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Neural network-based adaptive output feedback formation control for multi-agent systems

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Abstract This paper investigates the problem of output feedback formation tracking control for second-order multi-agent systems under an undirected connected graph and in the presence of dynamic uncertainties and bounded external disturbances. Two state tracking error measures (i.e., absolute and relative state tracking errors) are considered for each individual agent in the formation, and linear reduced-order observers are constructed based on the lumped state tracking errors which include absolute and relative state tracking errors. Chebyshev neural networks are used to approximate unknown nonlinear function in the agent dynamics on-line, and the implementation of the basis functions of Chebyshev neural networks depends only on the desired signals. The smooth projection algorithm is applied to guarantee that the estimated parameters remain in some known bounded sets. Numerical simulations are presented to illustrate the performance of the proposed controller.

Keywords Multi-agent systems · Formation control · Output feedback · Chebyshev neural network

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1 Introduction

Cooperative control for multi-agent systems has received great research interest in the past few years, and it has been applied in several diverse areas, such as multiple robots [1–4], unmanned air vehicles [5], autonomous underwater vehicles [6], and spacecraft [7–15]. Various strategies and approaches, which can be categorized according to their control architectures as leader–follower, virtual structure, and behavioral [16], have been proposed for the formation control. Most of the existing approaches on formation control of multi-agent systems require the assumption that all states of the agents are available for feedback and exchange among neighboring agents. In [9], a passivity-based formation control law, which is model-independent and requires a bidirectional ring topology, has been presented for maintaining attitude alignment among a group of spacecraft without velocity measurements. Ren [12] extended the work of Lawton and Beard [9] to the case of a general undirected connected communication topology, where modified Rodrigues parameters were used for the attitude representation. However, external disturbances were not taken into consideration in [9] and [12]. Abdessameud and Tayebi [15] proposed two velocity-free attitude coordination control schemes for a group of spacecraft, where exact knowledge of the spacecraft attitude dynamics was assumed to be known and external disturbances were not considered.

In practical applications, various uncertainties are acting on the agents due to imprecise measurements and external disturbances. In the current literature, most of the existing work on formation control for multi-agent systems has not taken these uncertainties into consideration. The presence of uncertainties in the multi-agent system makes the problem of output feedback formation control of a group of agents more challenging; therefore, it is highly desirable to develop a decentralized output feedback formation control approach for multi-agent systems in the presence of uncertain system dynamics and bounded external disturbances. Neural networks (NN) and/or fuzzy logic systems (FLS) have the capability to approximate any smooth functions over a compact set to arbitrary accuracy [17–21]. NN and FLS are very powerful techniques for control of systems when there are large uncertainties and strong nonlinearities. Hou et al. proposed a neural-network-based decentralized control algorithm for leaderless consensus control of multi-agent systems [22]. Cheng et al. presented a neural-network-based adaptive controller for leader–follower control of second-order multi-agent systems with uncertain dynamics and external disturbances [23]. An adaptive neural controller was proposed for consensus tracking control of second-order multi-agent systems in the presence of unknown nonlinearities and disturbances [24], and a neural-network-based distributed tracking control scheme was developed for unknown networked Lagrangian systems [25]. Dierks and Jagannathan studied the leader–follower formation control of mobile robots using neural networks [26]. In these works, full state measurements were assumed to be available to feedback and exchange among neighboring agents.

Chebyshev neural network (CNN) is a functional link network (FLN) whose input is generated by using a subset of Chebyshev polynomials, and it has been shown that CNN has powerful representation capabilities [21]. CNN has been applied for attitude control of a single spacecraft in the presence of both structured and unstructured uncertainties [27, 28].

This paper investigates the problem of output feedback formation tracking control for a class of second-order multi-agent systems in the presence of dynamic uncertainties and bounded external disturbances. Here the term *formation tracking* means that a group of agents tracks a time-varying reference trajectory while maintaining a certain desired geometric formation simultaneously. It is to be noted that the most notable gap of the existing approaches is the lack of a decentralized controller that could provide output feedback formation control for a group of agents in the presence of uncertain dynamics and bounded external disturbances; therefore, the research presented here focuses on the development of a decentralized output feedback controller for a group of agents to achieve the high-precision formation tracking performance, even in the presence of uncertain dynamics and bounded external disturbances. In contrast to the existing NN-based methods [22–25], the proposed approach does not require full state measurements; hence, the proposed method can reduce both the cost related to the onboard sensors and the communication requirements among neighboring agents.

This paper is organized as follows. Multi-agent systems, graph theory and Chebyshev neural network are briefly described in Sect. 2. Section 3 proposes a decentralized adaptive output feedback control algorithm for formation tracking control of second-order multi-agent systems using CNNs. Simulation results are presented in Sect. 4, and conclusions are drawn in Sect. 5.

2 Preliminaries

2.1 Multi-agent systems

Consider a class of multi-agent systems in which the ith ($i = 1, 2, \dots, n$) agent is described by the following second-order differential equation:

$$\dot{x}_i = v_i \tag{1}$$

$$\dot{v}_i = f_i(x_i, v_i) + u_i + \vartheta_i \tag{2}$$

$$y_i = x_i \tag{3}$$

where n denotes the total number of the agents, $x_i \in R^m$ and $v_i \in R^m$ are state vectors of the ith agent, $f_i(x_i, v_i) \in R^m$ represents the uncertain dynamics of the ith agent, which is assumed to be a smooth nonlinear function, $u_i \in R^m$ denotes the control input vector of the ith agent, $\vartheta_i \in R^m$ the bounded external disturbance, and $y_i \in R^m$ is the output vector of the ith agent. In this paper, it is considered that only the output signals y_i can be available for feedback and exchange among neighboring agents.

Let $x_d \in R^m$ be the desired trajectory for the formation of the multi-agent system and $h_i \in R^m$

$(i = 1, 2, \ldots, n)$ be some constant. The control objective is to design an output feedback control law for u_i such that the tracking error $x_i - (h_i + x_d)$ is as small as possible, even in the presence of dynamic uncertainties and external disturbances. Note that the vector $h = (h_1^T, h_2^T, \ldots, h_n^T)^T \in R^{nm}$ defines the basic frame of the expected formation. If $h_1 = h_2 = \cdots = h_n = 0$, then the problem becomes a consensus tracking problem, i.e., a group of agents reaches an agreement on a time-varying reference trajectory. To facilitate the controller design, the assumptions with respect to the desired trajectory x_d and external disturbances ϑ_i are stated as follows:

Assumption 1 The reference trajectory x_d, its first and second derivatives are considered to be in a compact set $\Omega_d \in R^{3m}$ defined by

$$\Omega_d \equiv \left\{(x_d, \dot{x}_d, \ddot{x}_d) \mid \|x_d\|^2 + \|\dot{x}_d\|^2 + \|\ddot{x}_d\|^2 \le c_d\right\} \tag{4}$$

where c_d is a positive constant.

Assumption 2 The disturbance ϑ_i $(i = 1, 2, \ldots, n)$ is bounded such that $\|\vartheta_i\| \le \vartheta_{Mi}$, where ϑ_{Mi} is a positive constant.

2.2 Graph theory

In this paper, the topology of the information flow between individual agents is described by an undirected connected graph. Let $G = (\Upsilon, E, A)$ be a weighted graph, where $\Upsilon = \{\upsilon_1, \upsilon_2, \ldots, \upsilon_n\}$ denotes the set of nodes, $E \subseteq \Upsilon \times \Upsilon$ the set of edges, and $A = (a_{ij}) \in R^{n\times n}$ the weighted adjacency matrix of the graph G with nonnegative elements. Node υ_i represents the ith agent, and an edge in G is denoted by an unordered pair (i, j). The pair $(i, j) \in E$ if and only if there is an information exchange between the ith agent and the jth agent, i.e., $(i, j) \in E \Leftrightarrow (j, i) \in E$. The adjacency element a_{ij} denotes the communication quality between the ith agent and the jth agent, i.e., $(i, j) \in E \Leftrightarrow a_{ij} > 0$. It is assumed that $a_{ij} = a_{ji}$ in this paper; that is, the weighted adjacency matrix A is a symmetric matrix.

