Optimal Power Flow Solution Incorporating Wind Power

Libao Shi, *Senior Member, IEEE,* Chen Wang, Liangzhong Yao,
Yixin Ni, *Senior Member, IEEE,* and Masoud Bazargan

***Abstract*—This paper presents a solution of optimal power flow (OPF) incorporating wind power. A paradigm for modeling the cost of wind-generated electricity from a wind farm is proposed. Based on the Weibull wind speed distribution and wind turbine model represented by function approximation, the frequency distribution of wind farm power output to be the basis for modeling wind generation cost is established via applying Monte Carlo simulation. The proposed wind generation cost model consists of the opportunity cost of wind power shortage and the opportunity cost of wind power surplus, which reflect the cost of dispatching additional reserve capacity and the cost of environmental benefit loss, respectively, and it is integrated into the conventional OPF program. Furthermore, the small signal stability constraints are considered simultaneously as well during optimization. A self-adaptive evolutionary programming method is employed to solve the OPF with wind power involved. A case study is conducted based on the IEEE New England test system (10-Generator-39-Bus) as a benchmark. The simulation results demonstrate the effectiveness and validity of the proposed model and method.**

***Index Terms*—Monte Carlo, optimal power flow (OPF), self-adaptive evolutionary programming, small signal stability, wind power.**

I. Introduction

THE AIM OF optimal power flow (OPF) is to minimize the total cost of generation while satisfying the system design and operational requirements. It is known that the conventional OPF problem only involved thermal energy power sources. With the introduction and development of renewable energy sources especially like wind energy [1], there is a need to incorporate wind generation cost into the classical OPF problem. Some published literature [2]–[10] have discussed the OPF problem incorporating wind generation cost. Liu and Shang [2] addressed the economic dispatch incorporating wind power plant. Zhou *et al.* [3] presented dynamic economic dispatch model with large-scale wind power penetration. Jabr and Pal [4] proposed a stochastic model of wind generation in an OPF dispatching program. The proposed model allows the coordination of wind and thermal power while accounting for: 1) the expected penalty cost for not using all available wind power, and 2) the expected cost of calling up power reserves because of wind power shortage. Castronuovo *et al.* [5] studied the optimal controllability of wind generators in a delegated dispatch. In the formulation, variations in the output restriction for wind provision, different wind turbines technologies and active and reactive controllability actions were considered. Siahkali and Vakilian [6] considered the generation scheduling including wind power generation. The impacts of wind generation were modeled by increasing the reserve requirement. The intermittency of wind generation in each period was substituted by wind energy speed of each period and power related to this speed. Sun *et al.* [7] proposed a new dynamic economic dispatch method based on the wind speed forecasting and stochastic programming theory. A biobjective economic dispatch problem considering wind penetration was formulated, which treated operational costs and security impacts as conflicting objectives in [8]. Miranda and Hang [9] described a new economic dispatch algorithm for systems with uncertain wind generation prediction, similar to the classical thermal dispatch model with load on a single bus. In [10], a model was developed to include the wind energy conversion system in the economic dispatch problem. Some factors to account for both overestimation and underestimation of available wind power were involved.

Generally speaking, the wind generation cost was not considered into the optimal objective function in some research work. The wind generation cost representing the intermittence and fluctuation of wind generation is generally considered as a kind of constraints [2]–[7]. Albeit some literature [8]–[10] introduced the wind generation cost into the objective function, the physical meanings of some elements in the model are ambiguous, and the probability distribution applied to reveal wind generation intermittence and fluctuation is too simplified. All in all, it is necessary and imperative to explore and exploit the OPF problem incorporating wind power further.

In this paper, a novel model of wind generation cost is proposed. On the basis of Weibull wind speed distribution [11] and wind turbine model represented by function approxima-

Manuscript received September 13, 2010; revised November 25, 2010; accepted May 25, 2011. Date of publication August 30, 2011; date of current version May 22, 2012. This work was supported in part by the Specialized Research Fund for the Doctoral Program of Higher Education, under Grant 200800031068, and in part by the Research Project of Science and Technology from the Shenzhen Government of China, under Grant JC200903180528A.

L. Shi and C. Wang are with the Graduate School at Shenzhen, Tsinghua University, Shenzhen 518055, China (e-mail: shilb@sz.tsinghua.edu.cn; wangchen05@mails.tsinghua.edu.cn).

L. Yao and M. Bazargan are with the ALSTOM Grid Research and Technology Center, Stafford ST17 4LX, U.K. (e-mail: liangzhong.yao@alstom.com; masoud.bazargan@alstom.com).

Y. Ni is with the Department of Electrical and Electronic Engineering, University of Hong Kong, Pokfulam, Hong Kong (e-mail: yxni@eee.hku.hk).

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Digital Object Identifier 10.1109/JSYST.2011.2162896

tion, the frequency distribution of wind farm power output to be the basis for quantifying wind generation cost is established via applying Monte Carlo simulation. The opportunity cost of wind power shortage and the opportunity cost of wind power surplus are proposed to establish the wind generation cost model. The proposed wind generation cost model is introduced into the conventional OPF program as objective function, and small signal stability constraints are considered as well during analysis. A self-adaptive evolutionary programming (SAEP) method [14] is applied to solve the OPF with wind power incorporated. A case study is conduced based on the IEEE New England test system (10-Generator-39-Bus) as benchmark. Some preliminary conclusions and comments are drawn based on the numerical results.

