Dynamic Surface Control Using Neural Networks for a Class of Uncertain Nonlinear Systems With Input Saturation

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***Abstract*— In this paper, a dynamic surface control (DSC) scheme is proposed for a class of uncertain strict-feedback nonlinear systems in the presence of input saturation and unknown external disturbance. The radial basis function neural network (RBFNN) is employed to approximate the unknown system function. To efficiently tackle the unknown external disturbance, a nonlinear disturbance observer (NDO) is developed. The developed NDO can relax the known boundary requirement of the unknown disturbance and can guarantee the disturbance estimation error converge to a bounded compact set. Using NDO and RBFNN, the DSC scheme is developed for uncertain nonlinear systems based on a backstepping method. Using a DSC technique, the problem of explosion of complexity inherent in the conventional backstepping method is avoided, which is specially important for designs using neural network approximations. Under the proposed DSC scheme, the ultimately bounded convergence of all closed-loop signals is guaranteed via Lyapunov analysis. Simulation results are given to show the effectiveness of the proposed DSC design using NDO and RBFNN.**

***Index Terms*— Backstepping control, dynamic surface control (DSC), nonlinear disturbance observer (NDO), robust control, uncertain nonlinear system.**

I. INTRODUCTION

IN PRACTICAL engineering, lots of plants possess nonlinear and uncertain characteristics. On the other hand, the magnitude of control signal is always limited due to actuator physical constraints. Thus, it is very important to develop effective robust control techniques for uncertain nonlinear systems with input saturation. Saturation as one of the common nonsmooth nonlinear constraint of control input should be explicitly considered in the control design to enhance robust control performance. If the input saturation is ignored in the control design, the closed-loop control performance will be severely degraded, and instability may occur. In recent years, there have been extensive studies on various systems with input saturation in [1]–[3]. Neural network (NN)-based near-optimal control was developed for a class of discrete-time affine nonlinear systems with control constraints in [4]. In [5], a robust adaptive control scheme was proposed for uncertain nonlinear systems in the presence of input saturation and external disturbance. Robust adaptive neural network control was proposed for a class of uncertain multi-input and multi-output (MIMO) nonlinear systems with input nonlinearities [6]. Backstepping control was studied for hovering unmanned aerial vehicle, including input saturations in [7]. In [8], an adaptive tracking control scheme was developed for uncertain MIMO nonlinear systems with input saturation. Adaptive control was studied for minimum phase single-input and single-output plants with input saturation [9]. However, there are few existing research results for the dynamic surface control (DSC) scheme of uncertain strict-feedback nonlinear systems with input saturation and unknown external disturbance.

On the other hand, robust adaptive backstepping control as an efficient control method has been extensively used for nonlinear control system design due to its design flexibility [10]–[13]. At the same time, NNs and fuzzy logical systems as the universal approximators have been widely employed to tackle the system uncertainty [14]–[19]. In [20], an adaptive sliding-mode control was proposed for nonlinear active suspension vehicle systems using Takagi–Sugeno fuzzy approach. Robust adaptive tracking control scheme was proposed for nonlinear systems based on the fuzzy approximator in [21]. A combined backstepping and small-gain approach was developed for the robust adaptive fuzzy output feedback control design in [22]. In [23], a globally stable adaptive backstepping fuzzy control scheme was studied for output-feedback systems with unknown high-frequency gain sign. Adaptive backstepping fuzzy control was proposed for nonlinearly parameterized systems with periodic disturbance in [24]. In [25], an observer-based adaptive decentralized fuzzy fault-tolerant control scheme was studied for nonlinear large-scale systems with actuator failures. Furthermore, backstepping control has been extensively used in many practical systems. Nonlinear adaptive flight control was proposed using backstepping method and NNs in [26]. In [27], a fuzzy adaptive control design was studied for hypersonic vehicles via backstepping method. Robust attitude control was developed for helicopters with actuator dynamics using NNs in [28]. In [29],

Manuscript received July 31, 2013; revised August 21, 2014; accepted September 21, 2014. Date of publication December 4, 2014; date of current version August 17, 2015. This work was supported in part by the Jiangsu Natural Science Foundation of China under Grant SBK20130033, in part by the National Natural Science Foundation of China under Grant 61374130 and Grant 61174102, in part by the Program for New Century Excellent Talents in the University of China under Grant NCET-11-0830, and in part by the Jiangsu Province Blue Project through the Innovative Research Team.

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Digital Object Identifier 10.1109/TNNLS.2014.2360933

an observer-based adaptive fuzzy backstepping control scheme was proposed for a class of stochastic nonlinear strict-feedback systems. However, there are few backstepping control results for uncertain nonlinear systems using disturbance observers. To tackle the unknown time-varying disturbance for effective backstepping control design, the robust adaptive backstepping control based on disturbance observation should be further developed.

With conventional backstepping, a possible issue is the problem of explosion of complexity. That is, the complexity of the controller grows drastically as the order n of the system increases. This explosion of complexity is caused by the repeated differentiations of certain nonlinear functions. To efficiently handle the system uncertainty in each subsystem, radial basis function NN (RBFNN) with the universal approximation capability is employed in [30] and [31]. Since RBFNN is used, we need to take derivatives of those radial basis functions, which further lead to heavier calculation burden in each step design. Recently, the DSC method was employed to solve this problem and many research results were presented [32]. In [33], an adaptive DSC design was proposed using adaptive backstepping for nonlinear systems. DSC was presented for a class of nonlinear systems in [34]. In [35], NN-based adaptive DSC was developed for nonlinear systems in strict-feedback form. A robust adaptive NN tracking control design was proposed for strict-feedback nonlinear systems using DSC approach in [36]. In [37], a NN-based adaptive DSC scheme was studied for uncertain nonlinear pure-feedback systems. Simultaneous quadratic stabilization was studied for a class of nonlinear systems with input saturation using DSC in [38]. In [39], an output feedback adaptive DSC scheme was developed for a class of nonlinear systems with input saturation. Recently, L_∞-type criteria are used in the DSC design to enhance the control performance [40]–[42]. However, DSC should be further investigated for uncertain strict-feedback nonlinear systems in the presence of input saturation and unknown external disturbance.

In recent years, disturbance observer design and application have attracted considerable interest for robust control of uncertain nonlinear systems. Thus, different disturbance observers have been developed [43]–[47] and robust control schemes were proposed using disturbance observers. A general framework was given for nonlinear systems using disturbance observer based control (DOBC) techniques in [48]. In [49], composite DOBC and terminal sliding mode control were investigated for uncertain structural systems. The disturbance attenuation and rejection problem was investigated for a class of MIMO nonlinear systems using a DOBC framework in [50]. In [51], composite DOBC and H_∞ control designs were proposed for complex continuous models. Adding robustness to nominal output feedback controllers was studied for uncertain nonlinear systems using a disturbance observer in [52]. Although significant progress has been made for the disturbance observer design, there are still some open problems that need to be solved. In almost all approaches reported in the literature, the unknown disturbance is assumed as a slowly changeable disturbance for the disturbance observer design that implies the derivative of the disturbance approaching to zero. It is apparent that this assumption is restrictive for a practical system. The NDO can provide the estimation of the bounded unknown disturbance and can be employed in the robust control design to compensate for the unknown disturbance. At the same time, the NDO does not rely on complete knowledge of the disturbance mathematical model, as an efficient disturbance observer. In this paper, the NDO is proposed for the uncertain nonlinear systems for which the known upper boundary assumption of the unknown disturbance is canceled and the convergence of the disturbance estimation error is proved.

This paper develops a new NDO-based DSC design for uncertain nonlinear systems with unknown external disturbance and input saturation. The control objective is that the proposed DSC can track a desired trajectory in the presence of unknown time-varying external disturbance and input saturation. The main contributions of this paper are as follows.

1) An NDO is developed to estimate the unknown disturbance. Especially, the known upper boundary requirement of the unknown disturbance is eliminated for the design of NDO.
2) DSC is implemented using the output of the developed NDO for uncertain nonlinear systems with input saturation and unknown external disturbances to enhance the robust control performance of the closed-loop system.
3) Closed-loop system stability is guaranteed using Lyapunov method, which shows that all closed-loop system signals are semiglobal uniformly ultimately bounded.

The organization of this paper is as follows. Section II details the problem formulation. Section III presents the DSC scheme with NDO. Simulation studies are presented in Section IV to demonstrate the effectiveness of the developed nonlinear disturbance observer-based DSC, followed by some concluding remarks in Section V.

Throughout this paper, $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)^*$, $||\cdot||$ denotes the l_2 norm, and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the smallest and largest eigenvalues of a square matrix $\cdot$, respectively.

