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Grey system theory-based models in time series prediction

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ABSTRACT

Being able to forecast time series accurately has been quite a popular subject for researchers both in the past and at present. However, the lack of ability of conventional analysis methods to forecast time series that are not smooth leads the scientists and researchers to resort to various forecasting models that have different mathematical backgrounds, such as artificial neural networks, fuzzy predictors, evolutionary and genetic algorithms. In this paper, the accuracies of different grey models such as GM(1,1), Grey Verhulst model, modified grey models using Fourier Series is investigated. Highly noisy data, the United States dollar to Euro parity between the dates 01.01.2005 and 30.12.2007, are used to compare the performances of the different models. The simulation results show that modified grey models have higher performances not only on model fitting but also on forecasting. Among these grey models, the modified GM(1,1) using Fourier series in time is the best in model fitting and forecasting.

1. Introduction

A time series is a collection of data points which are generally sampled equally in time intervals. Time series prediction refers to the process by which the future values of a system is forecasted based on the information obtained from the past and current data points. Generally, a pre-defined mathematical model is used to make accurate predictions. Time series prediction models are widely used in financial area, such as predicting stock price indexes, foreign currency exchange rates (FX rates) and so on. The ability to do prediction with a reasonable accuracy can change the economic policy of large companies and governments and ensure a more reasonable behavior by the financial actors.

Statistical and artificial intelligence (soft computing) based approaches are the two main techniques for time series prediction seen in the literature. While AR (Auto Regressive), MA (Moving Average), ARMA (Auto Regressive Moving Average), ARIMA (Auto Regressive Integrated Moving Average) and Box–Jenkins models (Box & Jenkins, 1976) can be mentioned as statistical models, neural network (NN) based models (Quah & Srinivasan, 1999; Rabiner, 1989; Roman & Jameel, 1996) are widely used as an artificial intelligence-based approach, back propagation being the most widely used technique for updating the parameters of the model. However, not only are the statistical models not as accurate as the neural network-based approaches for nonlinear problems, they may be too complex to be used in predicting future values of a time series. One major criticism about the NN model is that it demands a great deal of training data and relatively long training period for robust generalization (Jo, 2003). Other intelligent approaches seen in the literature for the analysis of time series include Linear regression, Kalman filtering (Ma & Teng, 2004), fuzzy systems (Kandel, 1991), hidden markov models (Rabiner, 1989) and the support vector machines (Cao, 2003). Some hybrid models are also seen in the literature: in Versace, Bhatt, Hinds, and Shiffer (2004), a combination of genetic algorithms and neural networks has been proposed. In Huang and Tsai (2009), support vector regression (SVR) and a self-organizing feature map (SOFM) technique have been hybridized to reduce the cost of training time and to improve prediction accuracies. High-order fuzzy logical relationships and genetic-simulated annealing techniques are combined in Lee, Wang, and Chen (2008) for temperature prediction and the Taiwan futures exchange (TAIFEX) forecasting, where genetic-simulated annealing techniques have been used to adjust the length of each interval in the universe of discourse to increase the forecasting accuracy.

FX rates are highly nonlinear, stochastic and highly non-stationary financial time series, and as such, it is very difficult to fit a model to them by the use conventional linear statistical methods or artificial neural networks. In this paper, the use of grey prediction theory is proposed to alleviate the problem.

Grey system theory is an interdisciplinary scientific area that was first introduced in early 1980s by Deng (1982). Since then, the theory has become quite popular with its ability to deal with the systems that have partially unknown parameters. As a superiority to conventional statistical models, grey models require only a limited amount of data to estimate the behavior of unknown systems (Deng, 1989).

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During the last two decades, the grey system theory has been developed rapidly and caught the attention of many researchers. It has been widely and successfully applied to various systems such as social, economic, financial, scientific and technological, agricultural, industrial, transportation, mechanical, meteorological, ecological, hydrological, geological, medical, military, etc., systems. Some research studies in financial area are as follows: In one study (Wang, 2002), the combination of fuzzification techniques and the grey system theory (GM(1,1) model with adaptive stepsize) is proposed to predict stock prices and it is shown that the approach is very efficient. In another study, the moving average autoregressive exogenous (ARX) prediction model is combined with grey predictors for time series prediction in Huang and Jane (2009), and it is proved that the hybrid method has a greater forecasting accuracy than the GM(1,1) method. Another study (Chang & Tsai, 2008) introduces a support vector regression grey model (SVRGM) which combines support vector regression (SVR) learning algorithm and grey system theory to obtain a better approach to time series prediction. In these studies and the others, it is seen that grey system theory-based approaches can achieve good performance characteristics when applied to real-time systems, since grey predictors adapt their parameters to new conditions as new outputs become available. Because of this reason, grey predictors are more robust with respect to noise and lack of modeling information when compared to conventional methods.