This paper is organized as follows. Section II details the paradigm for modeling wind generation cost. In Section III, the OPF problem incorporating wind generation cost and the corresponding solution are addressed. Section IV shows the preliminary results. Finally, the conclusions are summarized in Section V.

II. Paradigm for Modeling Wind Generation Cost

Inspired by [10], this paper proposes a method to quantify the cost of wind generation with clearer physical meaning and more practical application. The following schematic diagram as shown in Fig. 1 is used to describe the paradigm of modeling wind generation cost.

A. Wind Farm Power Output Probability Distribution

In our paper, the power output probability distribution of wind farm is established based on the Weibull wind speed distribution [11] and wind turbine model represented by function approximation. Fig. 2 gives the Weibull distribution of wind speed with $k = 2$ and $c = 10\,\text{m/s}$ [1]. By applying Monte Carlo simulation with sample size $N = 8000$, the frequency distribution of wind speed can be obtained as shown in Fig. 2.

In this paper, the relation between wind speed and mechanical power extracted from the wind is given as follows [1]:

$$P_m = \begin{cases} 0, & V_w \le V_{\text{cut-in}} \quad \text{or} \quad V_w \ge V_{\text{cut-off}} \\ 0.5\rho A_{wt} C_p(\beta,\lambda) V_w^3, & V_{\text{cut-in}} < V_w \le V_{\text{rated}} \\ P_{\text{rated}}, & V_{\text{rated}} < V_w < V_{\text{cut-off}} \end{cases} \tag{1}$$

where P_m is the power extracted from the wind, ρ is the air density, C_p is the performance coefficient, λ is the tip-speed ratio (v_t/v_w), the ratio between blade tip speed, v_t (m/s), and wind speed at hub height upstream of the rotor, v_w (m/s), $A_{wt} = \pi R^2$ is the area covered by the wind turbine rotor, R is the radius of the rotor, V_w denotes the wind speed, β is the blade pitch angle, $V_{\text{cut-in}}$ and $V_{\text{cut-off}}$ are the cut-in and cut-off wind speed of wind turbine, and V_{rated} is the wind speed at which the mechanical power output will be the rated power. When V_w is higher than V_{rated} and lower than $V_{\text{cut-off}}$, with a pitch angle control system, the mechanical power output of wind turbine will keep constant as the rated power P_{rated}.

According to (1), and combined with the frequency distribution of wind speed, for a wind farm with 200*2MVA

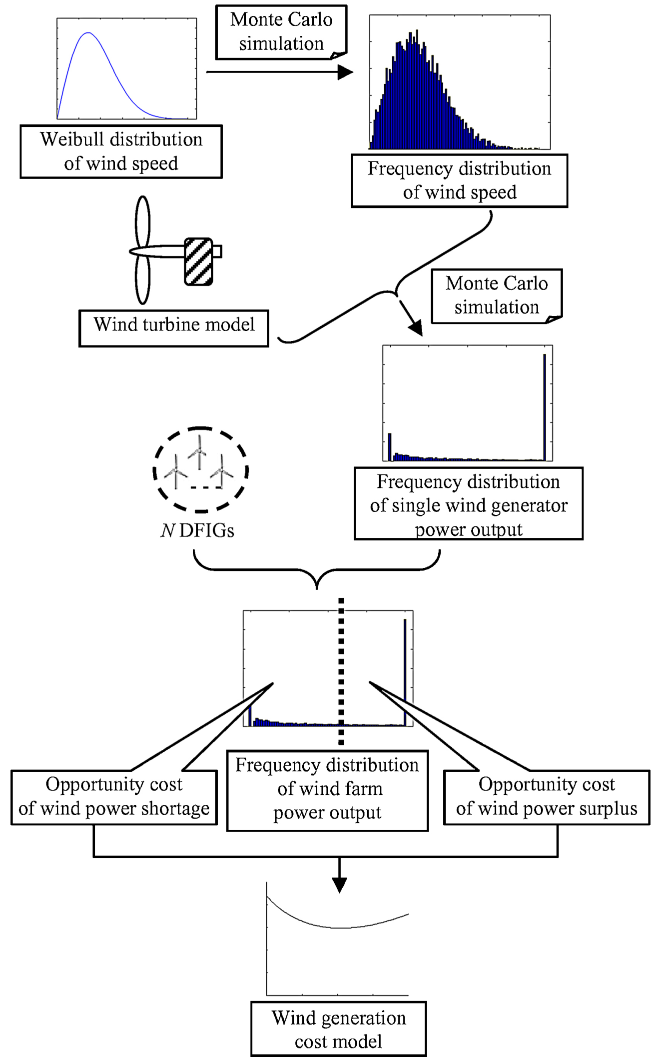

Fig. 1. Schematic diagram of modeling wind generation cost.

power output, the distribution of wind farm power output can be described by the frequency distribution histogram as shown in Fig. 3. There exist two concentrations of probability masses in the distribution: one corresponds to the value of zero, in which the wind farm is cut off, and the other corresponds to the value of 2 MW, in which the rated mechanical power output is generated by the wind turbine.