II. Problem Statement and Preliminaries

A. Problem Statement

Consider a class of uncertain strict-feedback nonlinear systems with input saturation and unknown disturbance which are described by

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}, \quad i = 1, \ldots, n-1 \\ \dot{x}_n &= f_n(\bar{x}_n) + g_{n0}(\bar{x}_n)u(v(t)) + d(t) \\ y &= x_1 \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i, i = 1, 2, \ldots, n$, are state vectors which are assumed to be measurable; $y \in R$ is the output of the uncertain nonlinear system; function terms $f_i(\bar{x}_i) : R^i \to R, i = 1, 2, \ldots, n$, $g_i(\bar{x}_i) : R^i \to R, i = 1, 2, \ldots, n-1$, and $g_{n0}(\bar{x}_n) : R^n \to R$ are unknown and continuous; $d \in R$ is an unknown and bounded disturbance; $v(t) \in R$ is the control input and $u(\cdot)$ denotes the plant input which is subject to saturation nonlinearity described by [5]

$$u(v(t)) = \mathrm{sat}(v(t)) = \begin{cases} \mathrm{sign}(v(t))u_M, & |v(t)| \geq u_M \\ v(t), & |v(t)| < u_M \end{cases} \tag{2}$$

where u_M is a bound of $u(t)$.

To efficiently tackle the saturation $u(v(t))$ in the DSC, it is approximated by the following smooth function [5]:

$$h(v) = u_M \tanh\left(\frac{v}{u_M}\right) = u_M \frac{e^{v/u_M} - e^{-v/u_M}}{e^{v/u_M} + e^{-v/u_M}}. \tag{3}$$

It is apparent that there exists a difference $\Delta(v)$ between $\mathrm{sat}(v(t))$ and $h(v)$. Then, we have

$$\Delta(v) = \mathrm{sat}(v(t)) - h(v). \tag{4}$$

Since the bounded property of the tan$h$ function and the sat function, we can see that the difference $\Delta(v)$ is bounded which satisfies the following condition [5]:

$$|\Delta(v)| = |\mathrm{sat}(v(t)) - h(v)| \le u_M(1 - \tanh(1)) = \bar{d}. \tag{5}$$

Consider the saturation characteristic and the corresponding approximation error, the uncertain nonlinear system (1) can be written as

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}, \quad i = 1, \ldots, n-1 \\ \dot{x}_n &= f_n(\bar{x}_n) + g_{n0}(\bar{x}_n)h(v(t)) + g_{n0}(\bar{x}_n)\Delta(v) + d(t) \\ y &= x_1. \end{aligned} \tag{6}$$

To facilitate the DSC design for the uncertain nonlinear system (6), invoking the mean-value theorem [53], we can express $h(v(t))$ in (6) as follows:

$$h(v(t)) = h(v^0) + \left.\frac{\partial h(v)}{\partial v}\right|_{v=v^\mu} (v - v^0) \tag{7}$$

where $v^\mu = \mu v + (1-\mu)v^0$ with $0 < \mu < 1$.

By choosing $v^0 = 0$, we obtain

$$h(v(t)) = h(0) + \left.\frac{\partial h(v)}{\partial v}\right|_{v=v^\mu} v. \tag{8}$$

Considering $h(0) = 0$, we have

$$h(v(t)) = \left.\frac{\partial h(v)}{\partial v}\right|_{v=v^\mu} v. \tag{9}$$

Define $g_n(\bar{x}_n) = g_{n0}(\bar{x}_n)(\partial h(v)/\partial v)|_{v=v^\mu}$ and $D(t) = d(t) + g_{n0}(\bar{x}_n)\Delta(v)$. Then, the uncertain nonlinear system (6) can be rewritten as

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}, \quad i = 1, \ldots, n-1 \\ \dot{x}_n &= f_n(\bar{x}_n) + g_n(\bar{x}_n)v + D(t) \\ y &= x_1. \end{aligned} \tag{10}$$

B. Neural Networks

In many references of robust adaptive control for uncertain nonlinear systems, RBFNNs are usually employed as approximation models for the unknown nonlinear and continuous function terms using their inherent approximation capabilities [54]. As a class of linearly parameterized NNs, RBFNNs are adopted to approximate the unknown and continuous function $f(Z): R^q \to R$ can be written as follows:

$$f(Z) = \hat{W}^T S(Z) + \varepsilon(Z) \tag{11}$$

where $Z = [z_1, z_2, \ldots, z_q]^T \in R^q$ is an input vector of NN, $\hat{W} \in R^p$ is a weight vector of the NN, $S(Z) = [s_1(Z), s_2(Z), \ldots, s_p(Z)]^T \in R^p$ is a basis function, ε is the approximation error which satisfies $|\varepsilon| \le |\bar{\varepsilon}|$, and $\bar{\varepsilon}$ is a bound unknown parameter.

In general, RBFNN can smoothly approximate any continuous function $f(Z)$ over the compact set $\Omega_Z \in R^q$ to any arbitrary accuracy as [55]

$$f(Z) = W^{*T} S(Z) + \varepsilon^*(Z) \quad \forall Z \in \Omega_Z \subset R^q \tag{12}$$

where W^* is the optimal weight value and $\varepsilon^*(Z)$ is the smallest approximation error. The Gaussian function is written in the form of

$$s_i(Z) = \exp[-(Z - c_i)^T (Z - c_i)/b_i^2], \quad i = 1, 2, \ldots, p \tag{13}$$

where c_i and b_i are the center and width of the neural cell of the ith hidden layer.

The optimal weight value of RBFNN is given by [55]

$$W^* = \arg \min_{\hat{W} \in \Omega_f} [\sup_{z \in S_Z} |\hat{f}(Z|\hat{W}) - f(Z)|] \tag{14}$$

where $\Omega_f = \{\hat{W} : \|\hat{W}\| \le M\}$ is a valid field of the parameter and M is a design parameter. $S_Z \subset R^n$ is an allowable set of the state vector.

Using the optimal weight value yields

$$|f(Z) - W^{*T} S(Z)| = |\varepsilon^*(Z)| \le |\bar{\varepsilon}|. \tag{15}$$

In this paper, the NDO is employed to estimate the unknown compounded disturbance $D(t)$ which consists of $d(t)$ and $g_{n0}(\bar{x}_n)\Delta(v)$. The RBFNNs are used to approximate the unknown continuous functions. Based on estimated outputs of the developed NDO and the RBFNN, the DSC scheme is proposed for uncertain nonlinear systems. The control objective is that the developed DSC scheme can make the system output follow a given desired system output y_d of the nonlinear system in the presence of the unknown external disturbance and the input saturation for all initial conditions satisfying $\Pi_i := \{\sum_{j=1}^{i}(z_j^2 + (\tilde{W}_j^T \Lambda_j \tilde{W}_j)) + \sum_{j=2}^{i} \eta_j^2 < 2p\}, i = 1, \ldots, n$ with $p > 0$, $z_1 = x_1 - y_d$, $z_i = x_i - \lambda_i$, $i = 2, \ldots, n$, $\tilde{W}_j = \hat{W}_j - W_j^*$, $j = 1, \ldots, n$, λ_i and η_i will be given. For the desired system output y_d, the proposed nonlinear disturbance observer-based DSC should ensure that all closed-loop signals are convergent.

To proceed with the design of the nonlinear disturbance observer-based DSC for the uncertain nonlinear system (1), the following assumptions are required.

Assumption 1 [56]: For all $t > 0$, the reference signal $y_d(t)$ is a sufficiently smooth function of t, and y_d, $\dot{y}_d$, and $\ddot{y}_d$ are bounded, that is, there exists a positive constant B_0 such that $\Pi_0 := \{(y_d, \dot{y}_d, \ddot{y}_d) : (y_d)^2 + (\dot{y}_d)^2 + (\ddot{y}_d)^2 \le B_0\}$.

Assumption 2 [57]: The signs of $g_i, i = 1, \ldots, n-1$ and g_{n0} are known. Furthermore, there exist positive constants $\underline{g}_i$ and $\bar{g}_i$, such that $\underline{g}_i \le |g_i| \le \bar{g}_i$. At the same time, there exist two positive constants $\underline{g}_0$ and $\bar{g}_0$ to render $\underline{g}_0 \le |g_{n0}| \le \bar{g}_0$ valid. Without losing generality, we shall assume that g_i and g_{n0} are positive in the DSC design.

Assumption 3: There exist the unknown positive constants β_0 and β_1 such that the external disturbance satisfy $|d| \le \beta_0$ and $|\dot{d}| \le \beta_1$.

Assumption 4 [57]: There exist constants $g_i^d > 0, i = 1, 2, \ldots, n$ such that $|\dot{g}_i(.)| \le g_i^d$ in the compact set Ω_j.

At the same time, there exists a positive constant g_{n0}^d such that $|\dot{g}_{n0}(.)| \le g_{n0}^d$.

Assumption 5: For a practical system described by the uncertain strict-feedback nonlinear system (1) subject to the input saturation (2) and the desired reference signal y_d, there should exist a feasible actual control input υ which can achieve the given tracking control objective.

