Robust Unknown Input Observer for Nonlinear Systems and Its Application to Fault Detection and Isolation

S. Mondal[1]
e-mail: sharifm123@gmail.com

G. Chakraborty
e-mail: goutam@mech.iitkgp.ernet.in

K. Bhattacharyya
e-mail: king@mech.iitkgp.ernet.in

Department of Mechanical Engineering,
Indian Institute of Technology,
Kharagpur-721302,
West Bengal, India

A robust unknown input observer for a nonlinear system whose nonlinear function satisfies the Lipschitz condition is designed based on linear matrix inequality approach. Both noise and uncertainties are taken into account in deriving the observer. A component fault detection and isolation scheme based on these observers is proposed. The effectiveness of the observer and the fault diagnosis scheme is shown by applying them for component fault diagnosis of an electrohydraulic actuator.

[DOI: 10.1115/1.2936857]

1 Introduction

A standard observer fails to estimate the states of a system when it is influenced by noise and uncertainties. Linear matrix inequality (LMI) based observers [1,2] are widely used for state estimation of uncertain and noisy systems. When the measurements of all the input signals are not possible, an unknown input observer (UIO), which is capable of estimating states irrespective of unknown inputs, is used to estimate the states. Researchers have developed different types of UIOs for ideal linear and nonlinear systems [3–8]. Significant research works have been carried out to include noise and uncertainties in the estimation process for the systems with unknown inputs [9–11]. Koenig [11] developed an UIO for nonlinear noisy systems via convex optimization. However, an UIO for a nonlinear system with both noise and uncertainties is still lacking.

The UIOs have become a multipurpose diagnosis tool in the field of fault detection and isolation (FDI) [2,5,7,9,10]. Different types of model based FDI techniques have been developed to diagnose various kinds of faults [1,2,5,7,9,10,12,13]. LMI based observers have been successfully used to diagnose different types of faults of nonlinear systems [1,2,9,10].

In this work, first, a robust UIO for nonlinear systems with noise and uncertainties is presented. The observer is designed for a nonlinear system whose nonlinear function satisfies Lipschitz condition. Second, a component fault detection and isolation (CFDI) technique, which consists of two steps, is developed using these observers. In Step 1, fault is detected and the faulty zone is isolated. In the next step, the faulty parameter is isolated. The main advantages of this FDI technique is that Step 2 is carried out only when a fault occurs in any of the subsystems. So the complexity of fault isolation is significantly reduced in comparison with standard parameter identification based FDI techniques [13] where all the system parameters are estimated at every time instant. The FDI algorithm is applied for detecting component fault of an electrohydraulic actuator. The simulated results show the effectiveness of the observer as well as the FDI technique.

2 Robust Unknown Input Observer

In this section, a robust UIO for nonlinear systems is derived. Consider a nonlinear system

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t) + f(\boldsymbol{x}(t)) + \boldsymbol{G}\boldsymbol{w}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{w}(t) \tag{2}$$

where $\boldsymbol{x}(t) \in \boldsymbol{R}^n$, $\boldsymbol{u}(t) \in \boldsymbol{R}^m$, $\boldsymbol{y}(t) \in \boldsymbol{R}^p$, $\boldsymbol{d}(t) \in \boldsymbol{R}^q$, and $\boldsymbol{w}(t) \in \boldsymbol{R}^r$ are state, input, output, unknown input, and disturbance vectors, respectively. The matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{E}$, and $\boldsymbol{G}$ of suitable dimensions are known and $f(\boldsymbol{x}(t))$ is a nonlinear function. Define spaces Γ_1 and Γ_2 for time-varying uncertainty matrices $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{B}$ as

$$\Gamma_1 = \{\Delta\boldsymbol{A}(t) | \Delta\boldsymbol{A}(t) = \boldsymbol{M}\Sigma(t)\boldsymbol{N}_1, \Sigma(t)^T\Sigma(t) \le \boldsymbol{I}\} \tag{3}$$

and

$$\Gamma_2 = \{\Delta\boldsymbol{B}(t) | \Delta\boldsymbol{B}(t) = \boldsymbol{M}\Sigma(t)\boldsymbol{N}_2, \Sigma(t)^T\Sigma(t) \le \boldsymbol{I}\} \tag{4}$$

The following assumptions are used in designing the observer:

(a) $(\boldsymbol{A} + \Delta\boldsymbol{A})$ is asymptotically stable for $\forall\, \Delta\boldsymbol{A} \in \Gamma_1$

(b) $(\boldsymbol{A},\boldsymbol{C})$ is detectable

$$\text{(c) } \operatorname{rank}(\boldsymbol{C}\boldsymbol{E}) = \operatorname{rank}(\boldsymbol{E}) = q \tag{5}$$

(d) $f(\boldsymbol{x}(t))$ satisfies the Lipschitz condition:

$$\|f(\boldsymbol{x}) - f(\hat{\boldsymbol{x}})\| \le \gamma\|\boldsymbol{x} - \hat{\boldsymbol{x}}\| \tag{6}$$

and

$$\text{(e) there exists a matrix } \boldsymbol{S} \text{ such that } \|f^T(\boldsymbol{x})f(\boldsymbol{x})\| \le \|\boldsymbol{x}^T\boldsymbol{S}^T\boldsymbol{S}\boldsymbol{x}\| \tag{7}$$