When the scheduled wind farm power output is confirmed, the actual wind farm power output may be lower or higher than the scheduled power output due to the intermittent and fluctuant nature of wind generation. For the former situation, the power shortage has to be tackled by purchasing power from the alternate sources or shedding load to maintain power balance. For the latter situation, the wind farm has to decrease power output, which leads to the waste of available renewable energy capacity and negative impact on the environment. In our paper, the aforementioned two situations are considered as the origin of wind generation cost. In Fig. 3, the frequency distribution of wind farm power output is divided into the left half-plane and the right half-plane by the scheduled power output, which correspond to the wind power shortage and

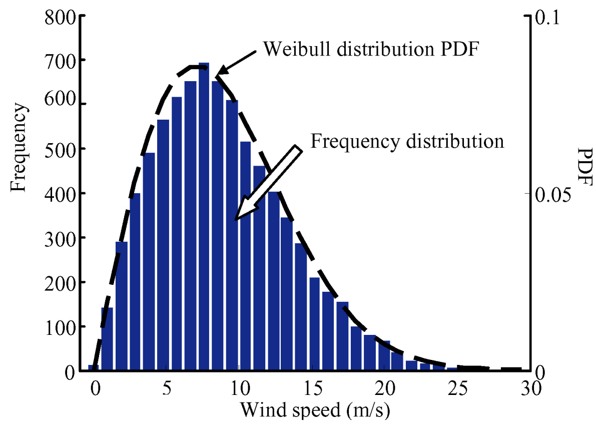

Fig. 2. Weibull distribution with $k = 2$ and $c = 10$.

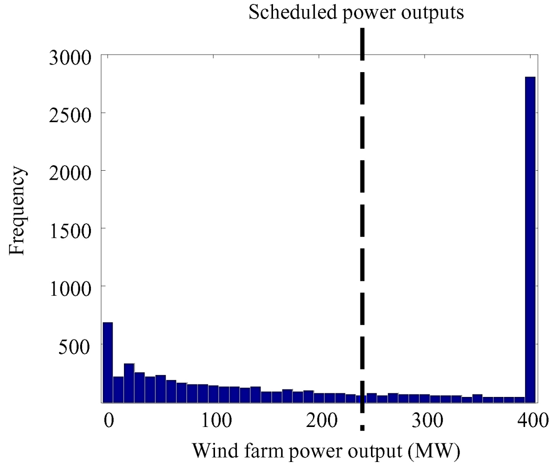

Fig. 3. Probability distribution of wind farm power output.

surplus, respectively. Since the scheduled wind power output is set artificially, the cost of wind-generated electricity from a wind farm can be considered as a kind of opportunity cost corresponding to the options of different scheduled power output. In this paper, two concepts called opportunity costs of wind power shortage and surplus are proposed to reveal the cost generated by wind generation.

B. Opportunity Cost of Wind Power Shortage

The opportunity cost of wind power shortage is defined as the cost generated by utilizing the system spinning reserve to deal with the situation in which the actual wind farm power output is lower than the scheduled power output. The following three factors must be considered to model the opportunity cost of wind power shortage: 1) the probability of wind power shortage occurrence; 2) the difference between actual wind power output and the scheduled wind power output; and 3) the adequacy of system spinning reserve. Finally, the opportunity cost of wind power shortage can be quantified as

$$C_L = K_L \cdot \Pr(P_{\mathrm{WF}} < P_{\mathrm{schedule}}) \cdot (P_{\mathrm{schedule}} - E_{P_{\mathrm{WF}} < P_{\mathrm{schedule}}}(P_{\mathrm{WF}})) \tag{2}$$

where C_L is the opportunity cost of wind power shortage ($/h), P_{schedule}, P_{WF} are the scheduled and actual wind farm power outputs (in kW), respectively, $\Pr(P_{\mathrm{WF}} < P_{\mathrm{schedule}})$ is the probability of wind power shortage occurrence, $E_{P_{\mathrm{WF}} < P_{\mathrm{schedule}}}(P_{\mathrm{WF}})$ is the expectation of wind farm power output under $P_{\mathrm{WF}} < P_{\mathrm{schedule}}$, i.e., the expectation value of left half-plane in Fig. 3 (in kW), and K_L is a coefficient representing the adequacy of system spinning reserve and the difficulty to dispatch the spinning reserve (in $/kWh).

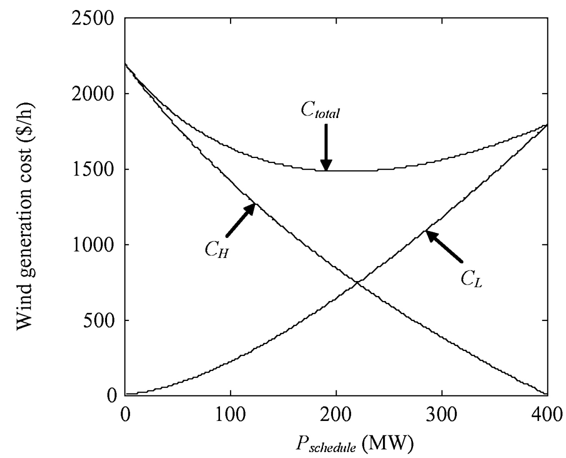

Fig. 4. Wind generation cost versus scheduled wind farm power output.