Remark 1: Due to the control input saturation $u(\upsilon(t))$ and the unknown external disturbance $d(t)$, the control design of the uncertain nonlinear system (1) becomes more complicated. In accordance with the characteristic of the DSC, the reference signal $y_d(t)$ and its time derivatives $\dot{y}_d(t)$, $\ddot{y}_d(t)$ are assumed to be bounded in Assumption 1. Assumption 2 implies that smooth functions are strictly either positive or negative. To Assumption 3, the external disturbance is assumed as bounded and the boundary is unknown. Since the time-dependent disturbance $d(t)$ can be largely attributed to the exogenous effects, it has finite energy. Hence, it is bounded and the time derivation is also bounded. On the other hand, the approximation error $\Delta(\upsilon)$ of the control input saturation is bounded which equals to $\Delta(\upsilon) = \mathrm{sat}(\upsilon(t)) - h(\upsilon)$. For a practical system, the time derivation of $\mathrm{sat}(\upsilon(t))$ is bounded when the actuator is determined. Furthermore, the time derivation of tan$h$ function $h(\upsilon)$ is also bounded. Thus, the time derivation of $\dot{\Delta}(\upsilon)$ is bounded. At the same time, $\dot{D}(t) = \dot{d}(t) + (\dot{g}_{n0}(\bar{x}_n)\Delta(\upsilon) + g_{n0}(\bar{x}_n)\dot{\Delta}(\upsilon))$. According to Assumptions 2 and 4, we know that g_{n0} and $\dot{g}_{n0}$ are bounded. From above analysis, we know that the compounded disturbance $D(t)$ satisfies $|D| \le \theta_0$ and $|\dot{D}| \le \theta_1$ with the unknown constants $\theta_0 > 0$ and $\theta_1 > 0$.

Remark 2: For a given practical system, the input saturation should meet the physical requirement of system control. In other words, there should exist a DSC that can track the given desired output of the nonlinear system in the presence of the unknown external disturbance and the input saturation for all given initial conditions. Many practical systems are controllable under the control input saturation, such as a flight control system. For an aircraft, the deflexion angles of control surfaces are limited, which lead to the bounded control forces and control moments. However, there usually exists a possible control to meet the flight control requirement under the limited control forces and control moments. Thus, for a given practical system, the input saturation should meet the physical requirement of system control. Namely, there should exist a DSC that can track the given desired output of the nonlinear system in the presence of the unknown external disturbance and the input saturation for all given initial conditions.

Remark 3: To tackle the control input saturation of the uncertain strict-feedback nonlinear system, the saturation function is approximated by the tan$h$ function in the DSC design. To facilitate the DSC design for the uncertain nonlinear system (6), we introduce $g_{n0}(\bar{x}_n)(\partial h(\upsilon)/\partial \upsilon)\,|_{\upsilon=\upsilon^\mu}$ to be as a control gain function by invoking the mean-value theorem. In general, $\partial h(\upsilon)/\partial \upsilon$ goes to zero as $\upsilon \to \infty$ which may lead to $g_n(\bar{x}_n) = g_{n0}(\bar{x}_n)(\partial h(\upsilon)/\partial \upsilon)|_{\upsilon=\upsilon^\mu}$ going to zero. However, from Assumption 5, we know that the difference between the designed control input υ and the actual control input u should be bounded to meet the controllable requirement. Due to the bounded actual control input u, the designed control input υ does not go to infinite which means g_n without going to zero in our DSC design.

III. DSC Using Nonlinear Disturbance Observer and Backstepping Technique

In this section, the NN-based DSC scheme will be developed for the uncertain strict-feedback nonlinear system (1) using the NDO. The detailed design process is described as follows.

Step 1: Consider the first equation in (10) when $n = 1$ and define the error variable as

$$z_1 = x_1 - y_d. \tag{16}$$

Invoking (10) and differentiating z_1 with respect to time yields

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_d = f_1(x_1) + g_1(x_1)x_2 - \dot{y}_d. \tag{17}$$

Assuming x_2 as a virtual control input, the desired feedback control α_2^* can be designed as

$$\alpha_2^* = -k_1 z_1 - \frac{1}{g_1}(f_1 - \dot{y}_d) \tag{18}$$

where k_1 is a positive design constant. f_1 and g_1 are unknown smooth functions of x_1.

Define $\rho_1(Z_1) = (1/g_1(x_1))(f_1(x_1) - \dot{y}_d)$ with $Z_1 = [x_1, \dot{y}_d]^T$. By employing the RBFNN to approximate $\rho_1(Z_1)$ and considering (12), α_2^* can be expressed as

$$\alpha_2^* = -k_1 z_1 - W_1^{*T} S_1(Z_1) - \varepsilon_1^*. \tag{19}$$

Since W_1^* and ε_1^* are unknown, the virtual control law α_2 is proposed as

$$\alpha_2 = -k_1 z_1 - \hat{W}_1^T S_1(Z_1) \tag{20}$$

where $\hat{W}_1$ is the estimation of W_1^* which is updated by

$$\dot{\hat{W}}_1 = \Lambda_1(S_1(Z_1)z_1 - \sigma_1 \hat{W}_1) \tag{21}$$

where $\Lambda_1 = \Lambda_1^T > 0$ and $\sigma_1 > 0$ are the design parameters.

To avoid repeatedly differentiating α_2, which leads to the so-called explosion of complexity in the sequel steps, the DSC technique can be employed to solve it. Introducing a first-order filter λ_2, and letting α_2 pass through it with time constant τ_2 yields

$$\tau_2\dot{\lambda}_2 + \lambda_2 = \alpha_2, \quad \lambda_2(0) = \alpha_2(0). \tag{22}$$

Defining $z_2 = x_2 - \lambda_2$ and $\eta_2 = \lambda_2 - \alpha_2$, we have $\dot{\lambda}_2 = -\eta_2/\tau_2$ and $x_2 = z_2 + \eta_2 + \alpha_2$. Considering (17) and (20), we obtain

$$\begin{aligned}\dot{z}_1 &= f_1 + g_1(z_2 + \eta_2 + \alpha_2) - \dot{y}_d \\ &= g_1\rho_1 + g_1(z_2 + \eta_2 + \alpha_2) \\ &= g_1\big(W_1^{*T} S_1(Z_1) + \varepsilon_1^*\big) \\ &\quad + g_1\big(z_2 + \eta_2 - k_1 z_1 - \hat{W}_1^T S_1(Z_1)\big) \\ &= g_1\big(z_2 + \eta_2 - k_1 z_1 - \tilde{W}_1^T S_1(Z_1) + \varepsilon_1^*\big)\end{aligned} \tag{23}$$

where $\tilde{W}_1 = \hat{W}_1 - W_1^*$.

For η_2, we have

$$\begin{aligned}\dot{\eta}_2 &= \dot{\lambda}_2 - \dot{\alpha}_2\\ &= -\frac{\eta_2}{\tau_2} + \left(-\frac{\partial\alpha_2}{\partial x_1}\dot{x}_1 - \frac{\partial\alpha_2}{\partial z_1}\dot{z}_1 - \frac{\partial\alpha_2}{\partial\hat{W}_1} - \frac{\partial\alpha_2}{\partial y_d}\dot{y}_d\right)\\ &= -\frac{\eta_2}{\tau_2} + M_2(z_1, z_2, \eta_2, \hat{W}_1, y_d, \dot{y}_d, \ddot{y}_d) \end{aligned} \tag{24}$$

where $M_2(z_1, z_2, \eta_2, \hat{W}_1, y_d, \dot{y}_d, \ddot{y}_d) = -(\partial\alpha_2/\partial x_1)\dot{x}_1 - (\partial\alpha_2/\partial z_1)\dot{z}_1 - \partial\alpha_2/\partial\hat{W}_1 - (\partial\alpha_2/\partial y_d)\dot{y}_d$ is a continuous function. For any B_0 and p, the sets $\Pi_0 := \{(y_d, \dot{y}_d, \ddot{y}_d) : (y_d)^2 + (\dot{y}_d)^2 + (\ddot{y}_d)^2 \le B_0\}$ and $\Pi_2 := \{\sum_{j=1}^{2} z_j^2 + \tilde{W}_1^T\Lambda_1\tilde{W}_1 + \eta_2^2 < 2p\}$ are compact in R^3 and R^{N_1+3}, respectively, where N_1 is the dimension of $\tilde{W}_1$. Thus, $\Pi_0 \times \Pi_2$ is also compact. Considering the continuous property, the function $M_2(.)$ has a maximum value B_2 for the given initial conditions in the compact set $\Pi_0 \times \Pi_2$ [35].

Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2g_1}z_1^2 + \frac{1}{2}\eta_2^2 + \frac{1}{2}\tilde{W}_1^T\Lambda_1^{-1}\tilde{W}_1. \tag{25}$$

Invoking (21), (23), and (24), the time derivative of V_1 is given by

$$\begin{aligned}\dot{V}_1 &= \frac{1}{g_1}z_1\dot{z}_1 - \frac{\dot{g}_1}{2g_1^2}z_1^2 + \eta_2\dot{\eta}_2 + \tilde{W}_1^T\Lambda_1^{-1}\dot{\hat{W}}_1\\ &\le z_1\big(z_2 + \eta_2 - k_1z_1 - \tilde{W}_1^TS_1(Z_1) + \varepsilon_1^*\big) + \frac{g_1^d}{2\underline{g}_1^2}z_1^2\\ &\quad + \eta_2\left(-\frac{\eta_2}{\tau_2} + M_2\right) + \tilde{W}_1^T\Lambda_1^{-1}\dot{\hat{W}}_1\\ &= -k_1z_1^2 + z_1z_2 + z_1\eta_2 + z_1\varepsilon_1^* + \frac{g_1^d}{2\underline{g}_1^2}z_1^2\\ &\quad -\frac{\eta_2^2}{\tau_2} + \eta_2M_2 - \sigma_1\tilde{W}_1^T\hat{W}_1 \end{aligned} \tag{26}$$

where M_2 denotes $M_2(z_1, z_2, \eta_2, \hat{W}_1, y_d, \dot{y}_d, \ddot{y}_d)$.