An UIO for systems (1) and (2) is described as

$$\dot{z}(t) = \boldsymbol{N}z(t) + \boldsymbol{L}\boldsymbol{y}(t) + \boldsymbol{J}\boldsymbol{u}(t) + \boldsymbol{P}f(\hat{\boldsymbol{x}}(t)) \tag{8}$$

$$\hat{\boldsymbol{x}}(t) = z(t) - \boldsymbol{H}\boldsymbol{y}(t) \tag{9}$$

Assuming $\boldsymbol{e}(t)=\boldsymbol{x}(t)-\hat{\boldsymbol{x}}(t)$, the error dynamics can be written as

$$\dot{\boldsymbol{e}} = \dot{\boldsymbol{x}} - \dot{\hat{\boldsymbol{x}}} = \boldsymbol{P}\dot{\boldsymbol{x}} + \boldsymbol{H}\boldsymbol{D}\dot{\boldsymbol{w}} - \dot{z} \tag{10}$$

$$\text{where } \boldsymbol{P} = \boldsymbol{I}_n + \boldsymbol{H}\boldsymbol{C} \tag{11}$$

Using Eqs. (1), (8), and (9), one gets

$$\dot{\boldsymbol{e}} = \boldsymbol{N}\boldsymbol{e} + (\boldsymbol{P}\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C} - \boldsymbol{N}\boldsymbol{P})\boldsymbol{x} + (\boldsymbol{P}\boldsymbol{B} - \boldsymbol{J})\boldsymbol{u} + \boldsymbol{P}f(\boldsymbol{x}) - \boldsymbol{P}f(\hat{\boldsymbol{x}}) + \boldsymbol{P}\boldsymbol{E}\boldsymbol{d} + \boldsymbol{P}\Delta\boldsymbol{A}\boldsymbol{x} + \boldsymbol{P}\Delta\boldsymbol{B}\boldsymbol{u} + (\boldsymbol{P}\boldsymbol{G} - \boldsymbol{N}\boldsymbol{H}\boldsymbol{D} - \boldsymbol{L}\boldsymbol{D})\boldsymbol{w} + \boldsymbol{H}\boldsymbol{D}\dot{\boldsymbol{w}} \tag{12}$$

Now, it is assumed that the following conditions are satisfied:

$$\boldsymbol{P}\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C} - \boldsymbol{N}\boldsymbol{P} = 0 \tag{13}$$

$$\boldsymbol{P}\boldsymbol{B} - \boldsymbol{J} = 0 \tag{14}$$

and

$$\boldsymbol{P}\boldsymbol{E} = 0 \tag{15}$$

Solving Eqs. (11), (14), and (15), one gets

$$\boldsymbol{H} = -\boldsymbol{E}(\boldsymbol{C}\boldsymbol{E})^+ + \boldsymbol{Y}_a(\boldsymbol{I}_p - (\boldsymbol{C}\boldsymbol{E})(\boldsymbol{C}\boldsymbol{E})^+) \tag{16}$$

[1]Corresponding author.

Contributed by the Dynamic Systems, Measurement, and Control Division of ASME for publication in the Journal of Dynamic Systems, Measurement, and Control. Manuscript received January 29, 2007; final manuscript received January 9, 2008; published online June 9, 2008. Assoc. Editor: Fen Wu.

$$\boldsymbol{P} = \boldsymbol{I}_n + (-\boldsymbol{E}(\boldsymbol{CE})^+ + \boldsymbol{Y}_a(\boldsymbol{I}_p - (\boldsymbol{CE})(\boldsymbol{CE})^+)\boldsymbol{C} \quad (17)$$

$$\boldsymbol{J} = (\boldsymbol{I}_n + (-\boldsymbol{E}(\boldsymbol{CE})^+ + \boldsymbol{Y}_a(\boldsymbol{I}_p - (\boldsymbol{CE})(\boldsymbol{CE})^+)\boldsymbol{C})\boldsymbol{B} \quad (18)$$

where $(\boldsymbol{CE})^+$ is the generalized inverse of $\boldsymbol{CE}$ and in order to preserve the detectability of the pair $(\boldsymbol{PA}, \boldsymbol{C})$, which is equivalent to Assumption (b), an arbitrary matrix $\boldsymbol{Y}_a$ of appropriate dimension is chosen by the designer such that the matrix $\boldsymbol{P}$ is of maximal rank.

From Eqs. (11) and (13), the matrix $\boldsymbol{N}$ can be written as

$$\boldsymbol{N} = \boldsymbol{PA} - \boldsymbol{KC} \quad (19)$$

$$\text{where } \boldsymbol{K} = \boldsymbol{L} + \boldsymbol{NH} \quad (20)$$

Using Eqs. (13)–(20) and assuming $\dot{\boldsymbol{w}} = \boldsymbol{v}$, the error dynamics [12] is rewritten as

$$\dot{\boldsymbol{e}} = (\boldsymbol{PA} - \boldsymbol{KC})\boldsymbol{e} + (\boldsymbol{PG} - \boldsymbol{KD})\boldsymbol{w} + \boldsymbol{HD}v + \boldsymbol{P}(f(\boldsymbol{x}) - f(\hat{\boldsymbol{x}})) + \boldsymbol{P}\Delta\boldsymbol{A}\boldsymbol{x} + \boldsymbol{P}\Delta\boldsymbol{B}\boldsymbol{u} \quad (21)$$

The H_∞-observer problem for a performance level λ $(\in \boldsymbol{R}^+)$ is to find the gain of the observer $\boldsymbol{K}$ that stabilizes asymptotically the state estimation error and ensures the following performance index:

$$J = \int_0^\infty (\boldsymbol{e}^T\boldsymbol{e} - \lambda^2 \boldsymbol{w}_d^T \boldsymbol{w}_d)dt < 0, \quad \forall\, 0 \neq \{\boldsymbol{w}_d(t)\} \quad (22)$$

where $\boldsymbol{w}_d = [\boldsymbol{u}^T\ \boldsymbol{w}^T\ \boldsymbol{v}^T\ \boldsymbol{d}^T]^T$. As the state variable $\boldsymbol{x}(t)$ (which depends on unknown inputs) appears in error dynamics, the unknown input term $\boldsymbol{d}(t)$ is included in $\boldsymbol{w}_d(t)$. To find out the gain matrix $\boldsymbol{K}$, the following theorem is proposed.

