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A Benders decomposition approach for a combined heat and power economic dispatch

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ABSTRACT

Recently, cogeneration units have played an increasingly important role in the utility industry. Therefore the optimal utilization of multiple combined heat and power (CHP) systems is an important optimization task in power system operation. Unlike power economic dispatch, which has a single equality constraint, two equality constraints must be met in combined heat and power economic dispatch (CHPED) problem. Moreover, in the cogeneration units, the power capacity limits are functions of the unit heat productions and the heat capacity limits are functions of the unit power generations. Thus, CHPED is a complicated optimization problem. In this paper, an algorithm based on Benders decomposition (BD) is proposed to solve the economic dispatch (ED) problem for cogeneration systems. In the proposed method, combined heat and power economic dispatch problem is decomposed into a master problem and subproblem. The subproblem generates the Benders cuts and master problem uses them as a new inequality constraint which is added to the previous constraints. The iterative process will continue until upper and lower bounds of the objective function optimal values are close enough and a converged optimal solution is found. Benders decomposition based approach is able to provide a good framework to consider the non-convex feasible operation regions of cogeneration units efficiently. In this paper, a four-unit system with two cogeneration units and a five-unit system with three cogeneration units are analyzed to exhibit the effectiveness of the proposed approach. In all cases, the solutions obtained using proposed algorithm based on Benders decomposition are better than those obtained by other methods.

1. Introduction

This paper presents an application of Benders decomposition (BD) [1,2] for a combined heat and power economic dispatch. Economic dispatch is used to determine the optimal schedule of online generating outputs so as to meet the load demand at the minimum operating cost. The conventional condensing plant delivers power at an efficiency of 35–55%. The waste heat can be captured and used to provide heating. Combined heat and power (CHP), also known as Cogeneration, is the simultaneous generation of usable heat, either for industrial use or space heating, and power, usually electricity, in a single process. Using efficient flue gas condensation, the total efficiency of CHP unit is found to be in the range of 80–111% (lower heating value base) [3–5].

As gas turbine development gathered pace in the 1970s, one response to the 1973 oil crisis was a growing chorus of opinion asking for the wider application of combined heat and power schemes. Combined heat and power will be an important contributor to energy efficiency and the reduction in energy costs for industry. Present-day worries about climate change suggest that high efficiency is required of all new thermal power plants. Combined heat and power is one solution [6]. Recently, cogeneration units have played an increasingly important role in the utility industry. Cogeneration units can provide not only electrical power but also heat to the customers. For most cogeneration units, the heat production capacities depend on the power generation and vice versa. Some complications arise in combined heat and power (CHP) systems because the dispatch has to find the set points of power and heat production with the minimum fuel cost in such a way that both demands were matched. Indeed, the CHP units should operate in a bounded power vs. heat plane [7–10].

Basically, the dispatch problem can be formulated as an optimization problem with a quadratic objective function and linear constraints. Such problems can be solved with a general-purpose package that is designed to solve quadratic programming problems, but the computational effort increases at least quadratically with the increasing number of units. The literature reports basically the traditional method to solve the ED problem adapted for CHP plants based on Lagrangian relaxation (LR) [7,8]. However, each author added some new feature to arrive to the optimal solution. For example [7] solves the combined heat and power economic dispatch (CHPED) problem based on the separability of the objective function of the problem. The separability, defined by

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the authors, is the fact that the objective function is the sum of the cost functions of separate units and most of the constraints are linked to one specific unit. In this method, a two-level strategy was adopted. The lower level solves the economic dispatch problems of the individual units for given power and heat lambdas and the upper level updates the lambda's by sensitivity coefficients. The procedure is repeated until the heat and power demands are met.

A new algorithm for CHPED problem was proposed in [8]. In this algorithm the problem was decomposed into heat and power dispatch subproblems. The two subproblems were connected by the heat-power feasible operation region constraints of cogeneration units. The analysis and interpretation of the connection have led to the development of the two-layer algorithm in which the outer layer used the LR technique to solve the power dispatch, and the inner layer used the gradient searching method to solve the heat dispatch with the unit heat capacities passed by the outer layer.

A genetic algorithm (GA) was successfully applied to solve the CHPED problem with an improved penalty function formulation [9]. A multi-objective method using a fuzzy decision index and a genetic algorithm was presented in [10]. This algorithm has been successfully applied to a sample seven-generator system. An improved genetic algorithm with multiplier updating (IGA_MU) for solving the CHPED problem was presented in [11]. This method has the merits of automatically adjusting the randomly given penalty to a proper value and requiring only a small-size population. In [12] the problem has been solved using Integrated Genetic–Tabu search algorithm. An improved algorithm based on sequential quadratic programming (SQP) method [13] to solve nonlinear constrained optimization problems, besides the logic of LR technique was proposed.

A harmony search (HS) algorithm has been applied to solve the CHPED problem. This algorithm is a new technique in the field of optimization, which does not require the strict continuity of classical search techniques. A new test system is proposed by the authors [14].

Bee colony optimization algorithm – a swarm-based algorithm – has been applied to solve the CHPED problem in [15]. This algorithm was illustrated on a test system consisting of four conventional power-only units, two cogeneration units and a heat-only unit, and the transmission loss and valve point effect have been considered.