Considering the following fact:

$$\begin{aligned}2\tilde{W}_1^T\hat{W}_1 &= \|\tilde{W}_1\|^2 + \|\hat{W}_1\|^2 - \|W_1^*\|^2\\ &\ge \|\tilde{W}_1\|^2 - \|W_1^*\|^2 \end{aligned} \tag{27}$$

we have

$$\begin{aligned}\dot{V}_1 \le& -\left(k_1 - 1.5 - \frac{g_1^d}{2\underline{g}_1^2}\right)z_1^2 - \left(\frac{1}{\tau_2} - 1\right)\eta_2^2 - \frac{\sigma_1}{2}\|\tilde{W}_1\|^2\\ &+ 0.5z_2^2 + 0.5\varepsilon_1^{*2} + 0.5B_2^2 + \frac{\sigma_1}{2}\|W_1^*\|^2. \end{aligned} \tag{28}$$

Step i ($2 \le i \le n-1$): In the ith step, we define the error variable as

$$z_i = x_i - \lambda_i \tag{29}$$

where λ_i is obtained from the $(i-1)$th step.

Considering (10) and differentiating z_i with respect to time yields

$$\dot{z}_i = \dot{x}_i - \dot{\lambda}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} - \dot{\lambda}_i. \tag{30}$$

Assuming x_{i+1} as a virtual control input, the desired feedback control α_{i+1}^* can be designed as

$$\alpha_{i+1}^* = -k_iz_i - \frac{1}{g_i}(f_i - \dot{\lambda}_i) \tag{31}$$

where k_i is a positive design constant. f_i and g_i are unknown smooth functions of $\bar{x}_i$.

Define $\rho_i(Z_i) = (1/g_i(\bar{x}_i))(f_i(\bar{x}_i) - \dot{\lambda}_i)$ with $Z_i = [\bar{x}_i, \dot{\bar{\lambda}}_i]^T$. By employing the RBFNN to approximate $\rho_i(Z_i)$ and considering (12), α_{i+1}^* can be expressed as

$$\alpha_{i+1}^* = -k_iz_i - W_i^{*T}S_i(Z_i) - \varepsilon_i^*. \tag{32}$$

Since W_i^* and ε_i^* are unknown, the virtual control law α_{i+1} is proposed as

$$\alpha_{i+1} = -k_iz_i - \hat{W}_i^TS_i(Z_i) \tag{33}$$

where $\hat{W}_i$ is the estimation of W_i^* which is updated by

$$\dot{\hat{W}}_i = \Lambda_i(S_i(Z_i)z_i - \sigma_i\hat{W}_i) \tag{34}$$

where $\Lambda_i = \Lambda_i^T > 0$ and $\sigma_i > 0$ are the design parameters.

To avoid repeatedly differentiating α_{i+1}, which leads to the so-called explosion of complexity in the sequel steps, the DSC technique can be employed to solve it. Introducing a first-order filter λ_{i+1}, and letting α_{i+1} pass through it with time constant τ_{i+1} yields

$$\tau_{i+1}\dot{\lambda}_{i+1} + \lambda_{i+1} = \alpha_{i+1}, \quad \lambda_{i+1}(0) = \alpha_{i+1}(0). \tag{35}$$

Defining $z_{i+1} = x_{i+1} - \lambda_{i+1}$ and $\eta_{i+1} = \lambda_{i+1} - \alpha_{i+1}$, we have $\dot{\lambda}_{i+1} = -\eta_{i+1}/\tau_{i+1}$ and $x_{i+1} = z_{i+1} + \eta_{i+1} + \alpha_{i+1}$. Considering (30) and (33), we obtain

$$\begin{aligned}\dot{z}_i &= f_i + g_i(z_{i+1} + \eta_{i+1} + \alpha_{i+1}) - \dot{\lambda}_i\\ &= g_i\rho_i + g_i(z_{i+1} + \eta_{i+1} + \alpha_{i+1})\\ &= g_i\big(W_i^{*T}S_i(Z_i) + \varepsilon_i^*\big)\\ &\quad + g_i\big(z_{i+1} + \eta_{i+1} - k_iz_i - \hat{W}_i^TS_i(Z_i)\big)\\ &= g_i\big(z_{i+1} + \eta_{i+1} - k_iz_i - \tilde{W}_i^TS_i(Z_i) + \varepsilon_i^*\big) \end{aligned} \tag{36}$$

where $\tilde{W}_i = \hat{W}_i - W_i^*$.

For η_{i+1}, we have

$$\begin{aligned}\dot{\eta}_{i+1} &= \dot{\lambda}_{i+1} - \dot{\alpha}_{i+1} = -\frac{\eta_{i+1}}{\tau_{i+1}}\\ &\quad + \left(-\frac{\partial\alpha}{\partial x_i}\dot{x}_i - \frac{\partial\alpha_{i+1}}{\partial z_i}\dot{z}_i - \frac{\partial\alpha_{i+1}}{\partial\hat{W}_i} - \frac{\partial\alpha_{i+1}}{\partial\lambda_i}\dot{\lambda}_i\right)\\ &= -\frac{\eta_{i+1}}{\tau_{i+1}} + M_{i+1} \end{aligned} \tag{37}$$

where M_{i+1} denotes $M_{i+1}(z_1, \ldots, z_{i+1}, \eta_1, \ldots, \eta_i, \hat{W}_1, \ldots, \hat{W}_i, y_d, \dot{y}_d, \ddot{y}_d)$ and $M_{i+1} = -(\partial\alpha/\partial x_i)\dot{x}_i - (\partial\alpha_{i+1}/\partial z_i)\dot{z}_i - \partial\alpha_{i+1}/\partial\hat{W}_i - (\partial\alpha_{i+1}/\partial\lambda_i)\dot{\lambda}_i$ is a continuous function. For any B_0 and p, the sets $\Pi_0 := \{(y_d, \dot{y}_d, \ddot{y}_d) : (y_d)^2 + (\dot{y}_d)^2 + (\ddot{y}_d)^2 \le B_0\}$ and $\Pi_i := \{\sum_{j=1}^{i}(z_j^2 + (\tilde{W}_j^T\Lambda_j\tilde{W}_j)) + \sum_{j=2}^{i}\eta_j^2 < 2p\}, i = 1, \ldots, n-1$ are compact in R^3 and $R^{\sum_{j=1}^{i}N_i + 2i - 1}$, respectively, where N_i is the dimension of $\tilde{W}_i$. Thus, $\Pi_0 \times \Pi_i$ is also compact. Considering the continuous property, the function M_{i+1} has a maximum value B_{i+1} for the given initial conditions in the compact set $\Pi_0 \times \Pi_i$ [35].

Consider the Lyapunov function candidate

$$V_i = \frac{1}{2g_i}z_i^2 + \frac{1}{2}\eta_{i+1}^2 + \frac{1}{2}\tilde{W}_i^T\Lambda_i^{-1}\tilde{W}_i. \tag{38}$$

Invoking (34), (36), and (37), the time derivative of V_1 is given by

$$\begin{aligned}\dot{V}_i &= \frac{1}{g_i}z_i\dot{z}_i - \frac{\dot{g}_i}{2g_i^2}z_i^2 + \eta_{i+1}\dot{\eta}_{i+1} + \tilde{W}_i^T\Lambda_i^{-1}\dot{\hat{W}}_i\\ &\le z_i\big(z_{i+1} + \eta_{i+1} - k_i z_i - \tilde{W}_i^T S_i(Z_i) + \varepsilon_i^*\big) + \frac{g_i^d}{2\underline{g}_i^2}z_i^2\\ &\quad + \eta_{i+1}\left(-\frac{\eta_{i+1}}{\tau_{i+1}} + M_{i+1}\right) + \tilde{W}_i^T\Lambda_i^{-1}\dot{\hat{W}}_i\\ &= -k_i z_i^2 + z_i z_{i+1} + z_i\eta_{i+1} + z_i\varepsilon_i^*\\ &\quad + \frac{g_i^d}{2\underline{g}_i^2}z_i^2 - \frac{\eta_{i+1}^2}{\tau_{i+1}} - \sigma_i\tilde{W}_i^T\hat{W}_i + \eta_{i+1}M_{i+1}.\end{aligned} \tag{39}$$

Considering the following fact:

$$\begin{aligned}2\tilde{W}_i^T\hat{W}_i &= \|\tilde{W}_i\|^2 + \|\hat{W}_i\|^2 - \|W_i^*\|^2\\ &\ge \|\tilde{W}_i\|^2 - \|W_i^*\|^2\end{aligned} \tag{40}$$

we have

$$\begin{aligned}\dot{V}_i \le& -\left(k_i - 1.5 - \frac{g_i^d}{2\underline{g}_i^2}\right)z_i^2 - \left(\frac{1}{\tau_{i+1}} - 1\right)\eta_{i+1}^2 - \frac{\sigma_i}{2}\|\tilde{W}_i\|^2\\ &+ 0.5z_{i+1}^2 + 0.5\varepsilon_i^{*2} + 0.5B_{i+1}^2 + \frac{\sigma_i}{2}\|W_i^*\|^2.\end{aligned} \tag{41}$$

Step n: In this step, the error variable is defined as

$$z_n = x_n - \lambda_n \tag{42}$$

where λ_n is obtained from the $(n-1)$th step.