THEOREM 1. *If* $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ *are symmetric, positive definite matrices,* $\boldsymbol{Y}$ *is a matrix, the constants* $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, *and* $\varepsilon_3 > 0$ *exist and the LMI* $[\boldsymbol{s}_{ij}]_{11\times 11} < 0$ *holds, then robust UIOs* (8) and (9) *for systems* (1) *and* (2) are *solvable and the observer gain matrix becomes* $\boldsymbol{K} = \boldsymbol{P}_2^{-1}\boldsymbol{y}$ *where* $\boldsymbol{s}_{11} = \boldsymbol{P}_1\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{P}_1 + 2\varepsilon_2 N_2^T N_2 + \varepsilon_1 \boldsymbol{S}^T\boldsymbol{S}$, $\boldsymbol{s}_{13} = \boldsymbol{P}_1\boldsymbol{B}$, $\boldsymbol{s}_{14} = \boldsymbol{P}_1\boldsymbol{G}$, $\boldsymbol{s}_{16} = \boldsymbol{P}_1\boldsymbol{E}$, $\boldsymbol{s}_{19} = \boldsymbol{P}_1$, $\boldsymbol{s}_{1,10} = \boldsymbol{P}_1\boldsymbol{M}$, $\boldsymbol{s}_{1,11} = \boldsymbol{P}_1\boldsymbol{M}$, $\boldsymbol{s}_{22} = (\boldsymbol{P}_2(\boldsymbol{PA}) - \boldsymbol{YC}) + ((\boldsymbol{PA})^T\boldsymbol{P}_2 - \boldsymbol{C}^T\boldsymbol{Y}^T)$, $\boldsymbol{s}_{24} = \boldsymbol{P}_2(\boldsymbol{PG}) - \boldsymbol{YD}$, $\boldsymbol{s}_{25} = \boldsymbol{P}_2(\boldsymbol{HD})$, $\boldsymbol{s}_{27} = \sqrt{2}\boldsymbol{I}$, $\boldsymbol{s}_{28} = \gamma\sigma\boldsymbol{P}_2$, $\boldsymbol{s}_{2,10} = \boldsymbol{P}_2(\boldsymbol{PM})$, $\boldsymbol{s}_{2,11} = \boldsymbol{P}_2(\boldsymbol{PM})$, $\boldsymbol{s}_{33} = -\lambda^2\boldsymbol{I} + 2\varepsilon_3 N_2^T N_2$, $\boldsymbol{s}_{44} = -\lambda^2\boldsymbol{I}$, $\boldsymbol{s}_{55} = -\lambda^2\boldsymbol{I}$, $\boldsymbol{s}_{66} = -\lambda^2\boldsymbol{I}$, $\boldsymbol{s}_{77} = -\boldsymbol{I}$, $\boldsymbol{s}_{88} = -\boldsymbol{I}$, $\boldsymbol{s}_{99} = -\varepsilon_1\boldsymbol{I}$, $\boldsymbol{s}_{10,10} = -\varepsilon_2\boldsymbol{I}$, $\boldsymbol{s}_{11,11} = -\varepsilon_3\boldsymbol{I}$, and $\boldsymbol{s}_{ij} = 0$ *otherwise, with* λ *a positive number*, γ *Lipschitz constant, and* σ*-largest singular value of* $\boldsymbol{P}$.

To prove the theorem, the following lemmas are presented first.

LEMMA 1. *If there exists a scalar* $\varepsilon > 0$ *and a symmetric positive definite matrix* $\boldsymbol{P}$, *then* $(\Delta X)^T\boldsymbol{P} + \boldsymbol{P}(\Delta X) \leqslant \varepsilon^{-1}\boldsymbol{PMM}^T\boldsymbol{P} + \varepsilon\boldsymbol{N}^T\boldsymbol{N}$ *where* $\Delta X = \boldsymbol{M}\Sigma(t)\boldsymbol{N}$ *with* $\Sigma^T(t)\Sigma(t) \leqslant \boldsymbol{I}$.

Proof. This lemma can be proved using simple mathematical relations, here omitted.

LEMMA 2. *If the nonlinear function* $f(\boldsymbol{x})$ *satisfies Eq.* (6), *then for a symmetric, positive definite matrix* $\boldsymbol{P}_s$, *the following inequality holds*:

$$2\boldsymbol{e}^T\boldsymbol{P}_s\boldsymbol{P}(f(\boldsymbol{x}) - f(\hat{\boldsymbol{x}})) \leq \gamma^2\sigma^2\boldsymbol{e}^T\boldsymbol{P}_s\boldsymbol{P}_s\boldsymbol{e} + \boldsymbol{e}^T\boldsymbol{e}$$

Proof. See Ref. [8].

LEMMA 3. *Consider a linear uncertain system*

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = \boldsymbol{Cx}(t) + \boldsymbol{Du}(t)$$

with $\Delta\boldsymbol{A} = \boldsymbol{M}\Sigma(t)N_1$, $\Delta\boldsymbol{B} = \boldsymbol{M}\Sigma(t)N_2$, and $\Sigma^T(t)\Sigma(t) \leqslant \boldsymbol{I}$ *for a given* $\lambda > 0$, *if there exist positive numbers* $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ *and a symmetric positive definite matrix* $\boldsymbol{P}$ *such that the LMI*

$$\begin{bmatrix} \boldsymbol{\Omega} & \boldsymbol{PB} & \boldsymbol{C}^T & \boldsymbol{PM} & \boldsymbol{PM} \\ \boldsymbol{B}^T\boldsymbol{P} & -\lambda^2\boldsymbol{I} + \varepsilon_2 N_2^T N_2 & \boldsymbol{D}^T & 0 & 0 \\ \boldsymbol{C} & \boldsymbol{D} & -\boldsymbol{I} & 0 & 0 \\ \boldsymbol{M}^T\boldsymbol{P} & 0 & 0 & -\varepsilon_1\boldsymbol{I} & 0 \\ \boldsymbol{M}^T\boldsymbol{P} & 0 & 0 & 0 & -\varepsilon_2\boldsymbol{I} \end{bmatrix} < 0$$

with $\boldsymbol{\Omega} = \boldsymbol{PA} + \boldsymbol{A}^t\boldsymbol{P} + \varepsilon_1 N_1^T N_1$ *holds, then the system is robust stable and* H_∞*-norm of the transfer function* $\boldsymbol{G}_{yu}$ *satisfies* $\|\boldsymbol{G}_{yu}\|_\infty < \lambda$, $\forall \Delta\boldsymbol{A} \in \Gamma_1$, and $\forall \Delta\boldsymbol{B} \in \Gamma_2$.