Let $D = \mathrm{diag}(d_1, d_2, \ldots, d_n)$ be the degree matrix of G, where $d_i = \sum_{j=1}^{n} a_{ij}$, $i = 1, 2, \ldots, n$. Then the Laplacian matrix L of the weighted graph G is defined by

$$L = D - A \tag{5}$$

For any two nodes i and j, if there exists a path between them, then G is called a connected graph. The lemma related to the Laplacian matrix and the graph theory is stated as follows:

Lemma 1 ([29]) *If $G = (\Upsilon, E, A)$ is an undirected connected graph, then the graph Laplacian matrix L is a symmetric matrix with n real eigenvalues in an ascending order*:

$$0 = \lambda_1 < \lambda_2 \le \lambda_3 \le \cdots \le \lambda_n \le 2d_M \tag{6}$$

where $d_M = \max_{1\le i\le n}\{d_i\}$ is the maximum degree *of a graph, and λ_2 is called the algebraic connectivity of a graph. Furthermore, $L1_n = 0_n$, where $1_n = (1, \ldots, 1)^T \in R^n$ and $0_n = (0, \ldots, 0)^T \in R^n$.*

2.3 Chebyshev neural network

The single-layer CNN, which is an FLN based on Chebyshev polynomials, is considered as the function approximator in this paper. Chebyshev polynomials are a set of orthogonal polynomials derived from the solution of the Chebyshev differential equation, and Chebyshev polynomials can be obtained by using the so-called two-term recursive formula as follows:

$$T_{i+1}(x) = 2xT_i(x) - T_{i-1}(x), \quad T_0(x) = 1 \tag{7}$$

where $x \in R$, and $T_1(x) \in R$ has several definitions, such as x, $2x$, $2x - 1$ and $2x + 1$. Here, $T_1(x)$ is considered to be x. For a given vector $X = (x_1, x_2, \ldots, x_m)^T \in R^m$, an enhanced pattern using Chebyshev polynomials is given by

$$\xi(X) = \big(1, T_1(x_1), \ldots, T_N(x_1), \ldots, T_1(x_m), \ldots, T_N(x_m)\big)^T \in R^{mN+1} \tag{8}$$

where $T_i(x_j)$ $(i = 1, \ldots, N;\ j = 1, \ldots, m)$ represents a Chebyshev polynomial, N denotes the order of the Chebyshev polynomials, and $\xi(X)$ is called Chebyshev polynomial basis function.

The CNN has been shown to be capable of approximating any smooth functions over a compact set to arbitrary accuracy. A general nonlinear function $F(x) \in R^m$ with $x \in R^m$ can be approximated by the CNN as

$$F(x) = W^*\xi(x) + \varepsilon \tag{9}$$

where ε is the bounded CNN approximation error, and W^* is the optimal weight matrix of the CNN.

3 Decentralized adaptive output feedback formation controller design using CNNs

This section proposes a decentralized adaptive output feedback formation controller using CNNs for multi-agent systems in the presence of unknown system dynamics and bounded external disturbances. The proposed control law can drive a group of agents to a time-varying reference trajectory while maintaining a certain desired geometric formation simultaneously.

3.1 State error

Here, two state tracking error measures, i.e., the absolute and relative state tracking errors, are considered for each individual agent in the formation. The absolute state tracking errors of the ith ($i = 1, 2, \dots, n$) agent are defined as:

$$e_{xi} = x_i - x_d - h_i \tag{10}$$

$$e_{vi} = v_i - \dot{x}_d \tag{11}$$

Using (1) and (2), the dynamic equations for the absolute state tracking errors of the ith agent are obtained:

$$\dot{e}_{xi} = e_{vi} \tag{12}$$

$$\dot{e}_{vi} = -\ddot{x}_d + f_i(x_i, v_i) + u_i + \vartheta_i \tag{13}$$

Defining

$$\begin{aligned} e_x &= \left(e_{x1}^T, e_{x2}^T, \dots, e_{xn}^T\right)^T \\ e_v &= \left(e_{v1}^T, e_{v2}^T, \dots, e_{vn}^T\right)^T \\ F &= \left(f_1^T, f_2^T, \dots, f_n^T\right)^T \\ X_d &= \left(\ddot{x}_d^T, \ddot{x}_d^T, \dots, \ddot{x}_d^T\right)^T \\ u &= \left(u_1^T, u_2^T, \dots, u_n^T\right)^T \\ \vartheta &= \left(\vartheta_1^T, \vartheta_2^T, \dots, \vartheta_n^T\right)^T \end{aligned} \tag{14}$$

where $X_d \in R^{mn}$, then the dynamic equations (12) and (13) can be re-expressed in terms of the aforementioned quantities as follows:

$$\dot{e}_x = e_v \tag{15}$$

$$\dot{e}_v = -X_d + F + u + \vartheta \tag{16}$$

The relative state tracking error is the difference of absolute state tracking errors between neighboring agents. The relative state tracking error between the ith and jth agents is defined by

$$r_{xij} = e_{xi} - e_{xj}, \quad i \neq j;\ i, j = 1, 2, \dots, n \tag{17}$$

$$r_{vij} = e_{vi} - e_{vj} \tag{18}$$

Let $\sigma_{1i} \in R^m$ and $\sigma_{2i} \in R^m$ ($i = 1, 2, \dots, n$) be the lumped state tracking errors including the absolute and relative state tracking errors for the ith agent. The lumped state tracking errors are given by

$$\sigma_{1i} = k_p \sum_{j=1}^{n} a_{ij} r_{xij} + k_i e_{xi} \tag{19}$$

$$\sigma_{2i} = k_p \sum_{j=1}^{n} a_{ij} r_{vij} + k_i e_{vi} \tag{20}$$

where a_{ij} is the element of the weighted adjacency matrix A, and k_p and k_i are positive constants. Note that

$$\sum_{j=1}^{n} a_{ij} r_{xij} = \sum_{j=1}^{n} a_{ij}(e_{xi} - e_{xj}) = \sum_{j=1}^{n} l_{ij} e_{xj} \tag{21}$$

$$\sum_{j=1}^{n} a_{ij} r_{vij} = \sum_{j=1}^{n} a_{ij}(e_{vi} - e_{vj}) = \sum_{j=1}^{n} l_{ij} e_{vj} \tag{22}$$

where l_{ij} is the element of the graph Laplacian matrix L. Applying (21) and (22) respectively to (19) and (20), the lumped tracking state errors σ_{1i} and σ_{2i} ($i = 1, 2, \dots, n$) can be rewritten as

$$\sigma_{1i} = k_p \sum_{j=1}^{n} l_{ij} e_{xj} + k_i e_{xi} \tag{23}$$

$$\sigma_{2i} = k_p \sum_{j=1}^{n} l_{ij} e_{vj} + k_i e_{vi} \tag{24}$$

Defining

$$\begin{aligned} &\Sigma_1 = \left(\sigma_{11}^T, \dots, \sigma_{1n}^T\right)^T, \qquad \Sigma_2 = \left(\sigma_{21}^T, \dots, \sigma_{2n}^T\right)^T \\ &K = \mathrm{diag}(k_1, k_2, \dots, k_n) \end{aligned} \tag{25}$$

then (23) and (24) can be written in terms of the aforementioned quantities as follows:

$$\Sigma_1 = M_1 e_x \tag{26}$$

$$\Sigma_2 = M_1 e_v \tag{27}$$

where $M_1 = (k_p L + K) \otimes I_m \in R^{mn\times mn}$, and $\otimes$ denotes the Kronecker product. Using (15) and (16), the dynamic equations for $\varSigma_1$ and $\varSigma_2$ are given by

$$\dot{\varSigma}_1 = \varSigma_2 \tag{28}$$

$$M_2 \dot{\varSigma}_2 = -X_d + F + u + \vartheta \tag{29}$$

where $M_2 = M_1^{-1}$.