Subbaraj et al. [16], Sadat Hosseini et al. [17] and Khorram and Jaberipour [18] applied a self-adaptive real-coded genetic algorithm (SARGA), a mesh adaptive direct search (MADS), and a harmony search algorithm to solve the CHPED problem, respectively. Also, evolutionary programming (EP) [19], multi-objective particle swarm optimization (MPSO) [20], improved ant colony search algorithm (ACSA) [21] and a customized branch-and-bound algorithm [22,23] were applied to solve this problem. In [24], a stochastic method, based on particle swarm optimization, for economic dispatch in a system that includes cogeneration units, is extended to a multi-objective formulation to include wind power and pollutant emissions constraints.

In this study, an algorithm based on Benders decomposition (BD) for solving the CHPED problem considering the cogeneration units with non-convex feasible operation region is proposed. Benders decomposition is a popular technique in the field of optimization. The BD algorithm has been successfully applied to a number of optimization problems in power systems operation and planning [25–31]. The structure of CHPED problems presents a natural decomposition scheme for the Benders approach: the variables representing the heat production are solved in the master problem while the ones representing the power production are kept in the subproblem. Therefore, at each iteration the master solution gives a heat production for which the subproblem finds the optimal power production. The Benders decomposition algorithm is an iterative algorithm. Due to the need to solve the master problem and the subproblems several times, the decomposition approach is only reasonable if these problems can be solved efficiently. This is the case for CHPED problems, where most of the times, it is much easier to solve the decomposed problems than the original one.

The four-unit system proposed in [8] which is a standard test case in this field has been used as a first case study and a new test system designed and proposed in [14] has been used as a second case study. The paper is organized as follows: The second section briefly describes the CHPED problem. The structure of the Benders decomposition algorithm and its application to the CHPED problem are explained in the third section. Section 4 presents and discusses a system consisting of a four-unit system with two cogeneration units and a five-unit system with three cogeneration units in detail. Finally, conclusions are provided in Section 5.

2. The CHP economic dispatch problem

The system under consideration has power-only units, combined heat and power units, and heat-only units. Fig. 1 shows the heat-power feasible operation region of a combined cycle cogeneration unit.

The feasible operation region is enclosed by the boundary curve ABCDEF. Along the boundary curve BC the heat capacity increases as the power generation decreases while the heat capacity decreases along the curve CD. The power output of the power units and the heat output of heat units are restricted by their own upper and lower limits.

Usually the power capacity limits of cogeneration units are functions of the unit heat productions and the heat capacity limits are functions of the unit power generations [8].

The combined heat and power economic dispatch (CHPED) problem is to determine the unit power and heat production so that the system production cost is minimized while the power and heat demands and other constraints are met [21]. The problem is formulated as:

$$\text{Min} \sum_{i=1}^{n_p} cost_i(p_i) + \sum_{j=n_p+1}^{n_p+n_c} cost_j(h_j, p_j) + \sum_{k=n_p+n_c+1}^{n_p+n_c+n_h} cost_k(h_k) \quad (1)$$

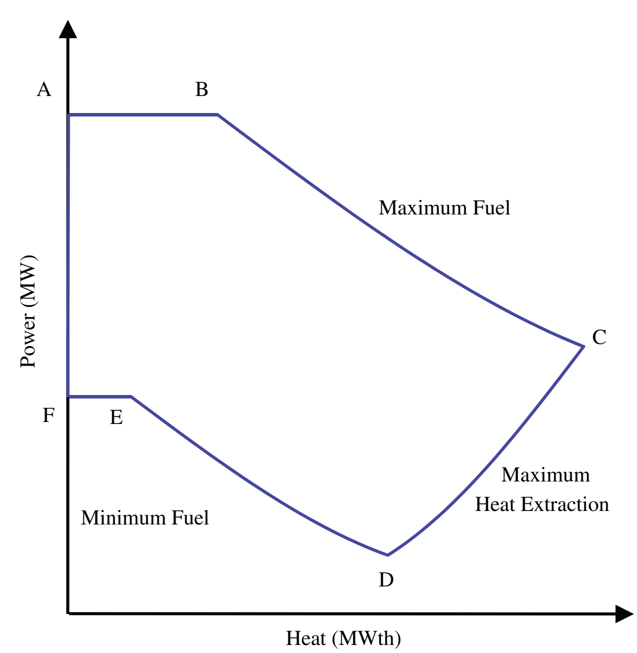

Fig. 1. Feasible operation region for a cogeneration unit.