The spread of this new theory has taken place as follows: In early 1990s, some universities located in Australia, China, Japan, Taiwan, USA, have started offering courses on grey system theory. Chinese Grey System Association (CGSA) was established in 1996. A conference on grey system theory and applications is held by CGSA every year. For dissemination of research results, an academic periodical; "The Journal of Grey System" is started to be published in England in 1989. Additionally, more than 300 different academic periodicals accept and publish the grey system related articles in the world (Liu & Lin, 1998). When all the literature above is investigated, it can be seen that grey system theory has aroused the interest of the scientists mostly from the far eastern countries since it was introduced into the scientific arena. Almost all the journal and conference papers have been published by eastern scientists; the scientists from the Western world have, to date, given only a limited attention to this theory. Although a number of academic books and lecture notes written in eastern languages can be found in the literature, there are only two books in English which are also written by eastern scientists.

In systems theory, a system can be defined with a color that represents the amount of clear information about that system. For instance, a system can be called as a black box if its internal characteristics or mathematical equations that describe its dynamics are completely unknown. On the other hand if the description of the system is, completely known, it can be named as a white system. Similarly, a system that has both known and unknown information is defined as a grey system. In real life, every system can be considered as a grey system because there are always some uncertainties. Due to noise from both inside and outside of the system of our concern (and the limitations of our cognitive abilities!), the information we can reach about that system is always uncertain and limited in scope (Lin & Liu, 2004).

There are many situations in which the difficulty of incomplete or insufficient information is faced. Even a simple motor control system always contains some grey characteristics due to the time-varying parameters of the system and the measurement difficulties. Similarly, it is difficult to forecast the electricity consumption of a region accurately because of the various kinds of social and economic factors. These factors are generally random and make it difficult to obtain an accurate model.

2. Fundamental concepts of grey system theory

2.1. Grey system based prediction

Grey models predict the future values of a time series based only on a set of the most recent data depending on the window size of the predictor. It is assumed that all data values to be used in grey models are positive, and the sampling frequency of the time series is fixed. From the simplest point of view, grey models which will be formulated below can be viewed as curve fitting approaches.

2.2. Generations of grey sequences

The main task of grey system theory is to extract realistic governing laws of the system using available data. This process is known as the generation of the grey sequence (Liu & Lin, 1998).

It is argued that even though the available data of the system, which are generally white numbers, is too complex or chaotic, they always contain some governing laws. If the randomness of the data obtained from a grey system is somehow smoothed, it is easier to derive any special characteristics of that system.

For instance, the following sequence that represents the price of a product might be given:

$$X^{(0)} = (820, 840, 835, 850, 890).$$

It is obvious that the sequence does not have a clear regularity. If accumulating generation suggested in grey system theory is applied to this sequence, $X^{(1)}$ is obtained which has a clear growing tendency.

$$X^{(1)} = (820, 1660, 2495, 3345, 4235).$$

2.3. GM(n,m) model

In grey systems theory, GM(n,m) denotes a grey model, where n is the order of the difference equation and m is the number of variables. Although various types of grey models can be mentioned, most of the previous researchers have focused their attention on GM(1,1) models in their predictions because of its computational efficiency. It should be noted that in real time applications, the computational burden is the most important parameter after the performance.

2.4. GM(1,1) model

GM(1,1) type of grey model is the most widely used in the literature, pronounced as "Grey Model First Order One Variable". This model is a time series forecasting model. The differential equations of the GM(1,1) model have time-varying coefficients. In other words, the model is renewed as the new data become available to the prediction model.

The GM(1,1) model can only be used in positive data sequences (Deng, 1989). In this paper, since all the primitive data points are positive, grey models can be used to forecast the future values of the primitive data points.

In order to smooth the randomness, the primitive data obtained from the system to form the GM(1,1) is subjected to an operator, named Accumulating Generation Operator (AGO) (Deng, 1989). The differential equation (i.e. GM(1,1)) is solved to obtain the n-step ahead predicted value of the system. Finally, using the predicted value, the Inverse Accumulating Generation Operator (IAGO) is applied to find the predicted values of original data.

