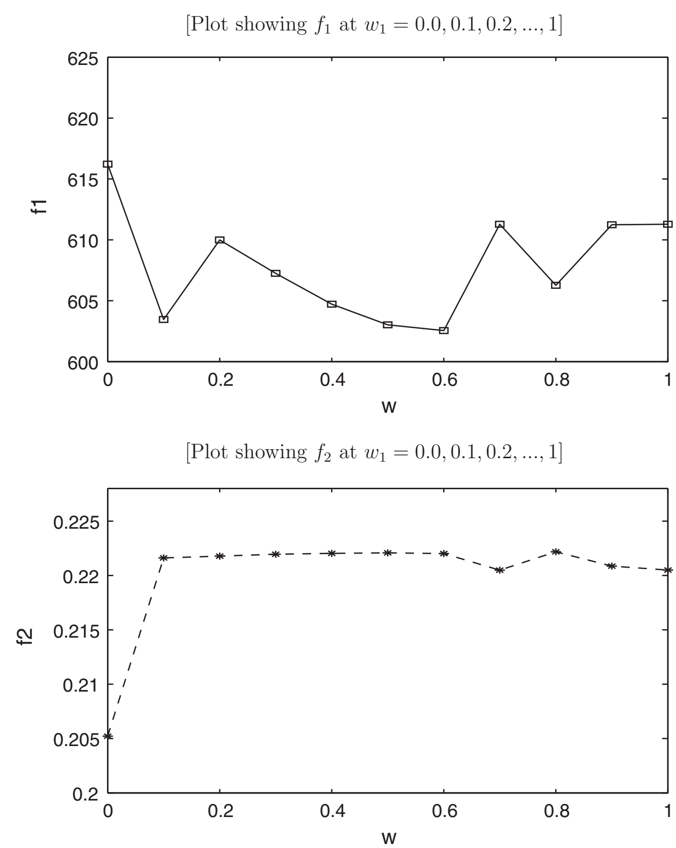

Fig. 1. Plot showing the values of f_1 and f_2 at different weights (w_1 is changed linearly).

7. Conclusions

The proposed approach was applied to economic emission load dispatch optimization problem which formulated as multi-objective optimization problem as well as computing the fuel cost and emission. The presented algorithm considered as globally search technique to get a pareto-optimal solution. The following points are the significant contributions of this paper.

- This approach is applied to solve multi-objective problem (EELD) with no limitation to the no of objective functions.
- The proposed approach is efficient for solving ill-defined systems and non-convex multi-objective optimization problems.
- The presented algorithm considered as globally search technique to get a pareto-optimal solution.
- The proposed approach is efficient for solving no convex multi-objective optimization problems where multiple Pareto-optimal solutions can be found in one run.
- The proposed approach helps the decision maker to choose the most prefer weights for the objective functions.
- For future work, there are many questions that should be answered. Although we have implemented the algorithm and tested it, we believe that the implementation of the algorithm should be refined with efficiency in mined. In particular, a better way of solving the trust-region subproblems that can handle large-scale problems should be used.

  Updating the Lagrangian multiplier is another point that needs to be refined. This indeed will reduce the cost of the computation of the steps.

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