C. Opportunity Cost of Wind Power Surplus

The opportunity cost of wind power surplus is defined as the cost generated by the environmental benefit loss caused by decreasing wind farm power output. Similarly, the following three factors must be considered to model the opportunity cost of wind power surplus: 1) the probability of wind power surplus occurrence; 2) the difference between actual wind power output and the scheduled wind power output; and 3) the concerns for local environmental loss. Finally, the opportunity cost of wind power surplus can be quantified as

$$C_H = K_H \cdot \Pr(P_{\mathrm{WF}} > P_{\mathrm{schedule}}) \cdot (E_{P_{\mathrm{WF}} > P_{\mathrm{schedule}}}(P_{\mathrm{WF}}) - P_{\mathrm{schedule}}) \tag{3}$$

where C_H is the opportunity cost of wind power surplus (in $/h), P_{schedule}, P_{WF} are the scheduled and actual wind farm power outputs (in kW), respectively, $\Pr(P_{\mathrm{WF}} > P_{\mathrm{schedule}})$ is the probability of wind power surplus occurrence, $E_{P_{\mathrm{WF}} > P_{\mathrm{schedule}}}(P_{\mathrm{WF}})$ is the expectation of wind farm power output under $P_{\mathrm{WF}} > P_{\mathrm{schedule}}$, i.e., the expectation value of right half-plane in Fig. 3 (in kW), and K_H is a coefficient representing the concerns for environment by local government (in $/kWh).

The total cost of wind-generated electricity from a wind farm can be represented as the sum of the opportunity costs of wind power shortage and surplus described above, that is

$$\begin{aligned} C_{\mathrm{total}} = C_H + C_L = K_L &\cdot \Pr(P_{\mathrm{WF}} < P_{\mathrm{schedule}}) \cdot \\ &(P_{\mathrm{schedule}} - E_{P_{\mathrm{WF}} < P_{\mathrm{schedule}}}(P_{\mathrm{WF}})) \\ &+ K_H \cdot \Pr(P_{\mathrm{WF}} > P_{\mathrm{schedule}}) \cdot \\ &(E_{P_{\mathrm{WF}} > P_{\mathrm{schedule}}}(P_{\mathrm{WF}}) - P_{\mathrm{schedule}}). \end{aligned} \tag{4}$$

From (4), when the probability distribution of wind farm power output, the coefficients K_L and K_H, are confirmed, the cost of alternate sources and the environmental benefit loss can be calculated via the scheduled power output. In other words, the proposed wind farm generation cost is considered as the function of wind farm scheduled power output. Fig. 4 depicts how the proposed wind generation cost changes with different scheduled wind farm power output ($K_H = K_L = 0.01$ \$/kWh, and the installed wind power capacity is 200*2MVA).

From Fig. 4, it can be seen that with increasing of P_{schedule}, the opportunity cost of wind power shortage, C_L, increases gradually, and the opportunity cost of wind power surplus, C_H, decreases gradually. The total wind generation cost, C_{total}, decreases at first and then turns to increase. When $P_{\text{schedule}} = 208$ MW, C_{total} reaches the minimum. It should be noted that here this optimal P_{schedule} only corresponds to standalone wind farms.

Currently, the proposed wind generation cost model mainly focuses on the classic dispatching of generation applied in supervisory control and data acquisition/energy management system, and this model just corresponds to the single time period (a snap shot in time) optimal operation issue of power system, i.e., the classic OPF problem rather than the unit commitment issue in multiple time slots. Actually, once the wind farm operator (not producer) participates in the power market, the independent system operator will be responsible for the dispatching of wind power and determine the corresponding electricity price. Consider that the conventional OPF problem incorporating wind power (not in power market) is the key issue to be studied in our paper, accordingly the scheduled wind power combined with the power output of coal-fired power plant are as the variables to be optimized. In order to integrate the wind generation cost into the objective function of conventional OPF reasonably, two coefficients K_L and K_H with the dimension of electricity price are introduced into the model. The dimension of electricity price of the coefficients can combine the wind generation cost with the cost of generation of conventional power plant well. On the other hand, the two coefficients also reveal the corresponding physical meaning of the proposed wind generation model as addressed in following section. Here, it should be noted that in our proposed model, the coefficients K_L and K_H are constant. It would not be reasonable in power market environment. Strictly speaking, K_L and K_H are the function of time t, and they should be optimized combining with the unit commitment issue simultaneously. All in all, the currently proposed model and some concepts are preliminary and very limited. Future work is under way to further enhance the model especially in power market environment.

D. Comments and Discussions on K_L and K_H

In (4), coefficients K_L and K_H reflect the weights of the opportunity costs of wind power shortage and surplus in total wind generation costs. As addressed above, K_L should be related to the adequacy of system spinning reserve and the difficulty to dispatch the available power. The lower the power system reserve capacity, the higher the cost to dispatch the available generation, the larger the value of K_L. It can be seen from (2) that the unit of K_L is same as the unit of electricity price. Therefore, it can be considered that the wind farm operator will purchase a certain amount of power to cope with the situation of wind power shortage. The corresponding purchase price is K_L.

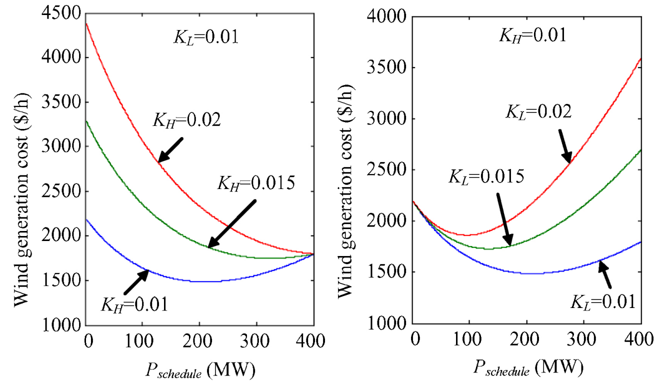

Fig. 5. Wind generation cost versus scheduled wind farm power output with changes of K_H and K_L.