The following lemma is used for the subsequent controller design.

Lemma 2 *The matrix $M_1 = (k_p L + K) \otimes I_m$ is symmetric and positive definite, and it satisfies the following bounded condition*:

$$m_1 \|x\|^2 \le x^T M_1 x \le m_2 \|x\|^2, \quad \forall x \in R^{mn} \tag{30}$$

where m_1 and m_2 are positive constants defined by

$$m_1 = \min_{1\le i\le n} \{k_i\} \tag{31}$$

$$m_2 = k_p \lambda_n + \max_{1\le i\le n} \{k_i\} \tag{32}$$

Proof It is obvious that there exists a set of orthogonal bases of R^{mn}, $p_1, p_2, \dots, p_{mn}$, such that

$$k_p L \otimes I_m = k_p P^T \varLambda P \tag{33}$$

where $P = (p_1, p_2, \dots, p_{mn}) \in R^{mn\times mn}$, and $\varLambda = \mathrm{diag}(0 I_m, \lambda_2 I_m, \dots, \lambda_n I_m) \in R^{mn\times mn}$. In addition, it follows that $P^T P = I_{mn}$ and $P^T = P^{-1}$.

Then, it is easy to obtain that

$$x^T M_1 x = x^T P^T (k_P \varLambda) P x + x^T K_m x \tag{34}$$

where $K_m = \mathrm{diag}(k_1 I_m, k_2 I_m, \dots, k_n I_m) \in R^{mn\times mn}$. Note that

$$0 \le x^T P^T (k_P \varLambda) P x \le k_p \lambda_n \|x\|^2 \tag{35}$$

$$\min_{1\le i\le n} \{k_i\} \|x\|^2 \le x^T K_m x \le \max_{1\le i\le n} \{k_i\} \|x\|^2 \tag{36}$$

Therefore, the matrix $M_1 = (k_p L + K) \otimes I_m$ is symmetric and positive definite, and it satisfies the bounded condition given by (30). □

3.2 Linear reduced-order observer

To facilitate the output feedback controller design, we define $s_i = \sigma_{2i} + \sigma_{1i} (i = 1, 2, \dots, n)$ and construct the following linear reduced-order observer for each individual agent in the formation:

$$\hat{s}_i = \eta_i + k_o \sigma_{1i} \tag{37}$$

$$\dot{\eta}_i = -k_o^2 \sigma_{1i} - (k_o + 1)\eta_i - \sigma_{1i} \tag{38}$$

where $\hat{s}_i \in R^m$ and $\eta_i \in R^m$ denote the output and state vectors of the observer, respectively, and $k_o > 3$ is a positive constant. The variable η_i used in (37) and (38) is employed to make the observer implementable using output signals of agents only. Let $\bar{s}_i = s_i - \hat{s}_i$ be the observation error; then the dynamic equations for $\hat{s}_i$ and $\bar{s}_i$ are respectively obtained as

$$\dot{\hat{s}}_i = -\hat{s}_i + k_o \bar{s}_i - \sigma_{1i} \tag{39}$$

$$\dot{\bar{s}}_i = \dot{\sigma}_{2i} + 2\hat{s}_i - (k_o - 1)\bar{s}_i \tag{40}$$

Define

$$\eta = \left(\eta_1^T, \dots, \eta_n^T\right)^T, \qquad s = \left(s_1^T, \dots, s_n^T\right)^T$$
$$\hat{s} = \left(\hat{s}_1^T, \dots, \hat{s}_n^T\right)^T, \qquad \bar{s} = \left(\bar{s}_1^T, \dots, \bar{s}_n^T\right)^T \tag{41}$$

and it follows that

$$\hat{s} = \eta + k_o \varSigma_1 \tag{42}$$

$$\dot{\eta} = -k_o^2 \varSigma_1 - (k_o + 1)\eta - \varSigma_1 \tag{43}$$

$$\dot{\hat{s}} = -\hat{s} + k_o \bar{s} - \varSigma_1 \tag{44}$$

$$\dot{\bar{s}} = \dot{\varSigma}_2 + 2\hat{s} - (k_o - 1)\bar{s} \tag{45}$$

Note that the lumped state tracking error $\varSigma_2$ can be expressed as

$$\varSigma_2 = \bar{s} + \hat{s} - \varSigma_1 \tag{46}$$

By premultiplying both sides of (45) by M_2 and then substituting (29) into the resulting expression, we have the following error dynamic equation:

$$\begin{aligned} M_2 \dot{\bar{s}} &= M_2 \dot{\varSigma}_2 + 2 M_2 \hat{s} - (k_o - 1) M_2 \bar{s} \\ &= -X_d + F + u + \vartheta + 2 M_2 \hat{s} - (k_o - 1) M_2 \bar{s} \end{aligned} \tag{47}$$

Remark 1 Note that s_i $(i = 1, 2, \dots, n)$ can be treated as a sliding mode surface. If $s_i = 0$ is achieved, according to the theory of sliding mode, the lumped state tracking error σ_{1i} is governed by

$$\dot{\sigma}_{1i} = -\sigma_{1i} \tag{48}$$

which in turn implies that the lumped state tracking error σ_{1i} converges to zero asymptotically.

3.3 Controller design

It is to be noted that if all agents' states x_i and v_i $(i = 1, 2, \dots, n)$ approach the desired trajectory, then the uncertain nonlinear function $f_i(x_i, v_i)(i = 1, \dots, n)$ converges to the *desired nonlinear function* f_{di} defined by

$$f_{di} = f_i(x_d + h_i, \dot{x}_d) \tag{49}$$

Using the approximation property of the CNN, the unknown function $f_i(x_d + h_i, \dot{x}_d) + \vartheta_i$ $(i = 1, 2, \dots, n)$ can be approximated over the compact set Ω_d by

$$f_i(x_d + h_i, \dot{x}_d) + \vartheta_i = W_i^* \xi_i(x_d, \dot{x}_d) + \varepsilon_i \tag{50}$$

where $W_i^* \in R^{m \times N_1}$, with $N_1 = mN_2 + 1$ and N_2 being the order of the Chebyshev polynomial, is the optimal weight matrix, and $\varepsilon_i \in R^m$ is the CNN approximation error. Now, the following assumptions are stated for the stability analysis of the overall closed-loop system.

Assumption 3 The optimal weight matrix W_i^* $(i = 1, 2, \dots, n)$ belongs to a known bounded set Ω_{Wi} defined by

$$W_i^* \in \Omega_{Wi} = \left\{ W_i^* : W_{i\,\min} \le W_{i,jk}^* \le W_{i\,\max} \right\} \tag{51}$$

where $j = 1, 2, \dots, m$, $k = 1, 2, \dots, N_1$, and $W_{i\,\min}$ and $W_{i\,\max}$ are known constants.

Assumption 4 The CNN approximation error ε_i is bounded such that $\|\varepsilon_i\| \le \varepsilon_{Mi}$ $(i = 1, 2, \dots, n)$, where ε_{Mi} is a positive constant.

To guarantee that the estimated CNN parameters remain within known bounded sets, the *smooth projection* [30, 31] is considered. Let W_i $(i = 1, 2, \dots, n)$ be the estimation of the optimal weight matrix W_i^*, and define a smooth projection of W_i as

$$\pi_i(W_i) = W_{i\pi} = \left(\pi_{i,jk}(W_{i,jk})\right) \tag{52}$$

where $j = 1, 2, \dots, m$ and $k = 1, 2, \dots, N_1$. Each projection operator $\pi_{i,jk}$: $R \to R$ is a real-valued smooth nondecreasing function defined by

$$\pi_{i,jk}(W_{i,jk}) = W_{i,jk}, \forall W_{i,jk} \in [W_{i\,\min}, W_{i\,\max}]$$

$$\pi_{i,jk}(W_{i,jk}) \in [W_{i\,\min} - \varepsilon_{Wi}, W_{i\,\max} + \varepsilon_{Wi}], \quad \forall W_{i,jk} \in R \tag{53}$$

where $\varepsilon_{Wi} > 0$ is a small constant.