subject to a heat and power balance constraints

$$\begin{aligned}
&\sum_{i=1}^{n_p} p_i + \sum_{j=n_p+1}^{n_p+n_c} p_j = p_d \\
&\sum_{j=n_p+1}^{n_p+n_c} h_j + \sum_{k=n_p+n_c+1}^{n_p+n_c+n_h} h_k = h_d
\end{aligned} \tag{2}$$

and the capacity limits constraints

$$\begin{aligned}
&p_i^{\min} \leqslant p_i \leqslant p_i^{\max}, \quad i = 1, \ldots, n_p \\
&p_j^{\min}(h_j) \leqslant p_j \leqslant p_j^{\max}(h_j), \quad j = n_p + 1, \ldots, n_p + n_c \\
&h_j^{\min}(p_j) \leqslant h_j \leqslant h_j^{\max}(p_j), \quad j = n_p + 1, \ldots, n_p + n_c \\
&h_k^{\min} \leqslant h_k \leqslant h_k^{\max}, \quad k = n_p + n_c + 1, \ldots, n_p + n_c + n_h
\end{aligned} \tag{3}$$

in which *cost* is the unit production cost; p is the unit power generation; h is the unit heat production; h_d and p_d are the system heat and power demands; i, j and k are the indices of conventional power units, cogeneration units and heat-only units, respectively; n_p, n_c and n_h are the numbers of the kinds of units mentioned above; $p^{\min}$ and $p^{\max}$ are the unit power capacity limits and $h^{\min}$ and $h^{\max}$ are the unit heat capacity limits. It is obvious that the complication arising in CHP economic dispatch is the mutual dependencies of extra constraints in contrast to the pure economic dispatch.

3. Benders decomposition algorithm

Benders decomposition [1] is one of the commonly used decomposition techniques for combinatorial optimization problems. Benders decomposition decomposes the original problem into a master problem and several subproblems. The lower bound solution of the master problem may involve fewer constraints. The subproblems will examine the solution of the master problem to see if the solution satisfies the remaining constraints. If the subproblems are feasible, the upper bound solution of the original problem will be calculated while forming a new objective function for the further optimization of the master problem solution. If any of the subproblems is infeasible, an infeasibility cut representing the least satisfied constraint will be introduced to the master problem. Then, a new lower bound solution of the original problem will be obtained by re-calculating the master problem with more constraints. The final solution based on the Benders decomposition algorithm may require iterations between the master problem and subproblems. When the upper bound and the lower bound are sufficiently close, the optimal solution of the original problem will be achieved.

To advantageously apply the Benders decomposition, the problem under consideration should have the appropriate structure. There are two such structures. The first is characterized by complicating constraints, and the second by complicating variables. The complicating constraints and variables are those that complicate the solution of the problem, or prevent a straightforward solution of the problem or a solution by blocks, i.e., they make the problem more difficult to solve [32].

In this section we describe the application of Benders decomposition to CHPED problem.

3.1. Complicating variables

The Benders decomposition is analyzed below to address CHPED problem as a nonlinear problem with decomposable structure and complicating variables.

The problem structure required to apply advantageously the Benders decomposition is:

$$\begin{aligned}
\text{minimize} \quad & Cost(\boldsymbol{h}, \boldsymbol{p}) \\
\text{subject to} \quad & \\
& b(\boldsymbol{p}) \leqslant 0 \\
& c(\boldsymbol{h}) \leqslant 0 \\
& d(\boldsymbol{h}, \boldsymbol{p}) \leqslant 0
\end{aligned} \tag{4}$$

where $\boldsymbol{h} = [h_{n_p+1}, \ldots, h_{n_p+n_c}, \ldots, h_{n_p+n_c+n_h}]$ is the heat production vector and $\boldsymbol{p} = [p_1, \ldots, p_{n_p}, \ldots, p_{n_p+n_c}]$ is the unit power generation vector. $b(\boldsymbol{p}) \leqslant 0, c(\boldsymbol{h}) \leqslant 0$ and $d(\boldsymbol{h}, \boldsymbol{p}) \leqslant 0$ are equality and inequality constraints of power only unit, heat only unit and cogeneration unit respectively. The problem includes both equality and inequality constraints. Variables $\boldsymbol{h}$ are considered simply as complicating variables, i.e., variables that if fixed to given values render a simple or decomposable problem. It should be noted that in the case of CHPED problem, we also can consider the variables $\boldsymbol{p}$ as complicating variables. The steps of the Benders decomposition approach are depicted in Fig. 2 and are described as follows:

Step 0: Initialization. Find feasible values for the heat production (complicating variables) $\boldsymbol{h}_0$, so that $c(\boldsymbol{h}_0) \leqslant 0$

Set $v = 1$, $\boldsymbol{h}^{(v)} = \boldsymbol{h}_0$ and $C_{down}^{(v)} = -\infty$.

The problem includes both equality and inequality constraints. $\boldsymbol{h}_0$ is an initial values of heat production vector, v is a number of iteration and $C_{down}^{(v)}$ is lower bound of the objective function at vth iteration.

Step 1: Subproblem solution. Solve subproblem

$$\begin{aligned}
\text{minimize} \quad & Cost(\boldsymbol{h}, \boldsymbol{p}) \\
\text{subject to} \quad & \\
& b(\boldsymbol{p}) \leqslant 0 \\
& d(\boldsymbol{h}, \boldsymbol{p}) \leqslant 0 \\
& \boldsymbol{h} = \boldsymbol{h}^{(v)} : \boldsymbol{\lambda}^{(v)}
\end{aligned} \tag{5}$$

where $\boldsymbol{h}^{(v)}$ and $\boldsymbol{p}^{(v)}$ show the values of heat production and power production vector at vth iteration. The solution of the problem above provides values for the $\boldsymbol{p}^{(v)}$ and the dual variable vector associated with those constraints that fix the complicating variables, $\boldsymbol{h}$, to given values. The values of the dual variables, also called shadow prices, give the sensitivity of the objective function optimal value to changes in the constraints. This sensitivity vector is denoted by $\boldsymbol{\lambda}^{(v)}$ $(\lambda_r = -(\partial Cost(\boldsymbol{h}, \boldsymbol{p})/\partial h_r)|_{(\boldsymbol{h}^{(v)}, \boldsymbol{p}^{(v)})} r = n_p + 1, \ldots, n_p + n_c + n_h)$.