Consider a time sequence $X^{(0)}$ that denotes the price of a product in USD (see Fig. 1)

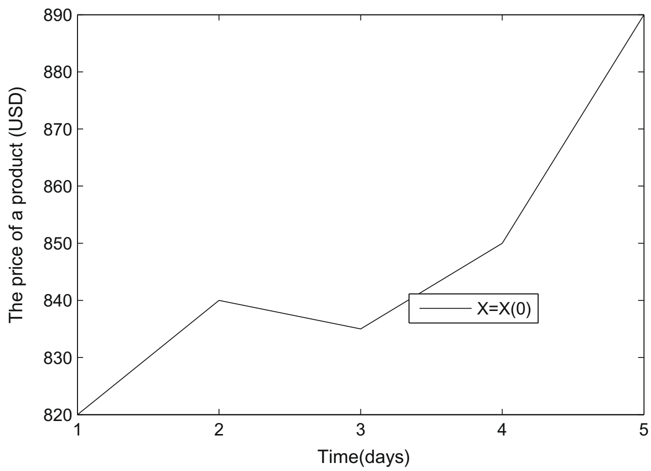

Fig. 1. The original data set.

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)), \quad n \geqslant 4, \tag{1}$$

where $X^{(0)}$ is a non-negative sequence and n is the sample size of the data. When this sequence is subjected to the Accumulating Generation Operation (AGO), the following sequence $X^{(1)}$ is obtained. It is obvious that $X^{(1)}$ is monotonically increasing (see Fig. 2).

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)), \quad n \geqslant 4, \tag{2}$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, 3 \ldots.., n. \tag{3}$$

The generated mean sequence $Z^{(1)}$ of $X^{(1)}$ is defined as:

$$Z^{(1)} = (z^{(1)}(1), z^{(1)}(2), \ldots, z^{(1)}(n)), \tag{4}$$

where $z^{(1)}(k)$ is the mean value of adjacent data, i.e.

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), \quad k = 2, 3, \ldots, n. \tag{5}$$

The least square estimate sequence of the grey difference equation of GM(1,1) is defined as follows (Deng, 1989):

$$x^{(0)}(k) + az^{(1)}(k) = b. \tag{6}$$

The whitening equation is therefore, as follows:

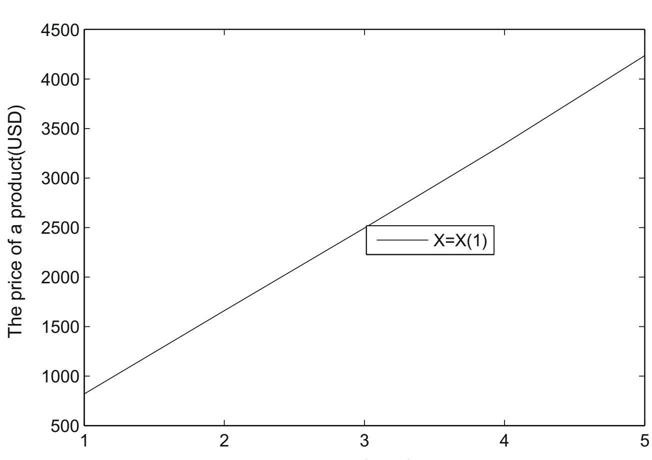

Fig. 2. The accumulated data set.

$$\frac{dx^1(t)}{dt} + ax^{(1)}(t) = b. \tag{7}$$

In above, $[a, b]^T$ is a sequence of parameters that can be found as follows:

$$[a, b]^T = (B^T B)^{-1} B^T Y, \tag{8}$$

where

$$Y = [x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)]^T, \tag{9}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(n) & 1 \end{bmatrix}. \tag{10}$$

According to Eq. (7), the solution of $x^{(1)}(t)$ at time k:

$$x_p^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a}. \tag{11}$$

To obtain the predicted value of the primitive data at time $(k+1)$, the IAGO is used to establish the following grey model.

$$x_p^{(0)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak}(1 - e^a) \tag{12}$$

and the predicted value of the primitive data at time $(k+H)$:

$$x_p^{(0)}(k+H) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-a(k+H-1)}(1 - e^a). \tag{13}$$

2.5. The grey Verhulst model

The Verhulst model was first introduced by a German biologist Pierre Franois Verhulst. The main purpose of Velhulst model is to limit the whole development for a real system and it is effective in describing some increasing processes, such as an *S*-curve which has a saturation region.