Now, the control law for the ith agent in the formation is defined as

$$u_i = \ddot{x}_d - W_{i\pi}\xi_i - \kappa_i \sigma_{1i} - k_o \kappa_i \hat{s}_i, \quad i = 1, 2, \dots, n \tag{54}$$

where κ_i is a positive constant. The adaptive law for W_i is given by

$$W_i = W_{0i} + \Phi_i \tag{55}$$
$$\dot{W}_{0i} = \delta_i \left(\sigma_{1i}\xi_i^T - \hat{s}_i \xi_i^T - \sigma_{1i}\dot{\xi}_i^T\right), \quad i = 1, 2, \dots, n$$

where $\Phi_i = \delta_i \sigma_{1i} \xi_i^T$, and δ_i is a positive constant. The adaptive law defined by (55) can be differentiated with respect to time as follows:

$$\dot{W}_i = \delta_i \bar{s}_i \xi_i^T, \quad i = 1, \dots, n \tag{56}$$

Define $\tilde{W}_i = W_i^* - W_i$ $(i = 1, 2, \dots, n)$, $\tilde{W}_{i\pi} = W_i^* - W_{i\pi}$, and

$$V_{Wi} = \frac{1}{\delta_i} \sum_{j=1}^{m} \sum_{k=1}^{N_1} \int_0^{\tilde{W}_{i,jk}} \left(W_{i,jk}^* - \pi_{i,jk}\left(W_{i,jk}^* - \omega_{i,jk}\right)\right) \times d\omega_{i,jk} \tag{57}$$

Then, V_{Wi} is positive definite with respect to $\tilde{W}_{i,jk}$ for $W_{i,jk}^* \in [W_{i\,\min}, W_{i\,\max}]$. Furthermore,

$$\dot{V}_{Wi} = -\frac{1}{\delta_i} \sum_{j=1}^{m} \sum_{k=1}^{N_1} \tilde{W}_{i\pi,jk} \dot{W}_{i,jk}, \quad i = 1, 2, \dots, n \tag{58}$$

Substituting the control law (54) into the error dynamics (47) yields the following dynamic equation for $\bar{s}$:

$$\begin{aligned} M_2 \dot{\bar{s}} &= F + \vartheta - W_\pi \xi - \bar{K}\sigma_1 - k_o K \hat{s} + 2M_2 \hat{s} \\ &\quad - (k_o - 1) M_2 \bar{s} \\ &= \chi + \tilde{W}_\pi \xi - K\sigma_1 - k_o \bar{K} \hat{s} - (k_o - 1) M_2 \bar{s} + \varepsilon \end{aligned} \tag{59}$$

where $\chi = F - F_d + 2M_2\hat{s} \in R^{mn}$, and

$$\bar{K} = \mathrm{diag}\{\kappa_1 I_m, \kappa_2 I_m, \dots, \kappa_n I_m\},$$

$$\varepsilon = \left(\varepsilon_1^T, \varepsilon_2^T, \dots, \varepsilon_n^T\right)^T$$

$$W^* = \mathrm{diag}\{W_1^*, W_2^*, \dots, W_n^*\}$$
$$W_\pi = \mathrm{diag}\{W_{1\pi}, W_{2\pi}, \dots, W_{n\pi}\}$$
$$\tilde{W}_\pi = W^* - W_\pi, \qquad \xi = (\xi_1^T, \xi_2^T, \dots, \xi_n^T)^T$$
$$F_d = (f_{d1}^T, f_{d2}^T, \dots, f_{dn}^T)^T$$

For a sufficiently large positive constant $V_{\max}$, we construct the following compact set:

$$\Omega_V = \left\{(\Sigma_1, \hat{s}, \bar{s}) \mid \Sigma_1^T \Sigma_1 + \hat{s}^T \hat{s} + \bar{s}^T \bar{s} \le \frac{2V_{\max}}{\lambda_M}\right\} \tag{60}$$

where $\lambda_M = \max(\lambda_{\max}(\bar{K}), \lambda_{\max}(M_2))$ with $\lambda_{\max}(\cdot)$ denoting the maximum eigenvalue of a matrix. Since the sets Ω_d and Ω_V are compact in R^{3m} and R^{3mn}, respectively, the variable χ has a maximum χ_M on the compact set $\Omega_d \times \Omega_V$. The stability analysis of the overall closed-loop system is stated in Sect. 3.4.

3.4 Stability analysis

Theorem 1 *Consider that a class of second-order multi-agent systems is described by (1) and (2), and Assumptions 1–4 are satisfied. The control laws are provided by (54), where the projection algorithm is defined by (52) and (53), and the adaptive laws are given by (55). For a sufficiently large positive constant $V_{\max}$, if the initial conditions satisfy*

$$\Sigma_1^T(0)\Sigma_1(0) + \hat{s}^T(0)\hat{s}(0) + \bar{s}^T(0)\bar{s}(0) \le \frac{2V_{\max}}{\lambda_M} \tag{61}$$

then Σ_1, $\hat{s}$ and $\bar{s}$ are uniformly ultimately bounded.

Proof Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}\Sigma_1^T \bar{K} \Sigma_1 + \frac{1}{2}\hat{s}^T \bar{K}\hat{s} + \frac{1}{2}\bar{s}^T M_2 \bar{s} + \sum_{i=1}^n V_{Wi} \tag{62}$$

The time derivative of the Lyapunov function (62) is given by

$$\begin{aligned}\dot{V} &= \bar{s}^T \tilde{W}_\pi \xi - \bar{s}^T \bar{K}\Sigma_1 - k_o \bar{s}^T \bar{K}\hat{s} - (k_o - 1)\bar{s}^T M_2 \bar{s} \\ &\quad + \bar{s}^T \chi + \bar{s}^T \varepsilon - \sum_{i=1}^n \left(\frac{1}{\delta_i}\sum_{j=1}^m \sum_{k=1}^{N_1} \tilde{W}_{i\pi,jk}\dot{W}_{i,jk}\right) \\ &\quad + \Sigma_1^T \bar{K}\Sigma_2 + \hat{s}^T K(-\hat{s} + k_o \bar{s} - \Sigma_1) \\ &= -\Sigma_1^T \bar{K}\Sigma_1 - \hat{s}^T K\hat{s} - (k_o - 1)\bar{s}^T M_2 \bar{s} \\ &\quad + \bar{s}^T \chi + \bar{s}^T \varepsilon \end{aligned} \tag{63}$$

$$\begin{aligned} &\le -\lambda_{\min}(\bar{K})\Sigma_1^T \Sigma_1 - \lambda_{\min}(\bar{K})\hat{s}^T \hat{s} \\ &\quad - \lambda_{\min}(M_2)(k_o - 1)\bar{s}^T \bar{s} + \bar{s}^T \chi + \bar{s}^T \varepsilon \end{aligned} \tag{64}$$

where $\lambda_{\min}(\cdot)$ represents the minimum eigenvalue of a matrix, and the relation

$$\bar{s}^T \tilde{W}_\pi \xi - \sum_{i=1}^n \left(\frac{1}{\delta_i}\sum_{j=1}^m \sum_{k=1}^{N_1} \tilde{W}_{i\pi,jk}\dot{W}_{i,jk}\right) = 0 \tag{65}$$

is applied.