The upper bound of the objective function at vth iteration is $C_{up}^{(v)} = Cost(\boldsymbol{h}^{(v)}, \boldsymbol{p}^{(v)})$. The information obtained solving the subproblem allows reproducing more and more accurately the original

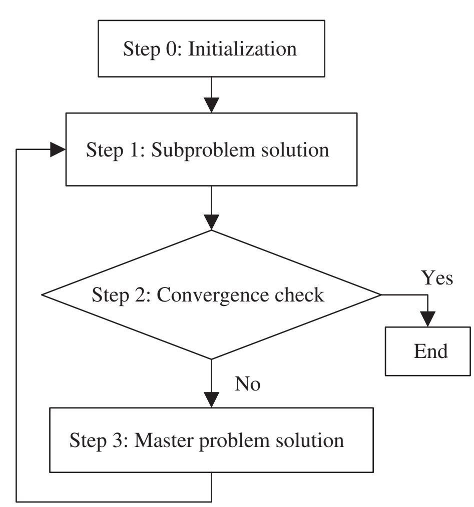

Fig. 2. The flow chart of the Benders decomposition algorithm.

problem. The solution of this subproblem provides $\boldsymbol{p}^{(\nu)}$, $Cost(\boldsymbol{h}^{(\nu)}, \boldsymbol{p}^{(\nu)})$ and $\lambda^{(\nu)}$.

Update the objective function upper bound, $C_{up}^{(\nu)} = Cost(\boldsymbol{h}^{(\nu)}, \boldsymbol{p}^{(\nu)})$.

Step 2: Convergence check. If the duality gap is smaller than the tolerance $\varepsilon(|C_{up}^{(\nu)} - C_{down}^{(\nu)}| \leqslant \varepsilon)$, the solution with a level of accuracy (small tolerance value to control convergence) of the objective function is:

$$\begin{aligned} \boldsymbol{h}^* &= \boldsymbol{h}^{(\nu)} \\ \boldsymbol{p}^* &= \boldsymbol{p}^{(\nu)} \end{aligned} \tag{6}$$

Otherwise, the algorithm continues with the next step.

Step 3: Master problem solution. Update the iteration counter: $\nu \leftarrow \nu + 1$. Solve the master problem:

$$\begin{aligned} &\text{minimize } \alpha \\ &\text{subject to} \\ &\alpha \geqslant Cost(\boldsymbol{h}^{(i)}, \boldsymbol{p}^{(i)}) + \sum_{r=n_p+1}^{n_p+n_c+n_h} \lambda_r^{(i)}(h_r - h_r^{(i)}); \forall i = 1, \ldots, \nu - 1 \\ &c(\boldsymbol{h}) \leqslant 0 \end{aligned} \tag{7}$$

The auxiliary function $\alpha(\boldsymbol{h})$ expresses the objective function of the original CHPED problem (total costs) as a function solely of the complicating variables ($\boldsymbol{h}$). Note that at every iteration a new constraint is added. The solution of the master problem provides $\boldsymbol{h}^{(\nu)}$ and $\alpha^{(\nu)}$.

Update objective function lower bound, $C_{down}^{(\nu)} = \alpha^{(\nu)}$.

The algorithm continues in Step 1.

3.2. Non-convex feasible operation region

The CHPED problem has equality and inequality constraints. Equality constraints arise from heat and power balance constraints and inequality constraints arise from feasible operation region of cogeneration units and limitations on power and heat generation of power-only units and heat-only units, respectively. The CHPED problem is convex, if the feasible operation region of each cogeneration unit is a convex polygon in terms of heat and power generation, and the production cost is a convex function of the generated heat and power. The feasible operation region of the cogeneration unit is convex, if any point on the line segment connecting the two feasible points is feasible. In case the operating cost at any point on the line segment is not higher than the corresponding linear combination of the operating costs at the end points, the production cost function is convex. Although in most cases the production cost is a convex function, the feasible operation region of advanced cogeneration units is non-convex. Traditional back-pressure CHP unit can be modeled as a convex unit. But the backpressure cogeneration unit with condensing and auxiliary cooling options, gas turbines, and combined gas and steam cycles can result in non-convex feasible operation regions. Therefore, CHPED problem is non-convex. One way to tide over this difficulty is to decompose the non-convex feasible operation region into a number of convex sub-regions and then used the conventional optimization approaches such as Lagrangian multiplier, nonlinear programming, quadratic programming, mixed interior programming, and interior point programming to solve the problem. In this way, the CHPED problem must be solved for all possible combination of convex sub-regions. However, this may require a large computational burden to obtain an optimal solution when a system has several cogeneration units with non-convex feasible operation regions.