Similarly, K_H should be related to the concerns for environmental benefit. The more the environment is concerned, the more the wind power should be utilized, the larger the value of K_H. The unit of K_H is also \$/kWh, namely, the unit of electricity price. Here, we consider K_H as a form of local subsidy price. Then the opportunity cost of wind power surplus means that it lost the subsidy.

Fig. 5 gives how the values of K_H and K_L affect the optimal P_{schedule}. It can be seen from Fig. 5 that the optimal P_{schedule} increases with increasing of K_H, and decreases with increasing of K_L. The reason is that the opportunity cost of wind power surplus will account for larger proportion of total wind generation cost with larger K_H, i.e., comparing with the situation of wind power shortage, the power system or government pays more attention to the environment benefit loss caused by the underutilization of existing wind power. For instance, for a power system with comparatively adequate reserve capacity, the local government provides relatively high subsides to the wind farm operator, the corresponding optimal P_{schedule} should be increased. In a similar way, with increasing of K_L, the opportunity cost of wind power shortage will account for larger proportion. At the same time, comparing with the environmental benefit, the power system is more worried about the impacts of wind power shortage on system operation. If the existing reserve capacity is insufficient, the corresponding optimal P_{schedule} should be decreased.

III. OPF Incorporating Wind Generation Cost

A. Mathematical Model

The objective of the OPF problem incorporating wind generation cost is stated as follows:

$$\text{Min}: f_1(P_G, P_{\text{schedule}}) = f_t(P_G) + f_w(P_{\text{schedule}}) \tag{5}$$

$$f_t(P_G) = \sum_{i=1}^{NG} \left(a_i + b_i P_{Gi} + c_i P_{Gi}^2\right) + \sum_{i=1}^{NG} \left|d_i \sin(e_i(\underline{P}_{Gi} - P_{Gi}))\right| \tag{6}$$

$$\begin{aligned} f_w(P_{\text{schedule}}) = {} & K_H \Pr(P_{\text{WF}} > P_{\text{schedule}}) \\ & \cdot (E_{P_{\text{WF}} > P_{\text{schedule}}}(P_{\text{WF}}) - P_{\text{schedule}}) \\ & + K_L \Pr(P_{\text{WF}} < P_{\text{schedule}}) \\ & \cdot (P_{\text{schedule}} - E_{P_{\text{WF}} < P_{\text{schedule}}}(P_{\text{WF}})). \end{aligned} \tag{7}$$

The minimization of the above function is subject to the following:

$$\Delta P_i = P_{Gi} - P_{Di} - \sum_{j \in i} V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) = 0 \ i \in N \tag{8}$$

$$\Delta Q_i = Q_{Gi} - Q_{Di} - \sum_{j \in i} V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) = 0 \ i \in N \tag{9}$$

$$\underline{P}_{Gi} \leq P_{Gi} \leq \bar{P}_{Gi} \ i \in NG \tag{10}$$

$$\underline{P}_{\text{wf}} \leq P_{\text{schedule}} \leq \bar{P}_{\text{wf}} \tag{11}$$

$$\underline{Q}_{Gi} \leq Q_{Gi} \leq \bar{Q}_{Gi} \ i \in NG \tag{12}$$

$$\underline{V}_i \leq V_i \leq \bar{V}_i \ i \in N \tag{13}$$

$$|T_{Li}| < \bar{T}_{Li} \ i \in NL \tag{14}$$

$$real(\lambda_i) < 0 \ i \in NE \tag{15}$$

where N, NG, ND, NL, and NE are the total number of buses, the total number of generators, the total number of loads, the total number of branches, and the number of eigenvalues of the state matrix, respectively, a, b, c, d, and e are the characteristic parameters of generation cost, P_{schedule}, $\underline{P}_{\text{wf}}$, and $\bar{P}_{\text{wf}}$ are the wind farm scheduled power output and its lower and upper limits, respectively, T_{Li}, and $\bar{T}_{Li}$ are the power of the ith line and its capacity limit, and real (λ_i) is the real part of the ith eigenvalue. The details of how to calculate the eigenvalues considering wind power can be found in [12]. Equation (15) denotes the small signal stability constraints. According to the theory of small signal stability [13], the eigenvalues and eigenvectors of the system state matrix can reflect the stability of the system at the operating point and the characteristics of the oscillation. Particularly, a positive real eigenvalue or real part of a complex pair of eigenvalues represents the small signal instability of test power system.

B. SAEP Computation Scheme of the OPF

Models (5)–(15) are a highly nonlinear and discrete programming problem. In our paper, a computational intelligence approach—SAEP [14], developed by author based on stochastic mechanism and evolutionary process—is applied to solving the nonlinear programming problem mentioned above. The main procedures are as follows.

1) *Construction of chromosome*: let V_i (except generators) and θ_i (except slack bus) denote the state variables, and P_{schedule}, V_{Gi}, P_{Gi} (except slack bus) the control variables. Q_{Gi} and the slack bus power output P_{GNG} are as the output variables. The values of elements in the chromosome are represented only by control variables as shown in Fig. 6.

P_{G1}	…	P_{Gi}	…	$P_{G(NG-1)}$	$P_{schedule}$	V_{G1}	…	V_{Gi}	…	V_{GNG}

Fig. 6. Construction of chromosome.