Applying the following inequalities,

$$\bar{s}^T \chi \le \lambda_{\min}(M_2)\bar{s}^T \bar{s} + \frac{\chi_M^2}{4\lambda_{\min}(M_2)}$$
$$\bar{s}^T \varepsilon \le \lambda_{\min}(M_2)\bar{s}^T \bar{s} + \frac{\varepsilon_M^2}{4\lambda_{\min}(M_2)}$$

into (64), we have

$$\begin{aligned}\dot{V} &\le -\lambda_{\min}(\bar{K})\Sigma_1^T \Sigma_1 - \lambda_{\min}(\bar{K})\hat{s}^T \hat{s} \\ &\quad - (k_o - 3)\lambda_{\min}(M_2)\bar{s}^T \bar{s} + c_2 \\ &\le -c_1 e^T e + c_2 \end{aligned} \tag{66}$$

where $c_1 = \min(\lambda_{\min}(\bar{K}), (k_o - 3)\lambda_{\min}(M_2))$ and $c_2 = (\chi_M^2 + \varepsilon_M^2)/(4\lambda_{\min}(M_2))$ are positive constants, and $e = (\Sigma_1^T, \hat{s}^T, \bar{s}^T)^T$. Thus, $\dot{V}$ is strictly negative outside the following compact set Ω_e:

$$\Omega_e = \left\{e(t) \,\middle|\, \|e(t)\| \le \sqrt{\frac{c_2}{c_1}}\right\} \tag{67}$$

which implies that $\|e\|$ decreases whenever e is outside the compact set Ω_e; hence, e is uniformly ultimately bounded. Using (46), it is concluded that Σ_2 is bounded. Since $W_{i\pi}$ and ξ_i are bounded, we conclude that the control input (54) is also bounded. □

Remark 2 In this paper, the Chebyshev polynomial is used for the basis function ξ_i $(i = 1, 2, \dots, n)$. Several other basis functions can be used for ξ_i, for example, radial basis function (RBF) [18], sigmoid function [17], and fuzzy basis function (FBF) [19]. As compared with sigmoid function, RBF and FBF, the key advantage of Chebyshev polynomial basis function lies in the fact that only one parameter (i.e., the order of the Chebyshev polynomial basis) is required to determine the Chebyshev polynomial basis. Considering this as well as other related issues, the CNN is considered in this paper.

Remark 3 It is to be noted that the time derivative of the Chebyshev polynomial basis function, $\dot{\xi}_i$ ($i = 1, 2, \ldots, n$), is required in the adaptive law (55). If the Chebyshev polynomial basis function ξ_i is determined, then it is easy to obtain $\dot{\xi}_i$ in an analytical manner. For example, let us consider $x = (x_1, x_2)^T$, and assume that the order of the Chebyshev polynomial is 3. Then the Chebyshev polynomial basis function ξ_i ($i = 1, 2, \ldots, n$) is computed as

$$\xi_i = \big(1, x_1, 2x_1^2 - 1, 4x_1^3 - 3x_1, x_2, 2x_2^2 - 1, 4x_2^3 - 3x_2\big)^T \quad (68)$$

Next, $\dot{\xi}_i$ ($i = 1, 2, \ldots, n$) can be easily derived from the preceding equation as follows:

$$\dot{\xi}_i = \big(0, \dot{x}_1, 4x_1\dot{x}_1, 12x_1^2\dot{x}_1 - 3\dot{x}_1, \dot{x}_2, 4x_2\dot{x}_2, 12x_2^2\dot{x}_2 - 3\dot{x}_2\big)^T \quad (69)$$

Note that the implementation of the Chebyshev polynomial basis functions depends only on the desired signals; therefore, the agent's actual state signals are not necessary for the computation of $\dot{\xi}_i$ ($i = 1, 2, \ldots, n$).

Remark 4 To prevent the parameter drift of the adaptive parameter W_{0i}, the σ-modification algorithms [32] are employed, i.e., the adaptive laws for W_{0i} ($i = 1, 2, \ldots, n$) are given as

$$\dot{W}_{0i} = \delta_{1i}\big(\sigma_{1i}\xi_i^T - \hat{s}_i\xi_i^T - \sigma_{1i}\dot{\xi}_i^T\big) - \delta_{1i}\delta_{2i}W_{0i} \quad (70)$$

where δ_{1i} and δ_{2i} are positive constants.

4 Simulation results

To verify the effectiveness of the proposed controller, numerical simulations are carried out using the multi-agent system described by (1) and (2) in conjunction with the control law (54), the projection algorithm (52) and (53), and the adaptive law (55). In the simulation, the multi-agent system consists of six agents, and the uncertain nonlinear function $f_i(x_i, v_i)$ and external disturbances ϑ_i in the agent's dynamics are given as

$$f_i(x_i, v_i) = \begin{pmatrix} 4x_{i2}\sin(\frac{\pi}{4} + \frac{g_{i1}v_{i1}}{2}) \\ 4x_{i1}\cos(\frac{g_{i2}v_{i2}}{2}) \end{pmatrix}$$

$$\vartheta_i = \begin{pmatrix} 0.1\sin(\frac{it}{2}) \\ 0.1\cos(\frac{it}{2}) \end{pmatrix}, \quad i = 1, 2, \ldots, 6 \quad (71)$$

where $x_i = (x_{i1}, x_{i2})^T$ and $v_i = (v_{i1}, v_{i2})^T$. The parameters g_{i1} and g_{i2} are given in Table 1. Note that the uncertain nonlinear function $f_i(x_i, v_i)$ ($i = 1, 2, \ldots, 6$) is a smooth function, and contains the nonlinear terms (i.e., $\cos(\cdot)$, $\sin(\cdot)$) and unknown parameters (i.e., g_{i1} and g_{i2}). Furthermore, the uncertain nonlinear function $f_i(x_i, v_i)$ ($i = 1, 2, \ldots, 6$) does not satisfy the assumption of "linearity in the parameters"; thus, the traditional adaptive control scheme cannot be applied to solve this problem. The communication graph for the multi-agent system is shown in Fig. 1. The information exchange among agents is represented by a graph with the weighted adjacency

Table 1 The parameters g_{i1} and g_{i2}

i	1	2	3	4	5	6
g_{i1}	0.6	−0.2	0.4	−0.3	0.5	0.5
g_{i2}	0.3	0.4	−0.4	−0.7	−0.2	−0.5

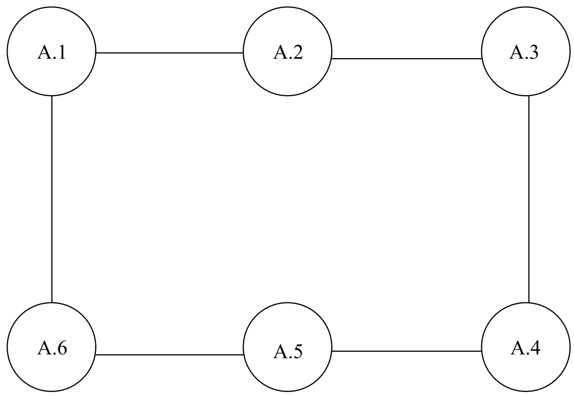


Fig. 1 Communication graph for the multi-agent system (A.i ($i = 1, 2, \ldots, 6$) represents the ith agent)

matrix A defined by

$$A = \begin{pmatrix} 0.0 & 0.5 & 0.0 & 0.0 & 0.0 & 0.8 \\ 0.5 & 0.0 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.6 & 0.0 & 0.7 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.7 & 0.0 & 0.8 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.0 & 0.9 \\ 0.8 & 0.0 & 0.0 & 0.0 & 0.9 & 0.0 \end{pmatrix} \tag{72}$$

The initial states of the individual agents are considered to be: $x_1(0) = (3, 0.5)^T$, $x_2(0) = (1.8, -0.2)^T$, $x_3(0) = (0.2, -1.2)^T$, $x_4(0) = (-0.1, 1.2)^T$, $x_5(0) = (-0.6, 1.8)^T$, $x_6(0) = (1, -1.7)^T$, and $v_i(0) = (0, 0)^T$ $(i = 1, 2, \ldots, 6)$. Furthermore, the projection operator $\pi_{i,jk}$ is given by [31]:

$$\pi_{i,jk}(W_{i,jk}) = \begin{cases} W_{i\max} + \varepsilon_{Wi}(1 - \exp(\frac{-(W_{i,jk} - W_{i\max})}{\varepsilon_{Wi}})) \\ \quad \text{if } W_{i,jk} > W_{i\max} \\ W_{i,jk} \quad \text{if } W_{i,jk} \in [W_{i\min}, W_{i\max}] \\ W_{i\min} - \varepsilon_{Wi}(1 - \exp(\frac{W_{i,jk} - W_{i\min}}{\varepsilon_{Wi}})) \\ \quad \text{if } W_{i,jk} < W_{i\min} \end{cases} \tag{73}$$

where $i = 1, 2, \ldots, 6$, $j = 1, 2$, and $k = 1, 2, \ldots, 13$. The controller parameters are taken as $k_p = 4$, $k_o = 8$, $k_i = 1$, $\kappa_i = 1$, $W_{i\max} = 1.5$, $W_{i\min} = -1.5$, $\delta_{1i} = 100$, $\delta_{2i} = 0.1$, where $i = 1, 2, \ldots, 6$, and the order of the Chebyshev polynomials is chosen as $N_2 = 3$. The initial weight matrix of the CNN is considered to be $W_{0i}(0) = 0_{2\times 13}$, $i = 1, 2, \ldots, 6$.