Proof. This lemma can be proved using the bounded real lemma [14], here omitted. Based on the above lemmas, the proof of Theorem 1 can be carried out (see Appendix).

Once observer gain $\boldsymbol{K}$ is found out using Theorem 1, the matrix $\boldsymbol{L}$ is obtained as

$$\boldsymbol{L} = \boldsymbol{K}(\boldsymbol{I}_p + \boldsymbol{CH}) - \boldsymbol{PAH} \quad (23)$$

As all coefficient matrices of the observers (8) and (9) are known, the UIO design is completed.

3 Fault Detection and Isolation Algorithm

In this section, a CFDI algorithm is presented. The FDI technique is devised with the assumptions that sensors and actuators are fault free.

Consider a time invariant nonlinear system

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + f(\boldsymbol{x}(t)) + \boldsymbol{Gw}(t) \quad (24)$$

$$\boldsymbol{y}(t) = \boldsymbol{Cx}(t) + \boldsymbol{Dw}(t) \quad (25)$$

The significance of the vectors and matrices are the same as described earlier.

Suppose a fault occurs in a single parameter of the system. The faulty system can be represented as

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A} + \Delta\boldsymbol{A}_f)\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B} + \Delta\boldsymbol{B}_f)\boldsymbol{u}(t) + f(\boldsymbol{x}(t)) + \Delta f_f(\boldsymbol{x}(t)) + \boldsymbol{Gw}(t) \quad (26)$$

where $\Delta\boldsymbol{A}_f$, $\Delta\boldsymbol{B}_f$, and Δf_f are the faulty parts of system matrix, input matrix, and nonlinear function, respectively.

Equation (26) can be rearranged as

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{Ed}(t) + f(\boldsymbol{x}(t)) + \boldsymbol{Gw}(t) \quad (27)$$

where $\boldsymbol{d}(t)$ is unknown signal and $\boldsymbol{E}$ is a known matrix, which satisfies the relation

$$\boldsymbol{Ed}(t) = \Delta\boldsymbol{A}_f\boldsymbol{x}(t) + \Delta\boldsymbol{B}_f\boldsymbol{u}(t) + \Delta f_f(\boldsymbol{x}(t)) \quad (28)$$

Now, the system is divided into N numbers of subsystems each characterized by few numbers of parameters. The FDI process is carried out in two steps as follows.

Step 1: Detection and partial isolation of fault. Assume that the ith subsystem is faulty. After considering all possible changes in the parameters of the ith subsystem, the system equations are written as

$$\dot{\boldsymbol{x}}_{(i)}(t) = (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}_{(i)}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{E}_{(i)}\boldsymbol{d}_{(i)}(t) + f(\boldsymbol{x}_i(t)) + \boldsymbol{Gw}(t) \quad (29)$$

$$\boldsymbol{y}_{(i)}(t) = \boldsymbol{C}_{(i)}\boldsymbol{x}_{(i)}(t) + \boldsymbol{D}_{(i)}\boldsymbol{w}(t) \quad (30)$$

where subscript (i) indicates that the fault occurs in the ith subsystem only. The matrix $\boldsymbol{E}_{(i)}$ is known and $\boldsymbol{d}_{(i)}(t)$ is unknown signal containing the changes of the system parameters.

As Eqs. (29) and (30) are in the form required to design a robust UIO, an observer is designed to estimate the states $\hat{\boldsymbol{x}}_{(i)}(t)$. The residuals are calculated as

$$\boldsymbol{r}_{(i)}(t) = \boldsymbol{y}_{(i)}(t) - \hat{\boldsymbol{y}}_{(i)}(t) = \boldsymbol{y}_{(i)}(t) - \boldsymbol{C}_{(i)}\hat{\boldsymbol{x}}_{(i)}(t) \quad (31)$$

Since an UIO, if properly designed, can estimate the states irrespective of the unknown inputs, it is obvious that the residual $\boldsymbol{r}_{(i)}(t)$ remains within a small bound, known as threshold value [12], if the fault occurs in the ith subsystem or if there is no fault. Otherwise, the residual crosses the threshold value. The magnitude of threshold values depends on noise, uncertainties, and inputs. Thus N number of robust UIOs can isolate the faulty subsystem. However, $(N-1)$ numbers of such observers are sufficient to isolate a faulty subsystem when $N>2$; as once $(N-1)$ subsystems are found fault free, the remaining subsystem is automatically identified as the faulty one. A decision table can be used to isolate the faulty subsystem.

Once the fault is detected and the faulty subsystem is isolated, the next step is carried out to isolate faulty parameter.

Step 2: Total isolation of fault. In this step, the effects of all the parameters of the faulty subsystem are replaced with an unknown input signal, say, $F_u(t)$, as

$$F_u(t) = f_u(s_u, \boldsymbol{x}(t)) \tag{32}$$

where s_u are the parameters of the faulty subsystem.

The state space model is then found out with $\boldsymbol{d}(t)=F_u(t)$. Now, choosing a suitable $\boldsymbol{C}$, a robust UIO is designed to estimate the states $\hat{\boldsymbol{x}}$. Knowing the states, $F_u(t)$ is estimated from a relation extracted from state equations. Then, $\hat{F}_u(t)$ is used to estimate s_u from Eq. (32).