Considering (10) and differentiating z_n with respect to time yields

$$\dot{z}_n = \dot{x}_n - \dot{\lambda}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)\upsilon + D - \dot{\lambda}_n. \tag{43}$$

The desired feedback control υ^* can be designed as

$$\upsilon^* = -k_n z_n - \frac{1}{g_n}(f_n - \dot{\lambda}_n) - D \tag{44}$$

where k_n is a positive design constant. f_n and g_n are unknown smooth functions of $\bar{x}_n$.

Define $\rho_n(Z_n) = 1/g_n(\bar{x}_n)(f_n(\bar{x}_n) - \dot{\lambda}_n)$ with $Z_n = [\bar{x}_n, \dot{\lambda}_n]^T$. By employing the RBFNN to approximate $\rho_n(Z_n)$ and considering (12), υ^* can be expressed as

$$\upsilon^* = -k_n z_n - W_n^{*T}S_n(Z_n) - \varepsilon_n^* - D. \tag{45}$$

Since W_n^*, ε_n^*, and D are unknown, the control law υ is proposed as

$$\upsilon = -k_n z_n - \hat{W}_n^T S_n(Z_n) - \hat{D} \tag{46}$$

where $\hat{D}$ is the estimation of D and $\hat{W}_n$ is the estimation of W_n^* which is updated by

$$\dot{\hat{W}}_n = \Lambda_n(S_n(Z_n)z_n - \sigma_n\hat{W}_n) \tag{47}$$

where $\Lambda_n = \Lambda_n^T > 0$ and $\sigma_n > 0$ are the design parameters.

Considering (43) and (46), we obtain

$$\begin{aligned}\dot{z}_n &= f_n + g_n\upsilon + D(t) - \dot{\lambda}_n = g_n\big(W_n^{*T}S_n(\bar{x}_n) + \varepsilon_n^*\big)\\ &\quad + g_n\big(-k_n z_n - \hat{W}_n^T S_n(Z_n) - \hat{D}\big) + D(t)\\ &= g_n\big(-k_n z_n - \tilde{W}_n^T S_n(Z_n) - \hat{D} + \varepsilon_n^*\big) + D(t)\end{aligned} \tag{48}$$

where $\tilde{W}_n = \hat{W}_n - W_n^*$.

To facilitate the design of the NDO, (43) can be also written as

$$\begin{aligned}\dot{z}_n &= l^{-1}\rho(\bar{x}_n, \upsilon) + D - \dot{\lambda}_n\\ &= l^{-1}W_\rho^{*T}S_\rho(\bar{x}_n) + l^{-1}\varepsilon_\rho + D - \dot{\lambda}_n\end{aligned} \tag{49}$$

where $\rho(\bar{x}_n, \upsilon) = l(f_n(\bar{x}_n) + g_n(\bar{x}_n)\upsilon)$, W_ρ^* is optimal weight value of the RBFNN, ε_ρ is the approximation error of the RBFNN, and $l > 0$ is a design parameter of the developed NDO.

Invoking (49), an auxiliary variable is given by

$$s = z_n - \xi \tag{50}$$

and the intermedial variable ξ is proposed as

$$\dot{\xi} = cs + l^{-1}\hat{W}_\rho^T S_\rho(\bar{x}_n) - \dot{\lambda}_n \tag{51}$$

where $c > 0$ is a designed parameter and $\hat{W}_\rho$ is the estimate of the optimal weight value W_ρ^*.

Differentiating (50) and considering (49) and (51), we have

$$\begin{aligned}\dot{s} &= \dot{z}_n - \dot{\xi} = l^{-1}W_\rho^{*T}S_\rho(\bar{x}_n) + l^{-1}\varepsilon_\rho + D - \dot{\lambda}_n\\ &\quad - \big(cs + l^{-1}\hat{W}_\rho^T S_\rho(\bar{x}_n) - \dot{\lambda}_n\big)\\ &= -cs - l^{-1}\tilde{W}_\rho^T S_\rho(\bar{x}_n) + l^{-1}\varepsilon_\rho + D\end{aligned} \tag{52}$$

where $\tilde{W}_\rho = \hat{W}_\rho - W_\rho^*$.

Considering (52) yields

$$\begin{aligned}s\dot{s} &= -cs^2 - l^{-1}s\tilde{W}_\rho^T S_\rho(\bar{x}_n) + l^{-1}s\varepsilon_\rho + sD\\ &\le -(c-1.0)s^2 - l^{-1}s\tilde{W}_\rho^T S_\rho(\bar{x}_n) + 0.5l^{-2}\varepsilon_\rho^2 + 0.5\theta_0^2.\end{aligned} \tag{53}$$

On the basis of the auxiliary variable s, the NDO is designed as

$$\hat{D} = l(s - \phi) \tag{54}$$

and the intermedial variable ϕ is given by

$$\dot{\phi} = -cs + \hat{D}. \tag{55}$$

Define $\tilde{D} = D - \hat{D}$. Differentiating (54), and considering (52) and (55) yield

$$\begin{aligned}\dot{\hat{D}} &= l(\dot{s} - \dot{\phi}) = l\Big(\big(-cs - l^{-1}\tilde{W}_\rho^T S_\rho(\bar{x}_n) + l^{-1}\varepsilon_\rho + D\big) - \big(-cs + \hat{D}\big)\Big)\\ &= -\tilde{W}_\rho^T S_\rho(\bar{x}_n) + \varepsilon_\rho + l(D - \hat{D})\\ &= -\tilde{W}_\rho^T S_\rho(\bar{x}_n) + \varepsilon_\rho + l\tilde{D}.\end{aligned} \tag{56}$$

Considering (56), we have

$$\dot{\tilde{D}} = \dot{D} - \dot{\hat{D}} = \dot{D} - l\tilde{D} + \tilde{W}_\rho^T S_\rho(\bar{x}_n) - \varepsilon_\rho. \tag{57}$$

Invoking (57), we obtain

$$\tilde{D}\dot{\tilde{D}} = \tilde{D}\dot{D} - l\tilde{D}^2 + \tilde{D}\tilde{W}_\rho^T S_\rho(\bar{x}_n) - \tilde{D}\varepsilon_\rho. \tag{58}$$

Considering the following fact:

$$2\tilde{D}\tilde{W}_\rho^T S_\rho(\bar{x}_n) \le 2|\tilde{D}|||\tilde{W}_\rho||||S_\rho(\bar{x}_n)|| \le \gamma_0\vartheta^2\tilde{D}^2 + \frac{1}{\gamma_0}||\tilde{W}_\rho||^2 \tag{59}$$

yields

$$\tilde{D}\dot{\tilde{D}} \le \tilde{D}^2 + 0.5\dot{D}^2 - l\tilde{D}^2 + \gamma\vartheta^2\tilde{D}^2 + \frac{1}{\gamma}||\tilde{W}_\rho||^2 + 0.5\varepsilon_\rho^2 \le -(l-(1.0+\gamma\vartheta^2))\tilde{D}^2 + \frac{1}{\gamma}||\tilde{W}_\rho||^2 + 0.5\theta_1^2 + 0.5\varepsilon_\rho^2 \tag{60}$$

where $||S_\rho(\bar{x}_n)|| \le \vartheta$, $\gamma = 0.5\gamma_0$ and $\gamma_0 > 0$ is a design parameter.

The parameter updated law $\hat{W}_\rho$ is designed as

$$\dot{\hat{W}}_\rho = \Lambda_\rho\big(l^{-1}S_\rho^T(\bar{x}_n)s - \sigma_\rho\hat{W}_\rho\big) \tag{61}$$

where $\Lambda_\rho = \Lambda_\rho^T > 0$ and $\sigma_\rho > 0$ are the design parameters.