The Grey Verhulst model can be defined as (Wen & Huang, 2004):

$$\frac{dx^{(1)}}{dx} + ax^{(1)} = b(x^{(1)})^2. \tag{14}$$

Grey difference equation of Eq. (14) is

$$x^{(0)}(k) + az^{(1)}(k) = b(z^{(1)}(k))^2, \tag{15}$$

$$x^{(0)}(k) = -az^{(1)}(k) + b(z^{(1)}(k))^2. \tag{16}$$

Similar to the GM(1,1) model

$$[a, b]^T = (B^T B)^{-1} B^T Y, \tag{17}$$

where

$$Y = [x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)]^T, \tag{18}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & (z^{(1)}(2))^2 \\ -z^{(1)}(3) & (z^{(1)}(3))^2 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(n) & (z^{(1)}(n))^2 \end{bmatrix}. \tag{19}$$

The solution of $x^{(1)}(t)$ at time k:

$$x_p^{(1)}(k+1) = \frac{ax^{(0)}(1)}{bx^{(0)}(1) + (a - bx^{(0)}(1))e^{ak}}. \tag{20}$$

Applying the IAGO, the solution of $x^{(0)}(t)$ at time k:

$$x_p^{(0)}(k) = \frac{ax^{(0)}(1)(a - bx^{(0)}(1))}{(bx^{(0)}(1) + (a - bx^{(0)}(1))e^{a(k-1)})} * \frac{(1 - e^a)e^{a(k-2)}}{(bx^{(0)}(1) + (a - bx^{(0)}(1))e^{a(k-2)})}. \tag{21}$$

As can be seen, in Eq. (21), if $a < 0$, then

$$\lim_{k\to\infty} x_p^{(1)}(k+1) \to \frac{a}{b}.$$

It means that the saturation point in Eq. (20) is $\frac{a}{b}$ which limits the prediction value. It is also the saturation point of $x_p^{(0)}(k)$ (Wen & Huang, 2004).

When k is sufficiently large, $x_p^{(1)}(k+1)$ and $x_p^{(1)}(k)$ will be very close. Because of this feature of grey Verhulst model, it is commonly used to describe and to predict processes with a saturation region.

2.6. GM(1,1) rolling model

GM(1,1) rolling model is based on the forward data of sequence to build the GM(1,1). For instance, using $x^{(0)}(k), x^{(0)}(k+1), x^{(0)}(k+2)$ and $x^{(0)}(k+3)$, the model predicts the value of the next point $x^{(0)}(k+4)$. In the next steps, the first point is always shifted to the second. It means that $x^{(0)}(k+1), x^{(0)}(k+2), x^{(0)}(k+3)$ and $x^{(0)}(k+4)$ are used to predict the value of $x^{(0)}(k+5)$. This procedure is repeated till the end of the sequence and this method is called rolling check (Wen, 2004).

GM(1,1) rolling model is used to predict the long continuous data sequences such as the outputs of a system, price of a specific product, trend analysis for finance statements or social parameters.

3. Error modification of grey models

In order to improve the modeling accuracy of grey models, several remedies have been discussed in the literature (Tan & Chang, 1996; Tan & Lu, 1996; Guo, Song, & Ye, 2005). In this study, fourier series have been used to modify the grey models.

3.1. Modification of GM(1,1) model using fourier series of error residuals

Consider the $X^{(0)}$ sequence in Eq. (1) and the predicted values given by the GM(1,1):

$$x_p^{(0)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right]e^{-ak}(1 - e^a) \tag{22}$$

then, the error sequence of $X^{(0)}$ can be defined as:

$$\epsilon^{(0)} = (\epsilon^{(0)}(2), \epsilon^{(0)}(3), \ldots, \epsilon^{(0)}(n)), \tag{23}$$

where

$$\epsilon^{(0)}(k) = x^{(0)}(k) - x_p^{(0)}(k), \quad k = 2, 3 \ldots, n. \tag{24}$$

The error residuals in (24) can be expressed in Fourier series as follows:

$$\epsilon^{(0)}(k) \cong \frac{1}{2}a_0 + \sum_{i=1}^{z}\left[a_i \cos\left(\frac{2\pi i}{T}k\right) + b_i \sin\left(\frac{2\pi i}{T}k\right)\right], \quad k = 2, 3 \ldots n, \tag{25}$$

$$T = n - 1 \quad \text{and} \quad z = \left(\frac{n-1}{2}\right) - 1.$$

It is obvious that T will be an integer number and z will be selected as an integer number (Guo et al., 2005).