Let the kth parameter s_{uk} be the faulty one and s_{uk} is estimated using the nominal values of the other parameters as

$$\hat{s}_{uk}(t) = g(s_{u1}, s_{u2}, \ldots, s_{uk-1}, s_{uk+1}, \ldots, s_{ul}, \hat{\boldsymbol{x}}(t), \hat{F}_u(t)) \tag{33}$$

Now, if the assumption is correct, then in steady state the estimated values remain within a bound. Otherwise, the estimated parameter differs widely with time. The moving average technique can be used to reduce the effect of noise and uncertainties in the estimated values. In this way, all the parameters of the faulty subsystem are estimated. Now, as the single fault case is considered, there will be only one case where estimated values remain within a small bound. The particular parameter for which this phenomenon appears is the faulty one. In this way, any parametric fault in any subsystem can be isolated.

4 Numerical Example

In this section, the proposed FDI technique is applied to detect and isolate the fault of an electrohydraulic actuator [15]. The system equation for the actuator can be written in state space form as

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} + \Delta \boldsymbol{A})\boldsymbol{x} + (\boldsymbol{B} + \Delta \boldsymbol{B})\boldsymbol{u} + f(\boldsymbol{x}) + \boldsymbol{G}w \tag{34}$$

$$\text{with } \boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1}{M_1} & -\frac{C_1}{M_1} & \frac{A_r}{M_1} & 0 \\ 0 & -\alpha & -\beta & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_v}{\tau} \end{bmatrix}, \quad \text{and } f(\boldsymbol{x}) = \begin{bmatrix} 0 \\ 0 \\ (\kappa\sqrt{P_s - \text{sgn}(\boldsymbol{x}_4)\boldsymbol{x}_3})\boldsymbol{x}_4 \\ 0 \end{bmatrix} \tag{35}$$

where $\boldsymbol{x}_1$ is the actuator piston position, $\boldsymbol{x}_2$ is the actuator piston velocity, $\boldsymbol{x}_3$ is the load pressure, $\boldsymbol{x}_4$ is the valve position, and $\boldsymbol{u}$ is the input current to servo valve. The numerical values of the different parameters [15] are listed in Table 1.

Table 1 Numerical values of the system parameters

Parameters	Numerical values
Mass, M_1	24 kg
Stiffness, K_1	16,010 N/m
Damping coefficient, C_1	310 N s/m
Constant, α	1.513×10^{10} N/m^3
Constant, β	1.0 1/s
Constant, κ	8.0×10^8
Actuator ram area, A_r	3.2673×10^{-4} m^2
Supply pressure, P_s	1.0344×10^7 N/m^2
Valve time constant, τ	0.0017 s
Valve gain, K_v	0.0017

Now, a fault is introduced in K_1. It is assumed that the magnitude of K_1 changes from 16,010 N/m to 8810 N/m at $t=30$ s. The FDI technique is now carried out.

The system is divided into three subsystems as SS1: K_1 and C_1; SS2: α, β, P_s, and κ; and SS3: K_v and τ.

It may be noted that some parameters (e.g., mass, actuator ram area, etc.) are less prone to faults than the others. These are considered as constant parameters. As there are three subsystems, so two observers are sufficient for Step 1.

Step 1. The observers are designed for SS1 and SS2 with the following unknown input matrices and signals $\boldsymbol{E}_{(1)}=[0\ 1\ 0\ 0]^T$, $\boldsymbol{E}_{(2)}=[0\ 0\ 1\ 0]^T$, $\boldsymbol{d}_{(1)}=-((\Delta K_1)_f/M_1)\boldsymbol{x}_{1(1)}-((\Delta C_1)_f/M_1)\boldsymbol{x}_{2(1)}$, and $\boldsymbol{d}_{(2)}=-(\Delta\alpha)_f\boldsymbol{x}_{2(2)}-(\Delta\beta)_f\boldsymbol{x}_{3(2)}+((\Delta\kappa)_f\sqrt{(\Delta P_s)_f-\text{sgn}(\boldsymbol{x}_{4(2)})\boldsymbol{x}_{3(2)}})$ $\times(\boldsymbol{x}_{4(2)})$. The term $(\Delta\cdots)_f$ is the faulty part of the parameter. The system and input matrices are the same as represented in Eq. (35). The uncertainty matrices and signals are $\boldsymbol{M}=\left[\begin{smallmatrix} 0 & \boldsymbol{\phi}_{1\times 3} \\ \boldsymbol{\phi}_{3\times 1} & \boldsymbol{I}_{3\times 3}\end{smallmatrix}\right]$, $\boldsymbol{N}_1=0.05\tilde{\boldsymbol{A}}$, $\boldsymbol{N}_2=0.05\boldsymbol{B}$, and $\Sigma(t)=\Sigma_0\sin(wt)$ with $\Sigma_0=\boldsymbol{I}$ and $w=1$ rad/s, where $\tilde{\boldsymbol{A}}=\boldsymbol{A}_{(a_{12}=0)}$. The input signal is chosen as $\boldsymbol{u}(t)=u_0\sin(w_1 t)$ with $u_0=20$ A and $w_1=1$ rad/s.

The other matrices are taken as follows: $\boldsymbol{G}_{(1)}=\boldsymbol{G}_{(2)}=[0.3\ 0.45\ 5\ 0.6]^T$, $\boldsymbol{D}_{(1)}=\boldsymbol{D}_{(2)}=[3\ 0.4]^T$, $\boldsymbol{C}_{(1)}=\left[\begin{smallmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1\end{smallmatrix}\right]$, and $\boldsymbol{C}_{(2)}=\left[\begin{smallmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0\end{smallmatrix}\right]$.