Consider the Lyapunov function candidate

$$V_n = \frac{1}{2g_n}z_n^2 + \frac{1}{2}\tilde{W}_n^T\Lambda_n^{-1}\tilde{W}_n + \frac{1}{2}\tilde{W}_\rho^T\Lambda_\rho^{-1}\tilde{W}_\rho + \frac{1}{2}s^2 + \frac{1}{2}\tilde{D}^2. \tag{62}$$

Invoking (48), (53), and (60), the time derivative of V_n is

$$\begin{aligned}\dot{V}_n &= \frac{1}{g_n}z_n\dot{z}_n - \frac{\dot{g}_n}{2g_n^2}z_n^2 + \tilde{W}_n^T\Lambda_n^{-1}\dot{\tilde{W}}_n \\ &\quad + \tilde{W}_\rho^T\Lambda_\rho^{-1}\dot{\tilde{W}}_\rho + s\dot{s} + \tilde{D}\dot{\tilde{D}} \\ &\le z_n\big(-k_nz_n - \tilde{W}_n^TS_n(Z_n) - \hat{D} + \varepsilon_n^*\big) + \frac{g_n^d}{2\underline{g}_n^2}z_n^2 \\ &\quad + \frac{z_nD}{g_n} + \tilde{W}_n^T\Lambda_n^{-1}\dot{\tilde{W}}_n + \tilde{W}_\rho^T\Lambda_\rho^{-1}\dot{\tilde{W}}_\rho \\ &\quad - (c-1.0)s^2 - l^{-1}s\tilde{W}_\rho^TS_\rho(\bar{x}_n) + 0.5l^{-2}\varepsilon_\rho^2 + 0.5\theta_0^2 \\ &\quad - (l-(1.0+\gamma\vartheta^2))\tilde{D}^2 + \frac{1}{\gamma}||\tilde{W}_\rho||^2 \\ &\quad + 0.5\theta_1^2 + 0.5\varepsilon_\rho^2. \end{aligned} \tag{63}$$

Considering $\tilde{D} = D - \hat{D}$ and Assumption 3, we have

$$\begin{aligned}\dot{V}_n &\le z_n\big(-k_nz_n - \tilde{W}_n^TS_n(Z_n) + \varepsilon_n^*\big) + \frac{g_n^d}{2\underline{g}_n^2}z_n^2 \\ &\quad + z_n\tilde{D} + z_nD\left(\frac{1}{g_n}-1\right) + \tilde{W}_n^T\Lambda_n^{-1}\dot{\tilde{W}}_n + \tilde{W}_\rho^T\Lambda_\rho^{-1}\dot{\tilde{W}}_\rho \\ &\quad - (c-1.0)s^2 - l^{-1}s\tilde{W}_\rho^TS_\rho(\bar{x}_n) + 0.5l^{-2}\varepsilon_\rho^2 + 0.5\theta_0^2 \\ &\quad - (l-(1.0+\gamma\vartheta^2))\tilde{D}^2 + \frac{1}{\gamma}||\tilde{W}_\rho||^2 + 0.5\theta_1^2 + 0.5\varepsilon_\rho^2 \\ &\le -\left(k_n - 1.5 - \frac{g_n^d}{2\underline{g}_n^2}\right)z_n^2 - z_n\tilde{W}_n^TS_n(Z_n) \\ &\quad + \tilde{W}_n^T\Lambda_n^{-1}\dot{\tilde{W}}_n + \tilde{W}_\rho^T\Lambda_\rho^{-1}\dot{\tilde{W}}_\rho - l^{-1}s\tilde{W}_\rho^TS_\rho(\bar{x}_n) \\ &\quad - (c-1.0)s^2 - (l-(1.5+\gamma\vartheta^2))\tilde{D}^2 \\ &\quad + \frac{1}{\gamma}||\tilde{W}_\rho||^2 + \left(0.5 + 0.5|\frac{1}{\underline{g}_n} - 1|^2\right)\theta_0^2 \\ &\quad + 0.5\theta_1^2 + 0.5\varepsilon_n^{*2} + (0.5+0.5l^{-2})\varepsilon_\rho^2. \end{aligned} \tag{64}$$

Substituting (47) and (61) into (64), we obtain

$$\begin{aligned}\dot{V}_n &\le -\left(k_n - 1.5 - \frac{g_n^d}{2\underline{g}_n^2}\right)z_n^2 - (c-1.0)s^2 \\ &\quad -(l-(1.5+\gamma\vartheta^2))\tilde{D}^2 - \sigma_n\tilde{W}_n^T\hat{W}_n - \sigma_\rho\tilde{W}_\rho^T\hat{W}_\rho \\ &\quad + \frac{1}{\gamma}||\tilde{W}_\rho||^2 + \left(0.5 + 0.5|\frac{1}{\underline{g}_n} - 1|^2\right)\theta_0^2 \\ &\quad + 0.5\theta_1^2 + 0.5\varepsilon_n^{*2} + (0.5+0.5l^{-2})\varepsilon_\rho^2. \end{aligned} \tag{65}$$

Considering the following facts:

$$2\tilde{W}_n^T\hat{W}_n = \|\tilde{W}_n\|^2 + \|\hat{W}_n\|^2 - \|W_n^*\|^2 \ge \|\tilde{W}_n\|^2 - \|W_n^*\|^2 \tag{66}$$

and

$$2\tilde{W}_\rho^T\hat{W}_\rho = \|\tilde{W}_\rho\|^2 + \|\hat{W}_\rho\|^2 - \|W_\rho^*\|^2 \ge \|\tilde{W}_\rho\|^2 - \|W_\rho^*\|^2 \tag{67}$$

we have

$$\begin{aligned}\dot{V}_n &\le -\left(k_n - 1.5 - \frac{g_n^d}{2\underline{g}_n^2}\right)z_n^2 - (c-1.0)s^2 \\ &\quad -(l-(1.5+\gamma\vartheta^2))\tilde{D}^2 - \frac{\sigma_n}{2}\|\tilde{W}_n\|^2 \\ &\quad -\left(\frac{\sigma_\rho}{2} - \frac{1}{\gamma}\right)\|\tilde{W}_\rho\|^2 + \left(0.5 + 0.5|\frac{1}{\underline{g}_n} - 1|^2\right)\theta_0^2 \\ &\quad + 0.5\theta_1^2 + 0.5\varepsilon_n^{*2} + (0.5+0.5l^{-2})\varepsilon_\rho^2 \\ &\quad + \frac{\sigma_n}{2}\|W_n^*\|^2 + \frac{\sigma_\rho}{2}\|W_\rho^*\|^2. \end{aligned} \tag{68}$$

The above DSC design procedure and stability analysis can be summarized in the following theorem, which contains the results for the uncertain strict-feedback nonlinear system (1) using the NDO and backstepping technique.

Theorem 1: Consider the uncertain strict-feedback nonlinear system (1) with the input saturation and the unknown external disturbance and suppose that full state information is available. The NDO is designed as (50), (51), (54), and (55). The updated laws of the NN weight values are chosen as (21), (34), (47), and (61). The nonlinear disturbance observer-based DSC is proposed in (46). Given any positive number p, for all initial conditions satisfying $\Pi_n := \{\sum_{j=1}^n(z_j^2 + (\tilde{W}_j^T\Lambda_j\tilde{W}_j)) + \sum_{j=2}^n\eta_j^2 < 2p\}$, the appropriate design parameters k_i, l, c, τ_i, Λ_i, Λ_ρ, σ_i, σ_ρ, and γ can be chosen according to (72) such that all closed-loop signals are uniformly bounded convergence under the proposed dynamic surface control based on the nonlinear disturbance observer. Furthermore, the tracking error $z_1 = x_1 - y_d$ can be made small by proper choice of the design parameters k_i, l, c, τ_i, Λ_i, Λ_ρ, σ_i, σ_ρ, and γ.

Proof: For considering the convergence of disturbance estimate error and closed-loop states, the Lyapunov function candidate of the whole closed-loop control system is considered as

$$V = \sum_{i=1}^n V_i. \tag{69}$$

Differentiating V and considering (28), (41), and (68), we obtain

$$\begin{aligned}\dot{V} \leq & -\left(k_1 - 1.5 - \frac{g_1^d}{2\underline{g}_1^2}\right)z_1^2 - \sum_{i=2}^{n}\left(k_i - 2.0 - \frac{g_i^d}{2\underline{g}_i^2}\right)z_i^2 \\ & - \sum_{i=2}^{n}\left(\frac{1}{\tau_i} - 1\right)\eta_i^2 - \sum_{i=1}^{n}\frac{\sigma_i}{2}\|\tilde{W}_i\|^2 \\ & - (c - 1.0)s^2 - (l - (1.5 + \gamma\vartheta^2))\tilde{D}^2 \\ & - \left(\frac{\sigma_\rho}{2} - \frac{1}{\gamma}\right)\|\tilde{W}_\rho\|^2 + 0.5\sum_{i=2}^{n}B_i^2 \\ & + \left(0.5 + 0.5|\frac{1}{\underline{g}_n} - 1|^2\right)\theta_0^2 + 0.5\theta_1^2 + 0.5\sum_{i=1}^{n}\varepsilon_i^{*2} \\ & + (0.5 + 0.5l^{-2})\varepsilon_\rho^2 + \sum_{i=1}^{n}\frac{\sigma_i}{2}\|W_i^*\|^2 + \frac{\sigma_\rho}{2}\|W_\rho^*\|^2 \\ \leq & -\varrho V + C \end{aligned} \tag{70}$$

where ϱ and C are given by

$$\varrho := \min\begin{pmatrix}\left(k_1 - 1.5 - \frac{g_1^d}{2\underline{g}_1^2}\right), \left(k_i - 2.0 - \frac{g_i^d}{2\underline{g}_i^2}\right) \\ (\frac{1}{\tau_i} - 1), (c - 1.0), (l - (1.5 + \gamma\vartheta^2)) \\ \frac{\sigma_i}{\lambda_{\max}(\Lambda_i^{-1})}, \frac{2(\frac{\sigma_\rho}{2} - \frac{1}{\gamma})}{\lambda_{\max}(\Lambda_\rho^{-1})}\end{pmatrix}$$

$$\begin{aligned} C := & \ 0.5\sum_{i=2}^{n}B_i^2 + \sum_{i=1}^{n}\frac{\sigma_i}{2}\|W_i^*\|^2 + \frac{\sigma_\rho}{2}\|W_\rho^*\|^2 \\ & + \left(0.5 + 0.5\left|\frac{1}{\underline{g}_n} - 1\right|^2\right)\theta_0^2 \\ & + 0.5\theta_1^2 + 0.5\sum_{i=1}^{n}\varepsilon_i^{*2} + (0.5 + 0.5l^{-2})\varepsilon_\rho^2. \end{aligned} \tag{71}$$