Eq. (25) can be rewritten as follows:

$$\epsilon^{(0)} \cong PC \tag{26}$$

P and C matrixes can be defined as follows:

$$P = \begin{bmatrix} 1/2 & \cos(2\frac{2\pi}{T}) & \sin(2\frac{2\pi}{T}) & \cos(2\frac{2\pi 2}{T}) & \sin(2\frac{2\pi 2}{T}) & \cdots & \cos(2\frac{2\pi z}{T}) & \sin(2\frac{2\pi z}{T}) \\ 1/2 & \cos(3\frac{2\pi}{T}) & \sin(3\frac{2\pi}{T}) & \cos(3\frac{2\pi 2}{T}) & \sin(3\frac{2\pi 2}{T}) & \cdots & \cos(3\frac{2\pi z}{T}) & \sin(3\frac{2\pi z}{T}) \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1/2 & \cos(n\frac{2\pi}{T}) & \sin(n\frac{2\pi}{T}) & \cos(n\frac{2\pi 2}{T}) & \sin(n\frac{2\pi 2}{T}) & \cdots & \cos(n\frac{2\pi z}{T}) & \sin(n\frac{2\pi z}{T}) \end{bmatrix}, \tag{27}$$

$$C = [a_0 \ a_1 \ b_1 \ a_2 \ b_2 \ \cdots \ a_n \ b_n]^T. \tag{28}$$

One can use least-squares method to solve the Eq. (26), and calculate the matrix C:

$$C \cong (P^T P)^{-1} P^T \epsilon^{(0)}. \tag{29}$$

Fourier series correction can be obtained as follows:

$$x_{p_f}^{(0)}(k) = x_p^{(0)}(k) - \epsilon_p^{(0)}(k), \quad k = 2, 3, \ldots n+1. \tag{30}$$

3.2. Modification of GM(1,1) model using fourier series in time

In order to denote the residual time series, the difference between the real time k and the model fitted $k_p^{(0)}(k)$ is obtained as follows (Guo et al., 2005):

$$q^{(0)} = (q^{(0)}(2), q^{(0)}(3), \ldots q^{(0)}(n)), \tag{31}$$

where

$$q^{(0)}(k) = k - k_p^{(0)}(k), \quad k = 2, 3 \ldots n, \tag{32}$$

$$k_p^{(0)}(k) = 1 - \frac{1}{a}\ln\left(\frac{x_p^{(0)}(k)}{\left[x^{(0)}(1) - \frac{b}{a}\right](1 - e^a)}\right), \quad k = 2, 3 \ldots n. \tag{33}$$

Eq. (32) can be expressed in the Fourier series as follows:

$$q^{(0)}(k) \cong \frac{1}{2}a_0 + \sum_{i=1}^{z}\left[a_i \cos\left(\frac{2\pi i}{T}k\right) + b_i \sin\left(\frac{2\pi i}{T}k\right)\right], \quad k = 2, 3 \ldots n. \tag{34}$$

Similar to the derivations in Section 3.2, Fourier series correction in time domain can be obtained as follows (Guo et al., 2005):

$$k_{p_f}^{(0)}(k) = k - q^{(0)}(k), \quad k = 2, 3, \ldots n. \tag{35}$$

4. Simulation results

4.1. The data

The prediction of the foreign currency exchange rates (FX rates) is a very important topic in financial area. The estimated daily trading volume of FX rates is about 1 trillion US dollars (Hussain, Knowles, Lisboa, & El-Deredy, 2008) On the other hand, it is very difficult to develop good mathematical models and thus make accurate predictions for FX rates, because of the fact that the statistical properties of the data change over time (Magdon-ismail, Nicholson, & Abu-mustafa, 1998).

Time series prediction in financial area is generally very difficult because of the factors listed below (Hussain et al., 2008):

1. The statistical properties of the data change over time (Nonstationary).
2. It is difficult to use mathematical prediction models with linear parameters (Nonlinearity).
3. Random, day-to-day variations (Highly noisy).