2) *Fitness function and handling of constraints*: models (5)–(15) can be rewritten as

$$\begin{cases} \min \quad F(x_i) \ \ \mathrm{x}_{i\min} \leq x_i \leq x_{i\max} \\ \text{s.t.} \ \ h_j(\mathbf{x}) = 0 \ \ j = 1, 2, \cdots, m \\ \qquad g_k(\mathbf{x}) \geq 0 \ \ k = 1, 2, \ldots, n \end{cases} \tag{16}$$

where the equality constraints are power flow equations which can be calculated directly. And the wind farm is considered as PQ bus with PF = 1 during calculation. The inequality constraints of control variables can be handled during initialization and mutation. Other inequality constraints are brought into the objective function by means of penalty mode. Particularly, the corresponding small signal stability analysis incorporating wind farm of doubly-fed induction generator (DFIG) type needs to be done with respect to a specific power flow solution in accordance with the given information of chromosome. Any calculated eigenvalue violates the small signal stability constraints as given in (15), the penalty term will be added. In this paper, an augmented objective function F_E is formed as follows:

$$F_E(x) = F(x) + \sum_{i}^{l} \left[\min(0, g_i(x))\right]^2 \cdot W_i \tag{17}$$

where $F(x)$ is the original objective function given in (5), $g_{i(\mathrm{x})}$ is the ith inequality constraint corresponding to the state variables and output variables, and W_i is the penalty factor.

The following fitness function is proposed to solve the aforementioned OPF problem:

$$f = K / F_E(x) \tag{18}$$

where K is the magnification factor.

IV. Application Example

The proposed model is examined with IEEE New England test system (10-Generator-39-Bus) shown in Fig. 7 on the MATLAB environment. The numerical data and parameters are taken from [15]. The characteristic parameters of each generator are given in the Appendix.

A. OPF Without Small Signal Stability Constraints

In this case studies, a wind farm with DFIG type is integrated to bus 1. This wind farm consists of 200 DFIGs with single 2 MW capacity. The parameters of wind turbine and DFIG can refer to the Appendix. The coefficients K_H and K_L are set to be 0.01\$/kWh and 0.02\$/kWh, respectively, during simulation. The environment parameters of SAEP are given as follows: population size 100, total generations 250.

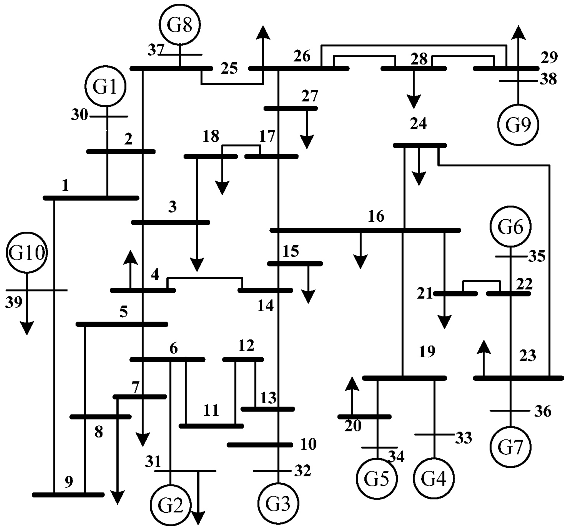

Fig. 7. IEEE New England 39-bus system.

TABLE I
RESULTS OF OPTIMAL GENERATION COST WITHOUT SMALL SIGNAL STABILITY

Generator No.	P_{Gi}	Q_{Gi}
G1	2.9968	0.9023
G2	5.9650	3.5203
G3	5.9743	2.1697
G4	5.1681	1.1665
G5	5.3772	1.4041
G6	6.4168	0.0069
G7	5.7766	1.5459
G8	5.2434	0.8532
G9	6.5601	−0.9039
G10	8.0651	0.2051
WF	3.7652	0
Total generation cost ($/h)	39 620	
Thermal generation cost ($/h)	36 237 (91.5%)	
Wind generation cost ($/h)	3383 (8.5%)	
N security	Satisfaction	

In order to illustrate the reliability and stability of SAEP, total 50 trials as shown in Fig. 8 are executed when applied to the test system. The best solution produced in the 50 trials is 39 620$/h. The worst solution is 39 689$/h. The average solution is 39 661$/h.

Table I gives the results of optimal generation cost.

It can be seen from Table I that the cost of wind-generated electricity from wind farm accounts for roughly 8.5% of total generation costs. Fig. 9 shows the dynamic optimization process applying SAEP method, which aims at illustrating the well optimization performance of the SAEP method. Two performance measures termed as online and offline performances are employed to quantitatively evaluating the dynamic optimization process of SAEP. The online performance represents the average fitness of the current population. Online performance is a measure designed to determine the ability of SAEP to perform well in optimization. It represents the on-going status for an optimization issue. The offline performance denotes the fitness of the best individual of the current population. Offline performance is a measure of convergence. It is intended to indicate the expected performance of SAEP's ability when applied in optimization issue. It can be seen from Fig. 9 that the SAEP method bears good convergence characteristics. With increasing of generation, the online performance increases rapidly at initial generations and then turns to increase smoothly. Similarly, the offline performance changes frequently at initial generations. Thereafter this change becomes smooth. Such a situation is closely related to the effects of the penalty factor given in (17) at initial generations. When more and more excellent individuals (or feasible solutions) with better fitness appear in the population, the effects of the penalty factor gradually decrease. The whole optimization process tends to be flat. In other words, the whole population is gradually reaching the global optimum with excellent convergence characteristics represented by the online and offline performances.