Two observers are designed to estimate the states $\hat{\boldsymbol{x}}_{(1)}$ and $\hat{\boldsymbol{x}}_{(2)}$. The residuals are plotted in Figs. 1 and 2. To find out the occurrence of a fault, suitable threshold values should be chosen. There are different ways of choosing threshold values. Here, the fixed threshold values $\varepsilon_{(1)}=\{0.011\ 0.014\}^T$ units and $\varepsilon_{(2)}=\{0.09\ 0.010\}^T$ units are chosen.

As $\boldsymbol{r}_{(1)}$ stays within $\varepsilon_{(1)}$ while $\boldsymbol{r}_{(2)}$ crosses $\varepsilon_{(2)}$, so the occurrence of fault is confirmed. To isolate the faulty subsystem, a decision table is drawn, as shown in Table 2.

It is seen from the decision table that fault is in SS1. The next step is now carried out to isolate the faulty parameter.

Step 2. The effect of all parameters of the faulty subsystem is replaced with an unknown force $F_u(t)$ as

$$F_u(t) = K_1\boldsymbol{x}_1(t) + C_1\boldsymbol{x}_2(t) \tag{36}$$

The system matrix, unknown input matrix, and signal become

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{A_r}{M_1} & 0 \\ 0 & -\alpha & -\beta & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, \quad \boldsymbol{E} = \begin{bmatrix} 0 \\ -\frac{1}{M_1} \\ 0 \\ 0 \end{bmatrix}, \quad \text{and } \boldsymbol{d} = F_u \tag{37}$$

The other matrices and signals are same as in the previous step. The output matrix is taken as $\boldsymbol{C}=\left[\begin{smallmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0\end{smallmatrix}\right]$. An observer is de-

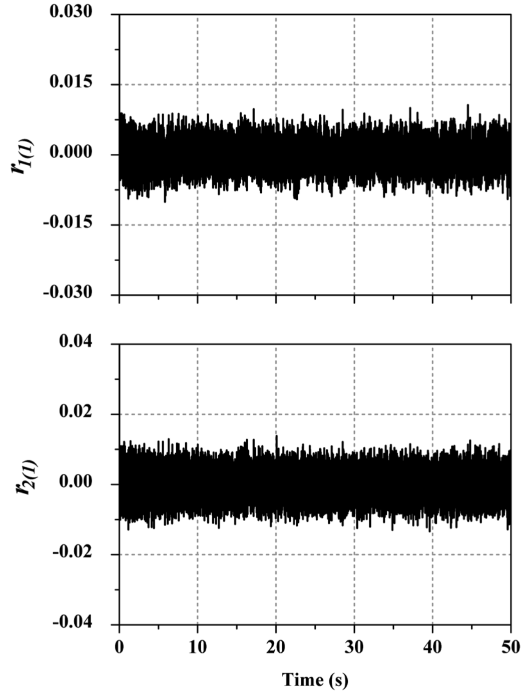

Fig. 1 Components of the residual $r_{(1)}(t)$

signed to estimate $\hat{x}(t)$. Then, $F_u(t)$ is estimated as

$$\hat{F}_u(t) = M_1\dot{\hat{x}}_2 - A_r\hat{x}_3 \tag{38}$$

where $\dot{\hat{x}}_2$ is calculated by differentiating $\hat{x}_2$ with respect to time.

Now, $\hat{F}_u(t)$ is used to estimate the suspected parameters. The estimated moving averaged values of K_1 and C_1 are plotted in Figs. 3 and 4. It can be seen that $\hat{K}_1$ varies within a small range (maximum variation of 17% from its mean value) while $\hat{C}_1$ varies widely (as large as 310% from its mean value). So the stiffness element is the faulty one. With this, the isolation process is completed.

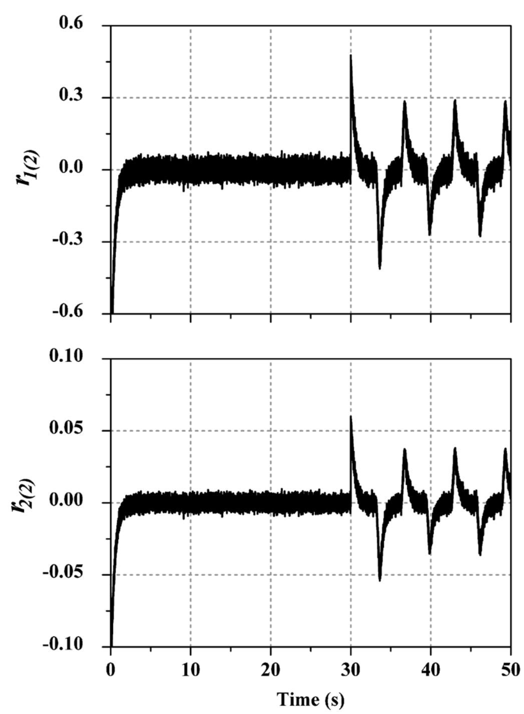

Fig. 2 Components of the residual $r_{(2)}(t)$

Table 2 Decision table

Residuals	Is $r_{(i)} > \varepsilon_{(i)}$?	Decisions	Remarks
$r_{(1)}$	No	Fault may be in SS1	Check $r_{(2)}$
$r_{(2)}$	Yes	Fault is in SS1	Go for Step 2

5 Conclusions

A robust UIO for nonlinear systems is presented. The observer is designed based on the LMI approach considering both noise and uncertainties of a nonlinear system whose nonlinear function satisfies Lipschitz condition. These observers may be useful in the field of robust control and fault diagnosis. Based on these observers, a component FDI algorithm is developed. The effectiveness of the observer as well as the FDI algorithm is shown with a numerical example.