To investigate the performance of the proposed controller, an absolute state error metric, a relative state difference metric and a relative velocity difference metric are considered. The absolute state error metric (ASEM) is defined as

$$\text{ASEM} = \sqrt{\sum_{i=1}^{n} \|e_{xi}\|^2} \tag{74}$$

The relative state difference metric (RSDM) is defined as

$$\text{RSDM} = \sqrt{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \|x_i - x_j\|^2} \tag{75}$$

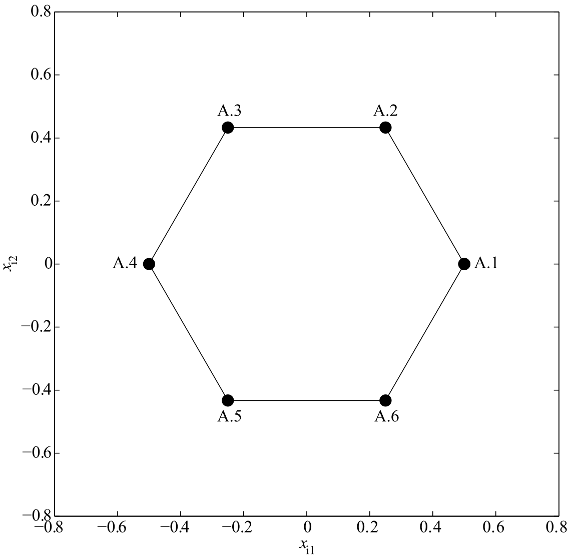


Fig. 2 Desired formation

and the relative velocity difference metric (RVDM) is defined as

$$\text{RVDM} = \sqrt{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \|v_i - v_j\|^2} \tag{76}$$

The desired geometric formation is shown in Fig. 2, and h_i $(i = 1, 2, \ldots, 6)$ is considered as

$$h_1 = \left(\frac{1}{2}, 0\right)^T, \qquad h_2 = \left(\frac{1}{4}, \frac{\sqrt{3}}{4}\right)^T$$

$$h_3 = \left(-\frac{1}{4}, \frac{\sqrt{3}}{4}\right)^T, \qquad h_4 = \left(-\frac{1}{2}, 0\right)^T$$

$$h_5 = \left(-\frac{1}{4}, -\frac{\sqrt{3}}{4}\right)^T, \qquad h_6 = \left(\frac{1}{4}, -\frac{\sqrt{3}}{4}\right)^T$$

The desired trajectory is defined as $x_d = 2(\cos(t), \sin(t))^T$. The centroid of six agents is given as

$$x_c = \frac{\sum_{i=1}^{6} x_i}{6} \tag{77}$$

and the desired centroid of six agents is defined as

$$x_{cd} = \frac{\sum_{i=1}^{6} (h_i + x_d)}{6} \tag{78}$$

Furthermore, the desired relative state difference metric is given as

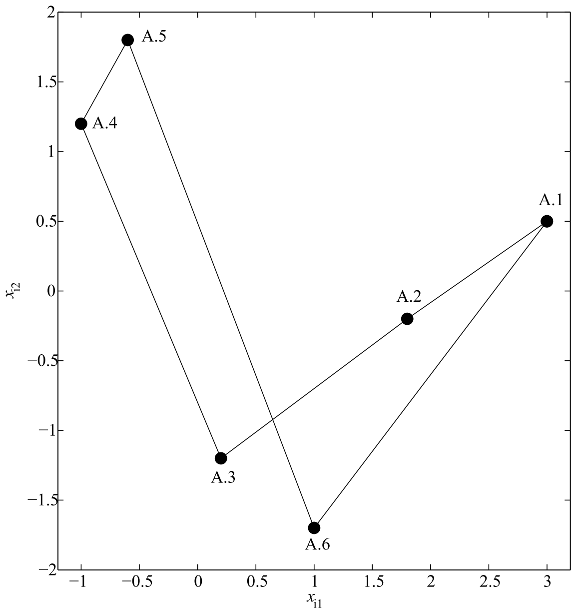


Fig. 3 Geometric formation of six agents at time $t = 0$ s

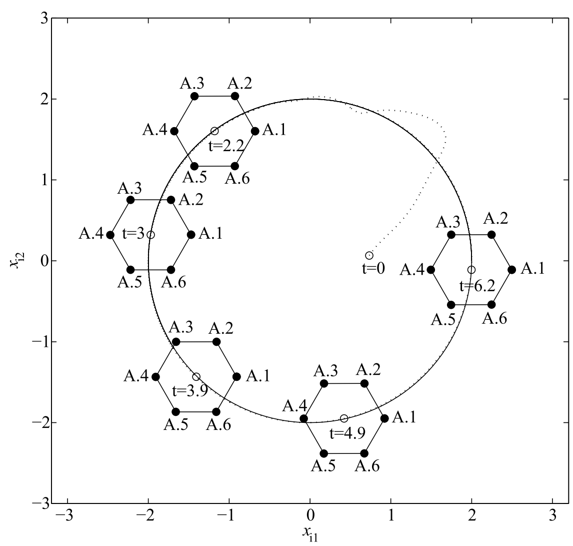
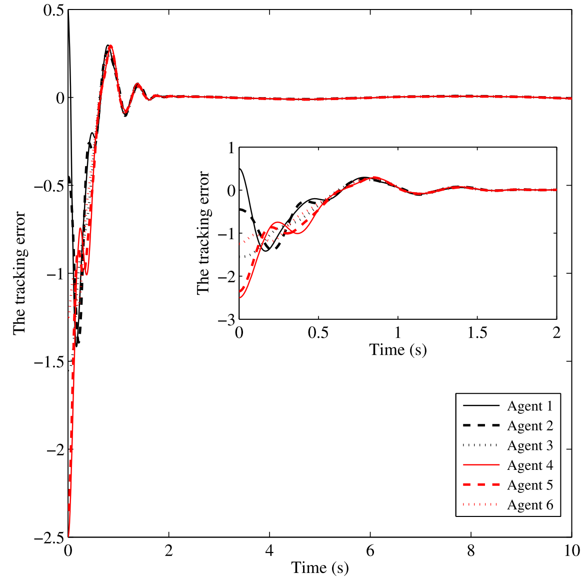