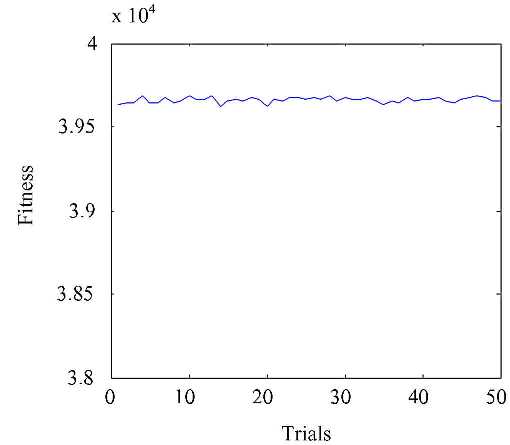

Fig. 8. Solutions over 50 trials.

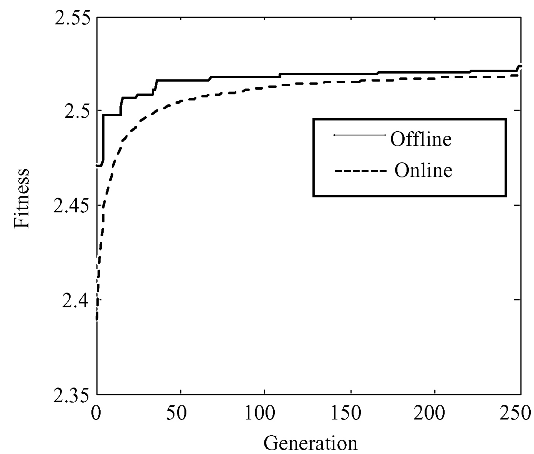

Fig. 9. Dynamic optimization curves.

Next, the wind farm is integrated to the bus2-10 with the same grid-connected conditions, respectively. The corresponding OPF results are given in Table II.

From Table II, it can be seen that the total system cost and the schedule wind farm power output are different with different integrated bus. When the wind farm is connected to Bus3, the solved scheduled wind farm power output is maximal. And the corresponding wind generation cost and the total cost are highest as well. From viewpoint of reducing

TABLE II
RESULTS OF OPTIMAL GENERATION COST WITH DIFFERENT BUS INTEGRATED

Integrated Bus No.	Scheduled Wind Farm Power Output (MW)	Total Cost ($/h)	Thermal Generation Cost ($/h)	Wind Generation Cost ($/h)
Bus1	376.52	39 620	36 237	3383
Bus2	**377.67**	**39 603**	**36 287**	**3316**
Bus3	381.12	39 729	36 258	3471
Bus4	360.63	39 697	36 419	3278
Bus5	357.77	39 686	36 468	3218
Bus6	339.90	39 714	36 626	3088
Bus7	362.73	39 655	36 409	3246
Bus8	348.05	39 646	36 542	3104
Bus9	357.26	39 698	36 448	3250
Bus10	358.94	39 709	36 496	3213

TABLE III
RESULTS OF OPTIMAL GENERATION COST WITH SMALL SIGNAL STABILITY

Generator No.	P_{Gi}	Q_{Gi}
G1	2.9406	2.0779
G2	5.9381	3.2134
G3	6.4409	4.9735
G4	5.6692	0.3675
G5	5.0409	2.7359
G6	6.5555	1.6528
G7	5.7860	−0.3239
G8	5.1754	−0.9157
G9	6.0905	0.2106
G10	8.0330	−1.0000
WF	3.6769	0
Total generation cost ($/h)	39 718	
Thermal generation cost ($/h)	36 415 (91.7%)	
Wind generation cost ($/h)	3303 (8.3%)	
N security	Satisfaction	
Small signal stability	Stable	

system total operational costs, Bus3 is not considered as a suitable integration position for the wind farm. When the wind farm is integrated into bus2, the corresponding system total cost is lowest compared with other integrated positions. In other words, Bus2 is the fittest integrated position from viewpoint of system total cost reduction. If reducing thermal generation cost is as the most concerned issue, Bus1 would be the first choice for wind farm to be connected. Similarly, Bus6 would be the most suitable integration position for wind farm from viewpoint of reducing wind generation cost.

B. OPF with Small Signal Stability Constraints

With the same simulation conditions given in scenario A, the corresponding results are shown in Table III.

Compared with the calculation results without considering small signal stability constraints, it can be found from Table III that the total generation cost increases after considering small signal stability constraints. In order to improve the small signal stability of the test system (two eigenvalues with positive real parts appear in scenario A), the wind farm has to reduce the power outputs.

Similarly, the wind farm is integrated to the bus2-10 with the same grid-connected conditions, respectively. The corresponding OPF results with consideration of small signal stability constraints are given in Table IV.