Su and Chiang [11] and Subbaraj et al. [16] expressed the feasible operation region of cogeneration units by means of inequality constraints, whereas the feasible operation region of unit 3 in Example 1 (second cogeneration unit) is a non-convex polygon and cannot be expressed in the form $g_i(x) \leqslant 0$. This means that the inequality constraints for the feasible operation region are not exactly correct. In other words, by considering a set of inequality constraints as a feasible operation region, some of the constraints eliminate parts of actual feasible operation region which may include a global operating point [33].

Benders decomposition algorithm, not only makes the CHPED problem easier to solve, but also enables us to consider the non-convex feasible operation region more accurately and more efficiently. As mentioned above, although the feasible operation regions of cogeneration units cannot be expressed as a set of inequality constraints, by choosing a feasible value for heat production in the master problem (h_{master}), there will be only bounds on the power generation. As a result, in the subproblem, inequality constraints for cogeneration units can be expressed in the form $p_{\min} \leqslant p \leqslant p_{\max}$, as shown in Fig. 3.

4. Simulation result

In this section, two examples which are taken from the previous literature are used to show the validity and effectiveness of the proposed algorithm. The first example has been previously solved using a variety of other techniques (both evolutionary and traditional mathematical programming methods), and the second example has been proposed in [14].

4.1. Example 1

A system which consists of a conventional power-only unit, two cogeneration units and a heat-only unit is considered. The diagrams of feasible operation regions of units 2 and 3 are illustrated in Fig. 4. This case was discussed in [8,9,11–14,16,17,19–21]. The cost functions and capacity limits of the aforementioned units are shown in Eq. (8). The evolution of the proposed method and the obtained result for this example are given in Appendix A, and to show how the proposed algorithm works, two iterations of BD on Example 1 are presented in detail in Appendix B. The results obtained for this example using BD algorithm are given in Table 1, and the results are compared with those of LR, GA, IGA_MU, GT, SQP, HS, SARGA, MADS, EP, MPSO and ACSA.

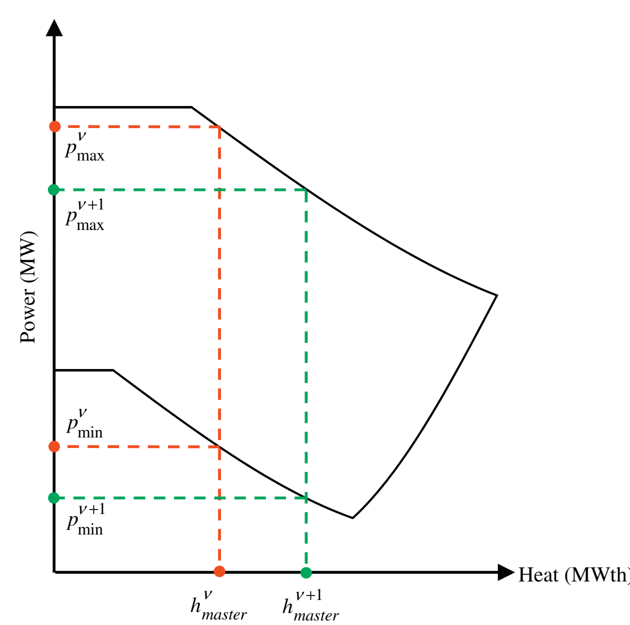

Fig. 3. The heat production in the master problem and the corresponding power capacity limits in the subproblem.