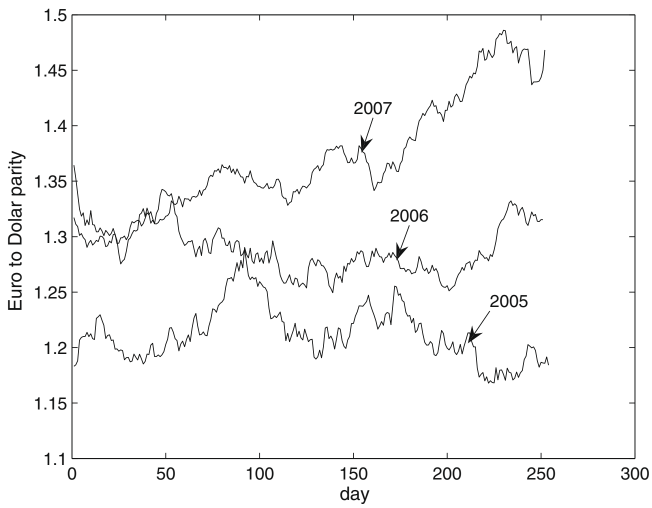

Fig. 3. The data set.

Fig. 3 shows Euro to the United States dollar parity between the dates 01.01.2005 and 30.12.2007. The time series have been formed using the daily rates which are the data points at the end of the each day. It can be seen that the data are highly nonlinear and nonstationary.

4.2. Moving Average Filter (MAF)

MAF is the most common filters in signal processing in order to reduce the random noise because of its simple structure. In this study, a MAF with different window sizes is used to make the primitive data smoother. By this way, the effect of MAFs on different grey models are investigated. The impulse response of the filter used in this study is as follows:

$$h[n] = \frac{1}{5}\delta[n] + \frac{1}{5}\delta[n-1] + \frac{1}{5}\delta[n-2] + \frac{1}{5}\delta[n-3] + \frac{1}{5}\delta[n-4]. \quad (36)$$

4.3. Model accuracy examination

To demonstrate the accuracy of the proposed forecasting models, the actual value $x^{(0)}(k)$ and the forecasted value $x_p^{(0)}(k)$ can be compared.

Eqs. (37)–(39) are the three accuracy evaluation standards that are used to examine the accuracy of the models in this study.

$$\epsilon = x^{(0)}(k) - x_p^{(0)}(k), \quad (37)$$

$$RPE = \frac{|\epsilon(k)|}{x^{(0)}(k)} 100\%, \quad (38)$$

$$ARPE = \frac{1}{n-1} \sum_{k=2}^{n} \frac{|\epsilon(k)|}{x^{(0)}(k)}, \quad (39)$$

where ϵ, RPE and $ARPE$ represent the error, the relative percentage error and the average relative percentage error, respectively.

4.4. The different grey models used in this study

In order to test the accuracies of different grey models, various models are formed in this paper:

- *GM(1,1)*: GM(1,1) model.
- *EFGM*: Modified GM(1,1) model using modeling errors and Fourier series.
- *TFGM*: Modified GM(1,1) model at time domain using Fourier series.
- *GVM*: Grey Verhulst model.
- *EFGVM*: Modified Grey Verhulst model using modeling errors and Fourier series.
- *TFGVM*: Modified Grey Verhulst model at time domain using Fourier series.

4.5. Simulation results

Table 1 shows that GM(1,1) model is better on both interpolation and extrapolation when compared to GVM model without using a filter. Modified GM(1,1) models, TFGM(1,1) and EFGM(1,1), are giving better performances when compared to GM(1,1) model as expected. However, the performance of interpolation has increased more than the performance of extrapolation.

Tables 2 and 3 show that while the performances of GM(1,1) models are decreasing, the performances of GVM models are increasing when a moving average filter (MAF) has been used. However, when the window size of the MAF is increased, the performance of GM(1,1) models has been decreased dramatically. This is because while GVM models give better performances *S*-type

Table 1
The accuracy of the models with GM and GVM window size = 5.