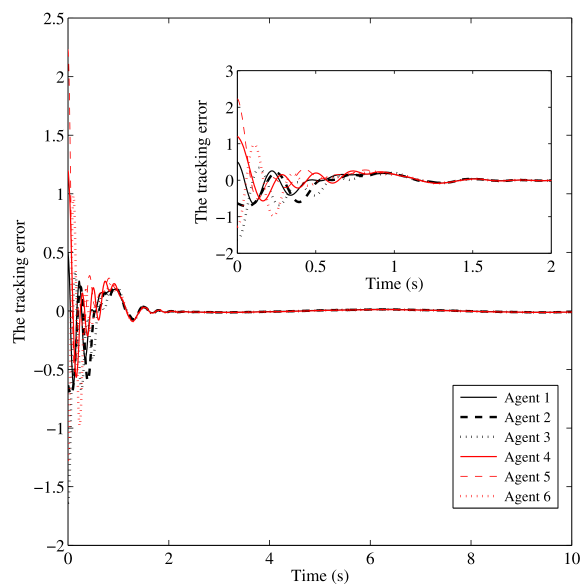
Fig. 4 Positions of six agents varying with time t: trajectory of x_c (*dotted line*) and trajectory of x_{cd} (*solid line*)

$$\mathrm{RSDM}_d = \sqrt{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left\| (h_i + x_d) - (h_j + x_d) \right\|^2} = 3 \tag{79}$$

and the desired relative velocity difference metric is defined as



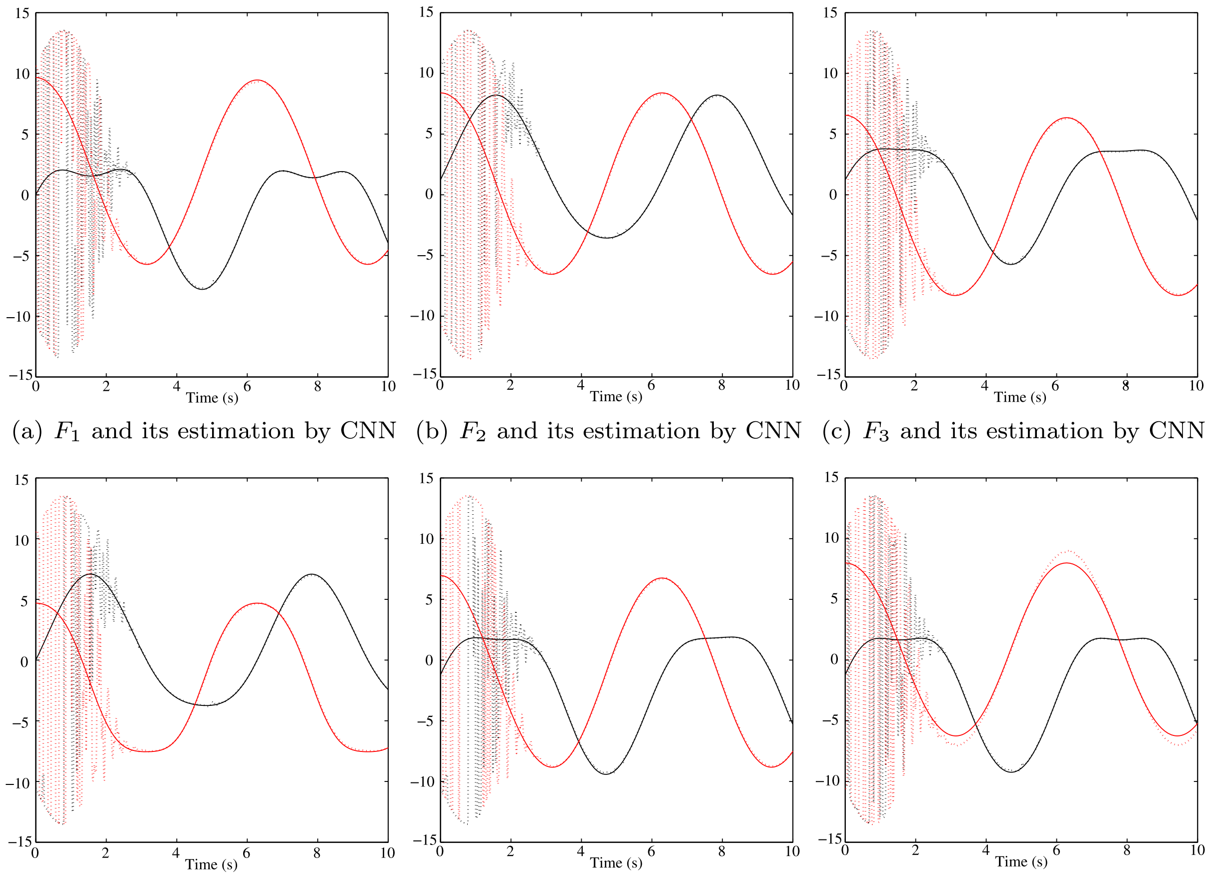
(a) e_{xi1}



(b) e_{xi2}

Fig. 5 The agent tracking error $e_{xi} = (e_{xi1}, e_{xi2})^T$ $(i = 1, 2, \ldots, 6)$

$$\mathrm{RVDM}_d = \sqrt{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left\| (\dot{h}_i + \dot{x}_d) - (\dot{h}_j + \dot{x}_d) \right\|^2} = 0 \tag{80}$$



(a) F_1 and its estimation by CNN (b) F_2 and its estimation by CNN (c) F_3 and its estimation by CNN

(d) F_4 and its estimation by CNN (e) F_5 and its estimation by CNN (f) F_6 and its estimation by CNN

Fig. 6 Capture of the unknown function $F_i = f_i(x_d + h_i, \dot{x}_d) + \vartheta_i$ $(i = 1, 2, \ldots, 6)$ by the CNN approximation: F_{i1} (*black solid line*) and its estimation (*black dotted line*), F_{i2} (*red solid line*) and its estimation (*red dotted line*)

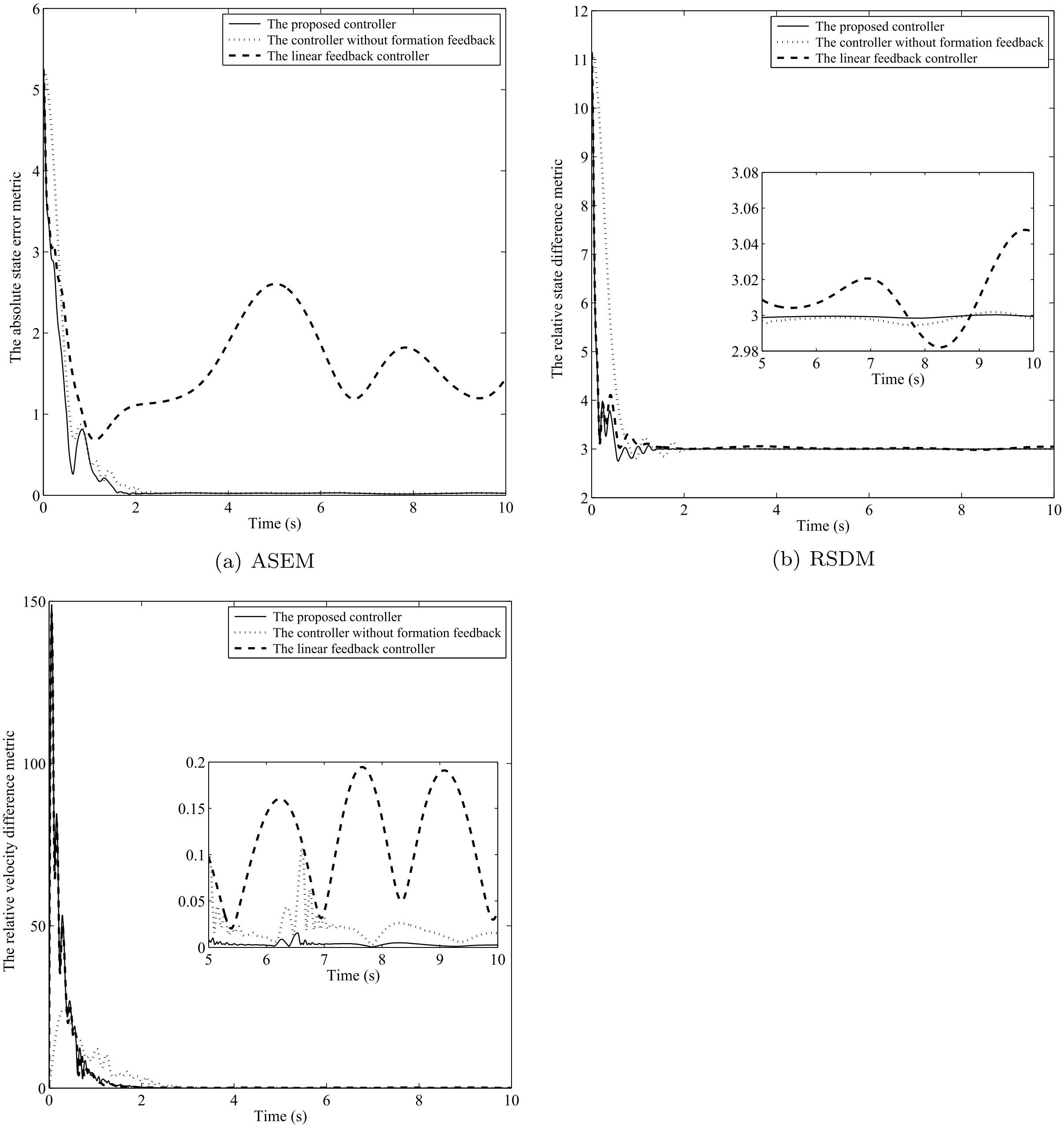
Furthermore, the input vector of the CNN is $(x_d^T, \dot{x}_d^T)^T$, and it is normalized as $(x_d^T, \dot{x}_d^T)^T/norm$ to obtain a good control performance, where *norm* is a positive constant which is considered to be 4 in this paper.