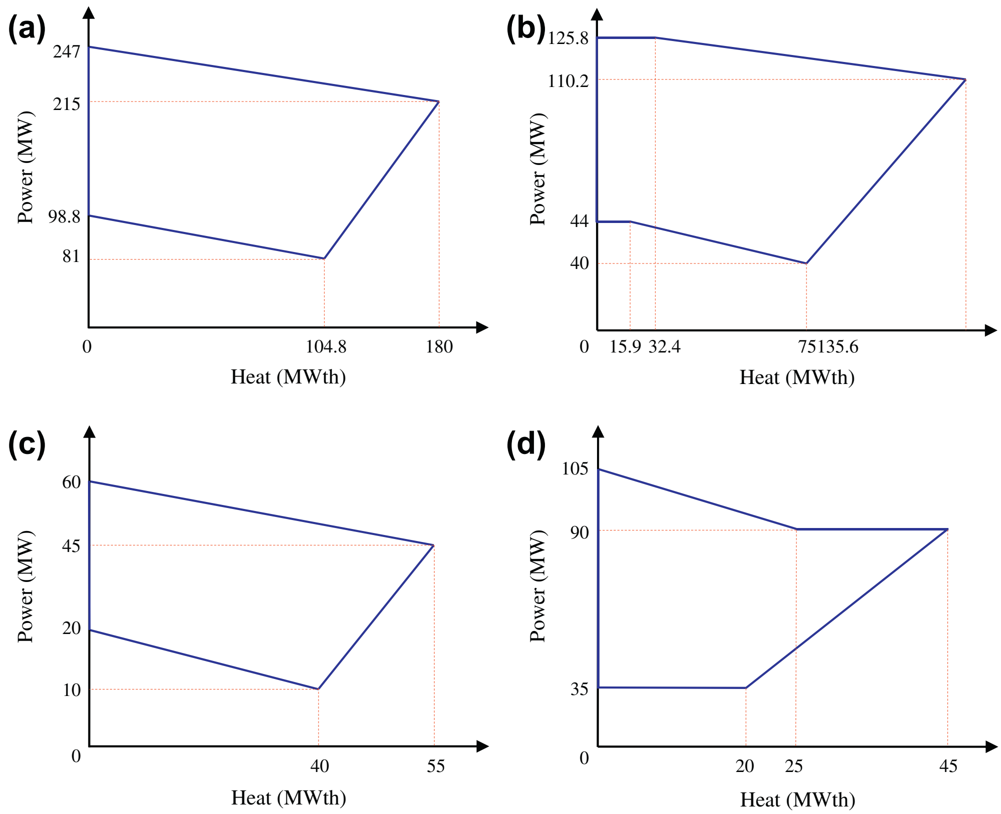

Fig. 4. Feasible operation region for (a) the second unit of Example 1, (b) the third unit of Example 1 and second unit of Example 2, (c) the third unit of Example 2, (d) the fourth unit of Example 2.

Table 1
Comparison of various methods for Example 1.

Methods	$P_d = 200$			$h_d = 115$			Cost ($)
	p_1	p_2	p_3	h_2	h_3	h_4	
LR [8]	0.0	160	40	40	75	0.0	9257.1
GA1 [9]	0.0	159.23	40.77	39.94	75.06	0.0	9267.2
GA2 [9]	0.08	150.93	49.00	48.84	65.79	0.37	9452.2
IGA_MU [11]	0.00	160.00	40.00	39.99	75.00	0.00	9257.07
GT [12]	0.00	157.92	42.08[a]	26.00	89.00[a]	0.00	9207.64
SQP [13]	0	160	40	40	75	0	9257.1
HS [14]	0.00	160.00	40.00	40.00	75.00	0.00	9257.07
SARGA [16]	0.00	159.99	40.01	39.99	75.00	0.00	9257.07
MADS [17]	0.00	160.00	40.00	40.00	75.00	0.00	9257.07
EP [19]	0.00	160.00	40.00	40.00	75.00	0.00	9257.10
MPSO [20]	0.05	159.43	40.57	39.97	75.03	0.00	9265.10
ACSA [21]	0.08	150.93	49.00	48.84	65.79	0.37	9452.2
BD (present study)	0.00	160.00	40.00	40.00	75.00	0.00	9257.07

[a] Outside the feasible operation region of cogeneration unit 3.

$$
\begin{aligned}
& min\ Cost = \sum_{i=1}^{4} cost_i \\
& cost_1 = 50p_1 \\
& cost_2 = 2650 + 14.5p_2 + 0.0345p_2^2 + 4.2h_2 + 0.03h_2^2 + 0.031p_2h_2 \\
& cost_3 = 1250 + 36p_3 + 0.0435p_3^2 + 0.6h_3 + 0.027h_3^2 + 0.011p_3h_3 \\
& cost_4 = 23.4h_4 \\
& 0 \leqslant p_1 \leqslant 150\ \text{MW} \\
& 0 \leqslant h_4 \leqslant 2695.2\ \text{MWth} \\
& p_1 + p_2 + p_3 = p_d \\
& h_2 + h_3 + h_4 = h_d
\end{aligned}
\tag{8}
$$

The system power and heat demands are 200 (MW) and 115 (MWth), respectively.

It can be observed from Table 1 that the result obtained using the proposed algorithm is the same as the best known solution reported previously in the literature. Although a better solution is reported in [12], this solution is not feasible due to violation of the constraints. It seems that this case is not a strong test to validate the presented algorithm for CHPED problem. The reason is that, in this case, the linear cost functions have been used for power and heat characteristics of the power-only unit (unit 1) and the heat-only unit (unit 4), respectively. In view of this fact, the productions of these units do not appear in the first equation of Karush–Kuhn–Tucker first-order conditions. Moreover, the linear coefficients in the cost function of the power-only unit and the

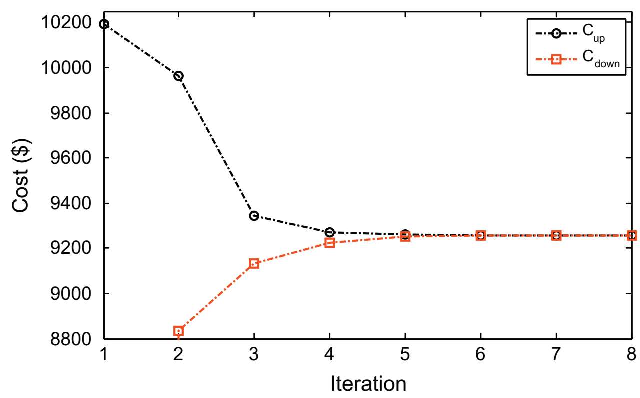

Fig. 5. The Benders algorithm evolution in Example 1.