The dates	The standards	GM	EFGM	TFGM	GVM	EFGVM	TFGVM
01.01.2005–01.01.2006	ARPE (%) (Int.)	0.1517	0.0804	0.0804	1.5634	0.8625	0.1682
	ARPE (%) (Ext.)	0.5396	0.5110	0.5110	29.8774	28.9847	4.9612
01.01.2006–01.01.2007	ARPE (%) (Int.)	0.1285	0.0699	0.0699	2.5034	1.4239	0.1418
	ARPE (%) (Ext.)	0.4386	0.4207	0.4206	12.2847	12.1073	6.0883
01.01.2007–01.01.2008	ARPE(%) (Int.)	0.0972	0.0543	0.0543	2.9236	1.8276	0.1052
	ARPE (%) (Ext.)	0.3439	0.3242	0.3241	15.2324	14.5963	5.1995

Table 2
The accuracy of the models MAF window size = 4 and GM and GVM window size = 5.

The dates	The standards	GM	EFGM	TFGM	GVM	EFGVM	TFGVM
01.01.2005–01.01.2006	ARPE (%) (Int.)	0.3413	0.1038	0.3378	2.1249	1.8136	0.3497
	ARPE (%) (Ext.)	0.6105	0.7297	0.5770	4.3561	6.3879	0.9807
01.01.2006–01.01.2007	ARPE (%) (Int.)	0.2741	0.0870	0.2699	1.1888	0.9265	0.2793
	ARPE (%) (Ext.)	0.4806	0.5961	0.4549	4.1610	4.2929	1.7863
01.01.2007–01.01.2008	ARPE (%) (Int.)	0.2175	0.0672	0.2165	0.7365	0.5162	0.2201
	ARPE (%) (Ext.)	0.3903	0.4762	0.3687	8.9730	8.5258	2.6013

Table 3
The accuracy of the models MAF window size = 10 and GM and GVM window size = 5.

The dates	The standards	GM	EFGM	TFGM	GVM	EFGVM	TFGVM
01.01.2005–01.01.2006	ARPE (%) (Int.)	0.5997	0.1153	0.5969	1.094	0.2767	0.6050
	ARPE (%) (Ext.)	0.8207	0.8099	0.8113	6.5260	6.6278	1.7241
01.01.2006–01.01.2007	ARPE (%) (Int.)	0.4657	0.0949	0.4632	0.5979	0.1812	0.4643
	ARPE (%) (Ext.)	0.6284	0.6143	0.6196	3.0732	2.9493	1.2397
01.01.2007–01.01.2008	ARPE (%) (Int.)	0.4117	0.0763	0.4105	0.8473	0.3287	0.4143
	ARPE (%) (Ext.)	0.5609	0.5101	0.5534	1.9357	1.5517	1.1361

Table 4
The accuracy of the models MAF window size = 10 and GM and GVM window size = 20.

The dates	The standards	GM	EFGM	TFGM	GVM	EFGVM	TFGVM
01.01.2005–01.01.2006	ARPE (%) (Int.)	0.7098	0.0817	0.7205	0.8640	0.0961	0.7252
	ARPE (%) (Ext.)	1.0977	1.1850	1.0369	1.7255	1.7582	1.4868
01.01.2006–01.01.2007	ARPE (%) (Int.)	0.5702	0.0590	0.5637	0.7281	0.0760	0.5671
	ARPE (%) (Ext.)	0.9002	0.9476	0.8018	1.5701	1.5638	1.0448
01.01.2007–01.01.2008	ARPE (%) (Int.)	0.4851	0.0480	0.4893	0.5725	0.0596	0.4908
	ARPE (%) (Ext.)	0.7733	0.8030	0.6669	1.2067	1.2309	0.7326

data, GM(1,1) models are good at monotonically increasing data sets.

Table 4 is used to check the performances of GVM models when the filter and prediction window size are increased. In this situation, GMV models give satisfactory performances both in interpolation and extrapolation. On the other hand, GM(1,1) models give dramatically unsuccessful results.

5. Conclusion and future works

This paper compares the performances of the various modified grey models in time series prediction. It is shown that the performance of the grey predictors can be further improved by taking into account the error residuals. Highly noisy data, the United States dollar to Euro parity, are used to show the efficiency of the various error corrected grey models for this purpose. The model accuracy examination results show that GM(1,1) model is able to make accurate predictions for forecasting of the monotonous type of processes. However, the model GM(1,1) cannot give the same performance when the primitive data sequence increases like as in an *S*-curve (like the data with a MAF used in this project) or it has a saturation region. The simulation results show that modified grey models have higher performances not only on model fitting but also on forecasting. Among these grey models, the modified GM(1,1) using Fourier series in time is the best in model fitting and forecasting.

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