To ensure the closed-loop system stability, the corresponding design parameters k_i, l, c, τ_i, σ_ρ, and γ should be chosen to make the following inequalities hold:

$$\begin{aligned} & k_1 - 1.5 - \frac{g_1^d}{2\underline{g}_1^2} > 0 \\ & k_i - 2.0 - \frac{g_i^d}{2\underline{g}_i^2} > 0, \quad i = 2, \ldots, n \\ & \frac{1}{\tau_i} - 1 > 0, \quad i = 2, \ldots, n-1 \\ & c - 1.0 > 0 \\ & l - (1.5 + \gamma\vartheta^2) > 0 \\ & \frac{\sigma_\rho}{2} - \frac{1}{\gamma} > 0. \end{aligned} \tag{72}$$

According to (70), we have

$$0 \leq V \leq \frac{C}{\varrho} + \left[V(0) - \frac{C}{\varrho}\right]e^{-\varrho t}. \tag{73}$$

From (73), we can know that V is convergent, i.e., $\lim_{t\longrightarrow\infty} V = C/\varrho$. According to (73), it may directly show that the signals e, z_i, $\tilde{W}_i$, $\tilde{W}_\rho$, and $\tilde{D}$ are semiglobally uniformly bounded when $t \to 0$. Hence, the tracking error e, the approximation errors $\tilde{W}_i$, $\tilde{W}_\rho$, and the disturbance estimation error $\tilde{D}$ of the closed-loop system are bounded. This concludes the proof. $\diamond$

Remark 4: To enhance the closed-loop system robustness of the uncertain nonlinear system, the nonlinear disturbance observer-based DSC scheme has been developed. For fully utilizing the dynamic information of the external disturbance, the NDO is proposed to estimate the unknown disturbance of the uncertain nonlinear system and the output of the NDO is used to design the DSC law, as shown in (46). Due to the introduction of the output of NDO, the control gain can be adjusted according to the variation of unknown disturbance and the disturbance rejection ability of the closed-loop system has been improved.

Remark 5: In this paper, the NDO is developed to estimate the unknown disturbance of the uncertain strict-feedback nonlinear system. In the developed NDO, the known boundary requirement of the disturbance is canceled and the bounded disturbance estimation error is guaranteed. At the same time, the slowly changeable assumption of the external disturbance is eliminated.

Remark 6: In the developed DSC, the design parameters k_i, l, c, τ_i, Λ_i, Λ_ρ, σ_i, σ_ρ, and γ need to be tuned to obtain a good transient performance and the closed-loop stable performance. If the tracking error is desired to be lower, we should increase k_1. Λ_i, Λ_ρ, σ_i, and σ_ρ are design parameters in adaptation law of NN weight value. Decreases in σ_i and σ_ρ or increases in the adaptive gain Λ_i and Λ_ρ will result in a better tracking performance. Furthermore, the L_∞ performance of system tracking error can be guaranteed by choosing the proper initial conditions for all closed-loop system signals according to (73) [13], [32], [40], [42].

IV. Simulation Study

In this section, simulation results are presented to illustrate the effectiveness of the developed nonlinear disturbance observer-based DSC and an example system is used in the simulation study. In the simulation, the NDO is designed as (50), (51), (54), and (55). The updated laws of the NN weight values are chosen as (21), (34), (47), and (61). The nonlinear disturbance observer-based DSC is proposed in (46).

Let us consider the one-link manipulator with the inclusion of motor dynamics. The model of the one-link manipulator is given by [36]

$$\begin{aligned}\bar{D}\ddot{q} + B\dot{q} + N\sin(q) &= \tau \\ M\dot{\tau} + H\tau &= u - K_m\dot{q}\end{aligned} \tag{74}$$

where q, $\dot{q}$, and $\ddot{q}$ denote the link angular position, velocity, and acceleration, respectively. τ is the motor current. u is the input control voltage. The parameter values with appropriate units are given by $\bar{D} = 1$, $M = 0.05$, $B = 1$, $K_m = 10$, $H = 10$, and $N = 10$.

Defining $x_1 = q$, $x_2 = \dot{q}$, and $x_3 = \tau$, and considering the input saturation, the above one-link manipulator model can be

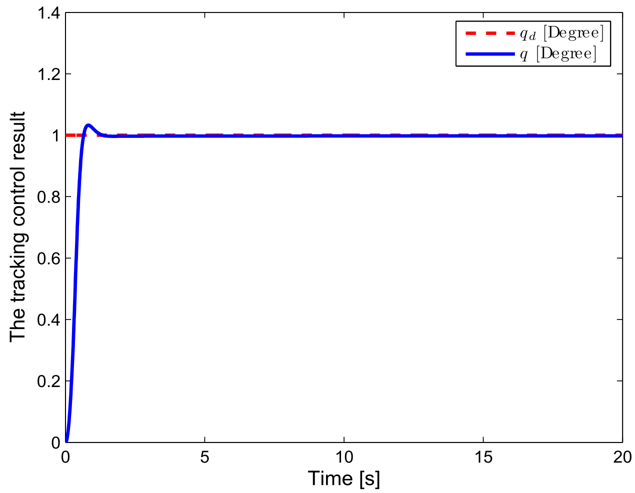

Fig. 1. Output x_1 (solid line) follows desired trajectory y_d (dashed line) of the single-link robot system for case 1.

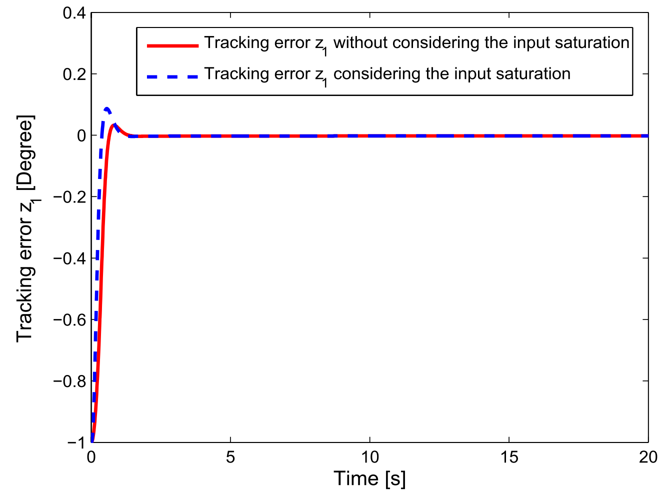

Fig. 2. Tracking error of the single-link robot system for case 1.

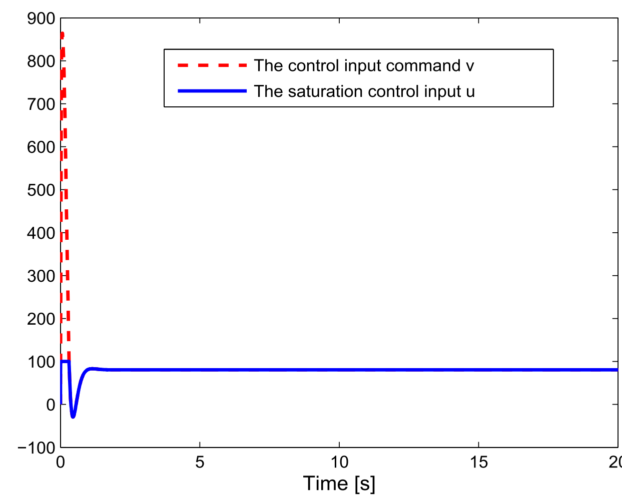

Fig. 3. Control input of the single-link robot system for case 1.

written as

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= \frac{x_3}{\bar{D}} - \frac{Bx_2}{\bar{D}} - \frac{N}{\bar{D}}\sin(x_1)\\ \dot{x}_3 &= \frac{1}{M}u(v(t)) - \frac{K_m}{M}x_2 - \frac{H}{M}x_3\\ y &= x_1.\end{aligned}$$

When uncertainty and disturbance are involved, the dynamics of one-link manipulator can be expressed as the following form:

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = x_2^2 e^{-x_1^2} - \frac{Bx_2}{\bar{D}} - \frac{N}{\bar{D}}\sin(x_1) + \frac{x_3}{\bar{D}}\\ \dot{x}_3 = -\frac{K_m}{M}x_2 - \frac{H}{M}x_3 + \frac{1}{M}u(v(t)) + d(t)\\ y = x_1.\end{cases} \tag{75}$$

Define $f_1(x_1) = 0$, $g_1(x_1) = 1$, $f_2(\bar{x}_2) = x_2^2 e^{-x_1^2} - Bx_2/\bar{D} - (N/\bar{D})\sin(x_1)$, $g_2(\bar{x}_2) = 1/\bar{D}$, $f_3(\bar{x}_3) = -(K_m/M)x_2 - (H/M)x_3$, and $g_3(\bar{x}_3) = 1/M$. It is apparent that the numerical example (75) is suitable for the case of system (1). In the simulation, the external disturbance is chosen as $d(t) = 0.4\cos(2t)$.