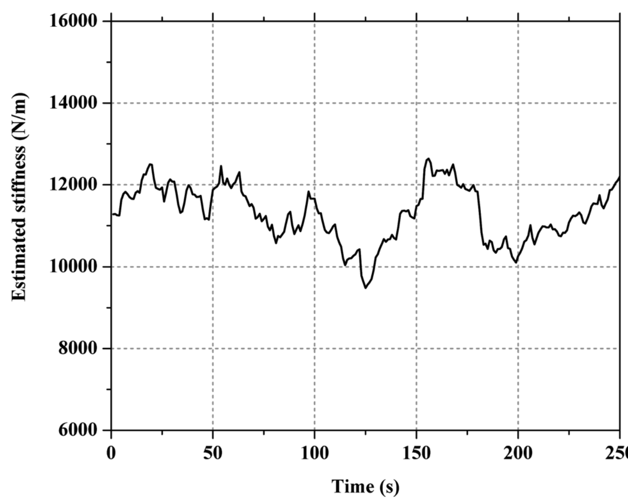

Fig. 3 Estimated stiffness ($\hat{K}_1$)

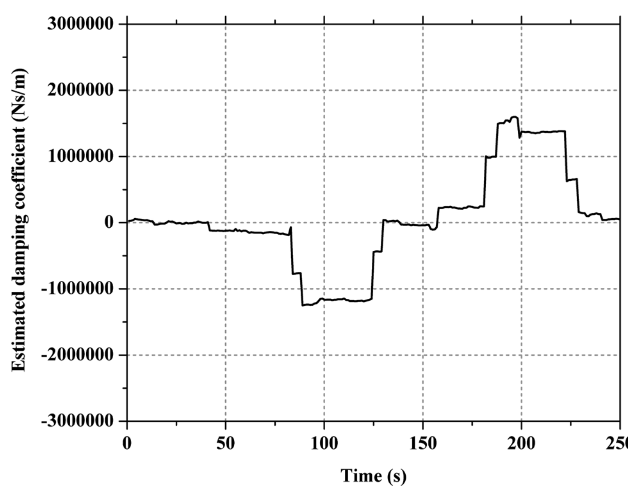

Fig. 4 Estimated damping coefficient ($\hat{C}_1$)

Appendix: Proof of Theorem 1

Consider the following Lyapunov candidate function:

$$V(t)=\boldsymbol{x}^T(t)\boldsymbol{P}_1\boldsymbol{x}(t)+\boldsymbol{e}^T(t)\boldsymbol{P}_2\boldsymbol{e}(t) \quad \text{where } \boldsymbol{P}_1>0 \quad \text{and } \boldsymbol{P}_2>0$$

In order to establish the sufficient conditions of the existence of observers (8) and (9), the Lyapunov method is applied. It requires that $\dot{V}$ is strictly negative to guarantee the asymptotic stability of system (21) and that implies

$$\dot{V}+\boldsymbol{e}^T\boldsymbol{e}-\lambda^2\{\boldsymbol{u}^T \quad \boldsymbol{w}^T \quad \boldsymbol{v}^T \quad \boldsymbol{d}^T\}\{\boldsymbol{u} \quad \boldsymbol{w} \quad \boldsymbol{v} \quad \boldsymbol{d}\}^T<0$$

Now, using relations (1) and (21) along with Lemmas 1 and 2, we get

$$\dot{V}+\boldsymbol{e}^T\boldsymbol{e}-\lambda^2\{\boldsymbol{u}^T \quad \boldsymbol{w}^T \quad \boldsymbol{v}^T \quad \boldsymbol{d}^T\}\{\boldsymbol{u} \quad \boldsymbol{w} \quad \boldsymbol{v} \quad \boldsymbol{d}\}^T$$
$$\leq \begin{Bmatrix} \boldsymbol{x} \\ \boldsymbol{e} \\ \boldsymbol{u} \\ \boldsymbol{w} \\ \boldsymbol{v} \\ \boldsymbol{d} \end{Bmatrix}^T \begin{bmatrix} \Omega_1+\Lambda_1 & 0 & \boldsymbol{P}_1\boldsymbol{B} & \boldsymbol{P}_1\boldsymbol{G} & 0 & \boldsymbol{P}_1\boldsymbol{E} \\ 0 & \Omega_2+\Lambda_2 & 0 & \boldsymbol{P}_2(\boldsymbol{PG}-\boldsymbol{KD}) & \boldsymbol{P}_2(\boldsymbol{HD}) & 0 \\ \boldsymbol{B}^T\boldsymbol{P}_1 & 0 & -\lambda^2 I+2\varepsilon_2 N_2^T N_2 & 0 & 0 & 0 \\ \boldsymbol{G}^T\boldsymbol{P}_1 & (\boldsymbol{PG}-\boldsymbol{KD})^T\boldsymbol{P}_2 & 0 & -\lambda^2\boldsymbol{I} & 0 & 0 \\ 0 & (\boldsymbol{HD})^T\boldsymbol{P}_2 & 0 & 0 & -\lambda^2\boldsymbol{I} & 0 \\ \boldsymbol{E}^T\boldsymbol{P}_1 & 0 & 0 & 0 & 0 & -\lambda^2\boldsymbol{I} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x} \\ \boldsymbol{e} \\ \boldsymbol{u} \\ \boldsymbol{w} \\ \boldsymbol{v} \\ \boldsymbol{d} \end{Bmatrix}$$

where $\Omega_1=\boldsymbol{P}_1\boldsymbol{A}+\boldsymbol{A}^T\boldsymbol{P}_1+\varepsilon_1\boldsymbol{S}^T\boldsymbol{S}+2\varepsilon_2 N_1^T N_1$, $\Lambda_1=\varepsilon_1^{-1}\boldsymbol{P}_1\boldsymbol{P}_1+\varepsilon_2^{-1}\boldsymbol{P}_1\boldsymbol{M}^T\boldsymbol{M}\boldsymbol{P}_1+\varepsilon_3^{-1}\boldsymbol{P}_1\boldsymbol{M}^T\boldsymbol{M}\boldsymbol{P}_1$, $\Omega_2=\boldsymbol{P}_2(\boldsymbol{PA}-\boldsymbol{KC})+(\boldsymbol{PA}-\boldsymbol{KC})^T\boldsymbol{P}_2$, and $\Lambda_2=\gamma^2\sigma^2\boldsymbol{P}_2\boldsymbol{P}_2+\varepsilon_2^{-1}\boldsymbol{P}_2(\boldsymbol{PM})^T((\boldsymbol{PM})\boldsymbol{P}_2)+\varepsilon_3^{-1}\boldsymbol{P}_2(\boldsymbol{PM})^T(\boldsymbol{PM})\boldsymbol{P}_2+2\boldsymbol{I}$.