Figure 3 shows the initial geometric formation of six agents. Using the proposed control law, Fig. 4 shows the positions of six agents at several times, and Fig. 5 illustrates the agent tracking error e_{xi} $(i = 1, 2, \ldots, 6)$. It is found that the six agents converge to the desired geometric formation and the centroid of the six agents converges to the desired trajectory simultaneously even in the presence of unknown agent dynamics and bounded external disturbances. Figure 6 shows that the CNNs used in the controller have the capability to simultaneously capture the unknown desired nonlinear functions $f_i(x_d + h_i, \dot{x}_d)$ and external disturbances ϑ_i $(i = 1, 2, \ldots, 6)$ after the learning phase.

Next, the performance comparison between the proposed controller and several other controllers is studied. The controllers used in the performance comparison are considered to be the controller without formation feedback (i.e., the weighted adjacency matrix $A = 0_n$), and the linear feedback controller defined by

$$u_i = \ddot{x}_d - \kappa_i \sigma_{1i} - k_o \kappa_i \hat{s}_i, \quad i = 1, 2, \ldots, n \tag{81}$$

The ASEM, RSDM and RVDM for the proposed controller, the controller without formation feedback, and the linear feedback controller are shown in Fig. 7. It is observed that the proposed controller can provide higher formation tracking performance for a group of agents than the controller without formation feedback and the linear feedback controller.



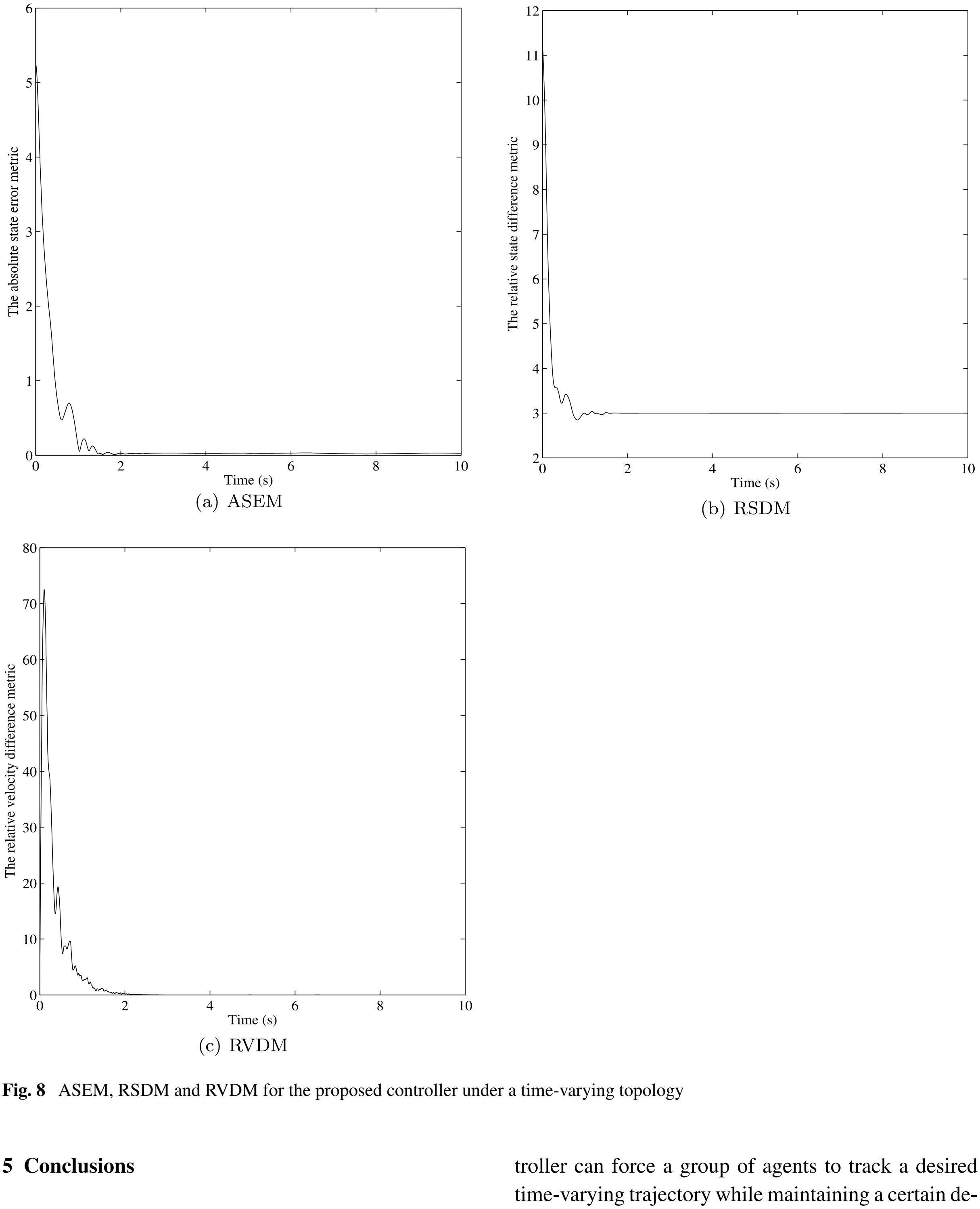
(a) ASEM

(b) RSDM

(c) RVDM

Fig. 7 ASEM, RSDM and RVDM for the proposed controller, the controller without formation feedback and the linear controller

Finally, the performance of the proposed controller is examined under a time-varying topology. For this example, the weighted adjacency matrix A is taken as $A = (a_{ij}/(1 + \|x_i - x_j\|))$, where the element a_{ij} $(i = 1, 2, \dots, n;\ j = 1, 2, \dots, n)$ is considered to be the same as that in (72). Using the proposed control law, the ASEM, RSDM and RVDM are shown in Fig. 8. It is observed that the proposed controller can provide a good formation tracking performance even under a time-varying topology.



(a) ASEM (b) RSDM (c) RVDM

Fig. 8 ASEM, RSDM and RVDM for the proposed controller under a time-varying topology

5 Conclusions

The primary contribution of this paper is the development of a neural-network-based decentralized adaptive output feedback formation controller for a class of uncertain multi-agent systems. The proposed controller can force a group of agents to track a desired time-varying trajectory while maintaining a certain desired geometric formation simultaneously, even in the presence of unknown agent's dynamics and external disturbances as demonstrated in the simulations; therefore, the proposed controller is robust against not only structured uncertainties but also unstructured uncertainties. Comparative studies between the proposed controller and several other controllers (i.e., the controller without formation feedback and the linear feedback controller) show that the performance of the proposed controller is superior to that of the other controllers.

In this paper, the desired trajectory is assumed to be available to all agents in the formation. The problem of output feedback consensus tracking and/or formation tracking control for second-order multi-agent systems with uncertainties is more challenging if only a subset of group agents has access to the common reference trajectory (or the leader), which needs further investigation.

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