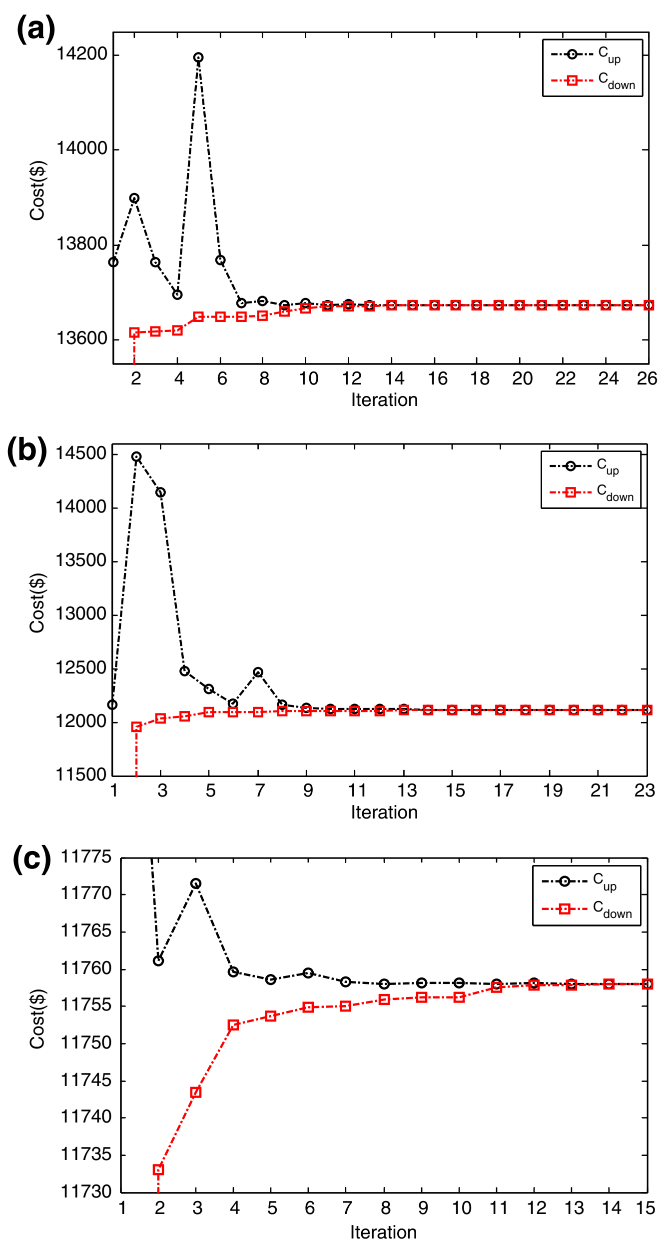

Fig. 6. The Benders algorithm evolution in Example 2 (a) case I, (b) case II and (c) case III.

heat-only unit have been selected to be larger than power and heat partial derivatives attributed to the cost functions of cogeneration units. Therefore, the power-only unit and the heat-only unit have been set at the minimum. It can be inferred from Table A.2 that the reason behind the easy solution of Example 1 is that the power generation of unit 1 and the heat generation of unit 4 are passive to the solution process (due to being at minimum output), and effectively reduce the number of variables to solve. Fig. 5 provides the convergence nature of the presented method for Example 1.

4.2. Example 2

This case study was originally proposed in [14]. This problem consists of a conventional power unit, three cogeneration units and a heat-only unit. The cogeneration unit 2 was taken from the previous case study and the diagrams of feasible operation regions of units 3 and 4 are illustrated in Fig. 4. The cost functions and capacity limits of the aforementioned units are shown in Eqs. (9)–(11). The test system is considered for three power and heat demands. The power and heat demands in cases I, II and III are 300 (MW) and 150 (MWth), 250 (MW) and 175 (MWth), and 160 (MW) and 220 (MWth), respectively.The objective function of the CHPED problem is:

$$Min\ Cost = \sum_{i=1}^{5} cost_i \tag{9}$$

where

$$\begin{aligned}
cost_1 &= 254.8863 + 7.6997p_1 + 0.00172p_1^2 + 0.000115p_1^3 \\
cost_2 &= 1250 + 36p_2 + 0.0435p_2^2 + 0.6h_2 + 0.027h_2^2 + 0.011p_2h_2 \\
cost_3 &= 2650 + 34.5p_3 + 0.1035p_3^2 + 2.203h_3 + 0.025h_3^2 + 0.051p_3h_3 \\
cost_4 &= 1565 + 20p_4 + 0.072p_4^2 + 2.3h_4 + 0.02h_4^2 + 0.04p_4h_4 \\
cost_5 &= 950 + 2.0109h_5 + 0.038h_5^2
\end{aligned} \tag{10}$$

Subject to:

$$\begin{aligned}
&35 \leqslant p_1 \leqslant 135\ \text{MW} \\
&0 \leqslant h_5 \leqslant 60\ \text{MWth} \\
&p_1 + p_2 + p_3 + p_4 = p_d \\
&h_2 + h_3 + h_4 + h_5 = h_d
\end{aligned} \tag{11}$$

The convergence behavior of proposed method for cases I, II and III of Example 2 are illustrated in Fig. 6. The evolution of the proposed method and obtained result for cases I, II and III of Example 2 are given in Appendix A. Optimal results obtained by proposed algorithm, GA and HS Algorithm [14] are shown in Table 2. It can be observed in this table that the best performance is obtained by the proposed algorithm for cases I, II and III of Example 2. In these cases, Benders decomposition algorithm provides superior performance compared to the available methods in the literature.