In order to satisfy Lyapunov stability criteria, the following condition should hold:

$$\begin{bmatrix} \Omega_1+\Lambda_1 & 0 & \boldsymbol{P}_1\boldsymbol{B} & \boldsymbol{P}_1\boldsymbol{G} & 0 & \boldsymbol{P}_1\boldsymbol{E} \\ 0 & \Omega_2+\Lambda_2 & 0 & \boldsymbol{P}_2(\boldsymbol{PG}-\boldsymbol{KD}) & \boldsymbol{P}_2(\boldsymbol{HD}) & 0 \\ \boldsymbol{B}^T\boldsymbol{P}_1 & 0 & -\lambda^2\boldsymbol{I}+2\varepsilon_2 N_2^T N_2 & 0 & 0 & 0 \\ \boldsymbol{G}^T\boldsymbol{P}_1 & (\boldsymbol{PG}-\boldsymbol{KD})^T\boldsymbol{P}_2 & 0 & -\lambda^2\boldsymbol{I} & 0 & 0 \\ 0 & (\boldsymbol{HD})^T\boldsymbol{P}_2 & 0 & 0 & -\lambda^2\boldsymbol{I} & 0 \\ \boldsymbol{E}^T\boldsymbol{P}_1 & 0 & 0 & 0 & 0 & -\lambda^2\boldsymbol{I} \end{bmatrix}<0$$

Now, using Schur complement [14], the above nonlinear matrix inequality can be written into a linear matrix inequality form which follows the form stated in theorem. Lemma 3 ensures the robust stability and guarantees that the H_∞-norm of the transfer function $[G_{eu}\ G_{ew}\ G_{ev}\ G_{ed}]_\infty<\lambda$. This completes the proof of the theorem.

References

[1] Rajamani, R., and Ganguli, A., 2004, "Sensor Fault Diagnosis for a Class of Non-Linear Systems Using Linear Matrix Inequalities," Int. J. Control, **77**(10), pp. 920–930.

[2] Wang, H. B., Wang, J. L., and Lam, J., 2007, "Robust Fault Detection Observer Design: Iterative LMI Approaches," ASME J. Dyn. Syst., Meas., Control, **129**, pp. 77–82.

[3] Chen, W., and Saif, M., 2006, "Unknown Input Observer Design for a Class of Nonlinear Systems: an LMI Approach," *Proceedings of the American Control Conference*, Minneapolis, MN, pp. 834–838.

[4] Darouach, M., Zasadzinski, M., and Xu, S. J., 1994, "Full Order Observers for Linear Systems With Unknown Inputs," IEEE Trans. Autom. Control, **39**(2), pp. 606–609.

[5] Koenig, D., and Mammar, S., 2001, "Design of a class of Reduced Order Unknown Inputs Nonlinear Observer for Fault Diagnosis," *Proceedings of the American Control Conference*, Arlington, VA, pp. 2143–2147.

[6] Pertew, A. M., Marquez, H. J., and Zhao, Q., 2005, "H_∞ Synthesis of Unknown Input Observers for Non-Linear Lipschitz Systems," Int. J. Control, **78**(15), pp. 1155–1165.

[7] Seliger, R., and Frank, P. M., 1991, "Robust Component Fault Detection and Isolation in Nonlinear Dynamic Systems Using Nonlinear Unknown Input Observer," *Proceedings of the IFAC/IMACS Symposium SAFEPROCESS'91*, Baden-Baden, Germany, pp. 313–318.

[8] Mondal, S., Chakraborty, G., and Bhattacharyya, K., 2007, "A Full Order Luenberger-Like Observer for Nonlinear Systems With Unknown Inputs," *Proceedings of ACODS'2007*, Bangalore, India, pp. 117–120.

[9] Zhong, M., Ding, S. X., Bingyong, T., Jeinsch, T., and Sader, M., 2002, "An LMI Approach to Design Robust Fault Detection Observers," *Proceedings of the Forth World Congress on Intelligent Control and Automation*, Shanghai, P.R.C., pp. 2705–2709.

[10] Zhong, M., Ding, S. X., Lam, J., and Wang, H., 2003, "An LMI Approach to Design Robust Fault Detection Filter for Uncertain LTI Systems," Automatica, **39**(3), pp. 543–550.

[11] Koenig, D., 2006, "Observer Design for Unknown Input Nonlinear Descriptor Systems Via Convex Optimization," IEEE Trans. Autom. Control, **51**(6), pp. 1047–1052.

[12] Frank, P. M., 1994, "On-Line Fault Detection in Uncertain Nonlinear Systems Using Diagnosis Observers: A Survey," Int. J. Syst. Sci., **25**(12), pp. 2155–2166.

[13] Bloch, G., Ouladsine, M., and Thomas, P., 1995, "On-Line Fault Diagnosis of Dynamic Systems Via Robust Parameter Estimation," Control Eng. Pract., **3**(12), pp. 1709–1717.

[14] Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V., 1994, *Linear Matrix Inequalities in Systems and Control Theory*, SIAM, Philadelphia, PA.

[15] Liu, R., and Alleyne, A., 2000, "Nonlinear Force/Pressure Tracking of an Electro-Hydraulic Actuator," ASME J. Dyn. Syst., Meas., Control, **122**, pp. 232–237.