To proceed with the design of nonlinear disturbance observer-based DSC scheme, all design parameters are chosen as $l = 200$, $c = 13$, $\sigma_i = 0.02$, $k_1 = 20$, $k_2 = 20$, $k_3 = 10$, and $\gamma = 20$. The initial state conditions are chosen as $x_0 = 0$, $x_2 = 0$, and $x_3 = 0$. The input saturation value is given as $u_M = 80$.

A. Case 1: For Constant Desired Trajectory

To illustrate the effectiveness of the developed nonlinear disturbance observer-based DSC design, the desired trajectory is taken as $y_d = 1$. Under the proposed nonlinear disturbance observer-based DSC scheme (46), the tracking control results are shown in Figs. 1–4. From Figs. 1 and 2, we note that the tracking performance is satisfactory and the tracking error quickly converge to zero for the uncertain one-link manipulator system (75) in the presence of the time-varying external disturbance and input saturation. Although the better tracking error is obtain without considering the input saturation, the accepted tracking performance is still maintained for the uncertain one-link manipulator system (75) in the presence of the time-varying external disturbance and input saturation under our developed DSC scheme. Using the output of the NDO, the control input is bounded and convergent, as shown in Fig. 3. The plots of the NN weight values are shown in Fig. 4, which are convergent.

B. Case 2: For Time-Varying Desired Trajectory

Here, the desired trajectory is taken as $y_d = \sin(t) + \cos(0.5t)$ to illustrate the effectiveness of the developed nonlinear disturbance observer-based DSC design. Using the proposed nonlinear disturbance observer to design DSC scheme (46), all tracking control results are given in Figs. 5–8. According to Figs. 5 and 6, the satisfactory tracking performance is obtained and the tracking error maintains in

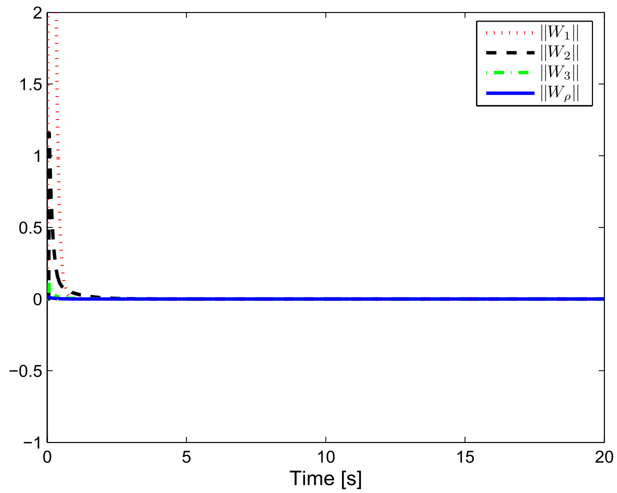

Fig. 4. Norms of NN weight values for case 1.

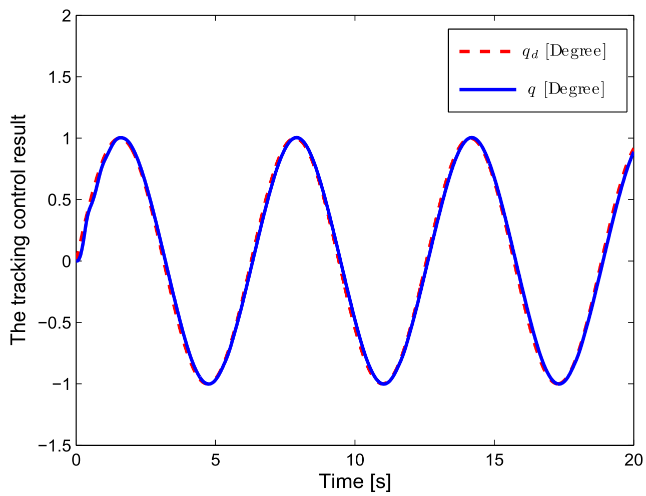

Fig. 5. Output x_1 (solid line) follows desired trajectory y_d (dashed line) of the single-link robot system for case 2.

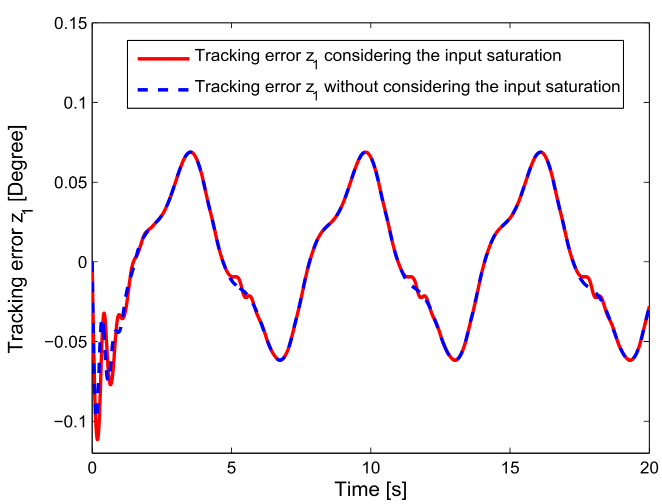

Fig. 6. Tracking error of the single-link robot system for case 2.

a small compact set for the uncertain one-link manipulator system (75) under the integrated effects of the time-varying external disturbance and input saturation. On the basis of the output of the NDO, the control input command is bounded and convergent, as shown in Fig. 7. From Fig. 8, the convergent plots of the NN weight values are noted.

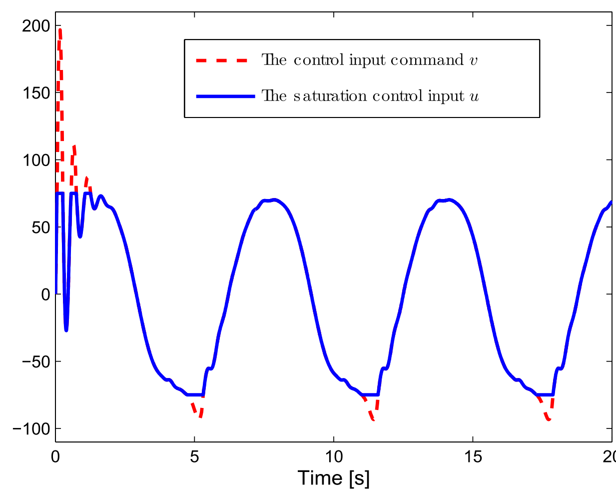

Fig. 7. Control input of the single-link robot system for case 2.

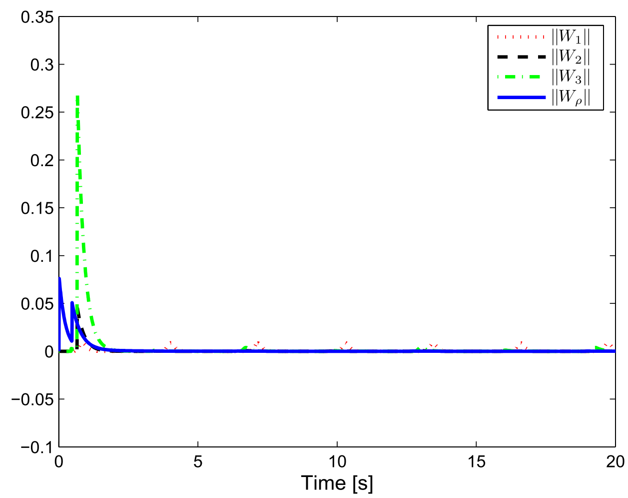

Fig. 8. Norms of NN weight values for case 2.

Based on above simulation results, we can obtain that the proposed disturbance observer-based DSC scheme is valid for the uncertain the one-link manipulator system with the time-varying unknown external disturbance and input saturation.

V. Conclusion

In this paper, the nonlinear disturbance observer-based NN DSC scheme has been developed for a class of uncertain nonlinear systems with input saturation. To improve the ability of the disturbance attenuation and system performance robustness, the NDO has been used to monitor the unknown compounded disturbance, and its output signal is utilized in the construction of nonlinear disturbance observer-based NN DSC scheme. Closed-loop system stability and tracking performance have been proved and analyzed using a rigorous Lyapunov analysis. Finally, simulation results of a one-link manipulator control system have been presented to illustrate the effectiveness of the proposed disturbance observer-based NN DSC scheme. In the future, the application of the developed nonlinear disturbance observer-based DSC scheme should be further studied. Furthermore, the DSC scheme can be developed using L_∞-type criteria to enhance the control performance when the control input saturation appear for the studied uncertain nonlinear systems.

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