TABLE IV
RESULTS OF OPTIMAL GENERATION COST WITH DIFFERENT BUS INTEGRATED AND SMALL SIGNAL STABILITY CONSTRAINTS INVOLVED

Integrated Bus No.	Scheduled Wind Farm Power Output (MW)	Total Cost ($/h)	Thermal Generation Cost ($/h)	Wind Generation Cost ($/h)
Bus1	**367.69**	**39 718**	**36 415**	**3303**
Bus2	374.27	39 866	36 503	3363
Bus3	377.54	39 990	36 598	3392
Bus4	365.96	39 852	36 564	3288
Bus5	348.06	39 854	36 722	3132
Bus6	375.30	39 859	36 487	3372
Bus7	399.61	39 872	36 274	3599
Bus8	390.33	39 856	36 345	3511
Bus9	372.67	39 726	36 377	3348
Bus10	394.68	39 801	36 250	3552

It can be seen from Table IV that compared with the results of Table II the total system costs with different integrated bus all increase after involving small signal stability constraints. Particularly, when the wind farm is integrated into the bus1, the corresponding system total cost is lowest compared with other integrated positions. In other words, bus1 becomes the fittest integrated position after considering small signal stability constraints. Furthermore, Bus10 would be the fittest integrated position for wind farm with consideration of thermal generation cost reduction (different from the integrated position in scenario A). Similarly, Bus5 would be the first choice for wind farm to be integrated from viewpoint of reducing wind generation cost with consideration of small signal stability constraints.

V. CONCLUSION

The OPF problem incorporating wind generation cost was explored and exploited in this paper. A novel model for quantifying the wind generation cost with consideration of intermittent and fluctuant characteristics of wind power has been presented. The proposed model can make the probability distribution of wind farm power output more realistic and practical, and can simulate different wind speed distributions easily due to the utilization of Monte Carlo technique. In this proposed model, the wind farm generation cost consists of two components: the opportunity cost of wind power shortage and the opportunity cost of wind power surplus. The former is the cost of the additional reserve capacity in the case of wind power shortage and the latter is related to the environmental benefit loss caused by decreasing wind farm power outputs. Both of the two components could be attributed to the intermittence and uncertainties of wind power. The proposed wind generation cost model is integrated in the conventional OPF program as objective function with small signal stability constraints involved. A computational intelligence method-SAEP is employed to solve the OPF with wind power incorporated. The simulation results demonstrate the effectiveness and validity of the proposed model and method.

It has to be pointed out that there is not a single model which can provide a complete description of a true physical system. Every model has its pros and cons including the proposed model in this paper. The model presented in this paper fulfils the objectives of realizing the single time period based optimal operation issue of power system. Future work is under way to further enhance the model proposed involving how to introduce the factor of multiple time slots and apply in a market environment.

APPENDIX

The parameters of the wind turbine are as follows:

Parameters	Value
ρ	$1.2235\,\mathrm{kg/m^3}$
R	45 m
C_p	0.473
$V_{\text{cut-in}}$	3 m/s
$V_{\text{cut-off}}$	25 m/s
V_{rated}	10.28 m/s

The parameters of a 2 MW DFIG with rated voltage 690 V in p.u. are as follows:

Parameters	Value
R_s	0.00488
X_{ls}	0.09241
X_{lr}	0.09955
X_m	3.95279
R_r	0.00549
H	3.5 s

The characteristic parameters of the synchronous generator are given as follows:

No.	a	b	c	d	e
1	43	350	50	300	0.035
2	53	320	50	200	0.042
3	41	360	45	200	0.042
4	40	340	50	100	0.084
5	43	330	50	150	0.084
6	60	350	40	100	0.063
7	43	366	40	150	0.035
8	53	350	50	200	0.045
9	43	360	40	200	0.045
10	43	360	50	150	0.037

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Libao Shi (M'04–SM'09) received the Ph.D. degree in electrical engineering from Chongqing University, Chongqing, China, in 2000.

He is currently an Associate Professor with the National Key Laboratory of Power Systems, Graduate School at Shenzhen, Tsinghua University, Shenzhen, China. His current research interests include wind power in power systems, computational intelligence, power system optimal operation and control, energy management systems/document management systems, and power system stability analysis.

Chen Wang received the B.S. degree in electrical engineering from Tianjin University, Tianjin, China, in 2005. Currently, he is pursuing the Ph.D. degree in electrical engineering with the Graduate School at Shenzhen, Tsinghua University, Shenzhen, China.

His current research interests include impacts of wind power on power system dynamics and power system optimal operation.

Liangzhong Yao received the M.S. and Ph.D. degrees in electrical power system engineering from Tsinghua University, Beijing, China, in 1989 and 1993, respectively.

He is currently the Department Manager with Network Solutions and the Renewable Energy Technologies Programme, and is also a Technology Consultant and Senior Expert with the ALSTOM Grid Research and Technology Center, Stafford, U.K.

Dr. Yao is a fellow of the Institution of Engineering and Technology and a member of the International Council on Large Electric Systems.

Yixin Ni (SM'92) received the B.E., M.E., and Ph.D. degrees from Tsinghua University, Shenzhen, China.

She was a Professor and the Director with the National Power System Laboratory, Tsinghua University. She is currently an Associate Professor with the Department of Electrical and Electronic Engineering, University of Hong Kong, Pokfulam, Hong Kong. Her current research interests include power system stability and control, high-voltage, direct current transmission, flexible alternating current transmission systems, and power markets.

Masoud Bazargan received the B.S. degree in electrical engineering in 1983 and the M.S. degree in systems engineering in 1985, both from City University, London, U.K.

Since joining the industry, he has been mainly working on the area of power system modeling and simulation. During his career, he has been with manufacturing as well as utility and consultancy industries. He is currently the Managing Director with the ALSTOM Grid Research and Technology Center, Stafford, U.K.