5. Conclusion

In this paper, the problem of combined heat and power economic dispatch (CHPED) is approached. The difficulty of solving this highly complex and intricate problem can be overcome using Benders decomposition technique, which allows its analysis in a complete and practical way. Real-world problems by employing our method are solved. Simulation results confirm the validity of the intuition supporting the algorithm. The tests on the Example 2

Table 2
Compared results of the previous methods and the BD (present study) for Example 2.

Case	Method	p_d	h_d	p_1	p_2	p_3	p_4	h_2	h_3	h_4	h_5	Cost ($)
I	GA [14]	300	150	135.00	70.81	10.84	83.28	80.54	39.81	0.00	29.64	13779.50
	HS [14]			134.74	48.20	16.23	100.85	81.09	23.92	6.29	38.70	13723.20
	BD (present study)			135.0000	40.7687	19.2313	105.0000	73.5957	36.7759	0.0000	39.6284	13672.83
II	GA [14]	250	175	119.22	45.12	15.82	69.89	78.94	22.63	18.40	54.99	12327.37
	HS [14]			134.67	52.99	10.11	52.23	85.69	39.73	4.18	45.40	12284.45
	BD (present study)			135.0000	40.0000	10.0000	65.0000	75.0000	40.0000	14.4029	45.5971	12116.60
III	GA [14]	160	220	37.98	76.39	10.41	35.03	106.0	38.37	15.84	59.97	11837.40
	HS [14]			41.41	66.61	10.59	41.39	97.73	40.23	22.83	59.21	11810.88
	BD (present study)			42.1454	64.6296	10.0000	43.2250	96.2614	40.0000	23.7386	60.0000	11758.06

Table A.1
Evolution of the BD algorithm for the Example 1.

Iteration (v)	λ_2	λ_3	λ_4	C_{down}	C_{up}	Error
1	11.1446	1.8845	23.4000	$-\infty$	10194.569380	∞
2	7.7236	33.7470	23.3999	8836.519384	9961.496037	1124.976653
3	10.7816	23.3932	23.4000	9131.768281	9345.577724	213.809443
4	11.6561	4.0898	23.4000	9222.291672	9269.481304	47.189632
5	11.4949	20.9781	23.3999	9251.786100	9263.418662	11.632562
6	11.5602	4.1744	23.4000	9256.948437	9257.098005	0.149568
7	11.5594	20.7594	23.3999	9257.032526	9257.127896	0.095370
8	11.5600	20.7576	23.4000	9257.074996	9257.075003	0.000007

Table A.2
Power and heat economic dispatch for Example 1.

Iteration (v)	p_1	p_2	p_3	h_2	h_3	h_4	Cost ($)
1	0.0000	157.1023	42.8977	34.5737	32.1871	48.2392	10194.5694
2	0.0000	113.6634	86.3366	0.000	115.0000	0.0000	9961.4960
3	0.0000	150.5982	49.4018	31.8839	83.1161	0.0000	9345.5777
4	0.0000	159.8877	40.1123	41.6595	73.3405	0.0000	9269.4813
5	0.0000	159.2132	40.7868	39.3208	75.6792	0.0000	9263.4187
6	0.0000	159.9998	40.0002	40.0031	74.9969	0.0000	9257.0980
7	0.0000	159.9933	40.0067	39.9942	75.0058	0.0000	9257.1279
8	0.0000	160.0000	40.0000	40.0000	75.0000	0.0000	9257.0750

Table A.3
Evolution of the BD algorithm for the Case 1 of Example 2.

Iteration (v)	λ_2	λ_3	λ_4	λ_5	C_{down}	C_{up}	Error
1	5.0791	5.2791	10.6844	3.6607	$-\infty$	13764.854714	∞
2	38.2460	-2.8809	5.9000	6.5709	13615.970456	13899.872528	283.902072
3	13.9764	2.3229	6.1920	6.5709	13617.343984	13763.716798	146.372814
4	4.8165	4.1694	9.1646	6.5709	13619.812959	13696.142708	76.329749
5	1.9178	84.8393	5.9001	5.4685	13647.540472	14195.527965	547.987493
6	3.7204	28.5360	6.0618	5.3363	13648.705775	13767.437645	118.731870
7	4.4910	6.2149	9.5942	5.2415	13649.540390	13676.897760	27.357370
8	7.6463	4.4581	8.8117	4.9776	13651.865407	13680.862637	28.997230
9	4.9652	4.8666	9.6032	5.3783	13659.671801	13673.984938	14.313137
10	5.1843	5.3373	9.7136	4.2985	13667.081775	13677.463278	10.381503
11	4.8451	5.2346	9.6738	4.8849	13670.410146	13673.832827	3.422681
12	6.0449	4.8362	9.3510	4.8763	13671.063332	13674.210062	3.146730
13	5.0975	4.9926	9.6413	4.9747	13671.500484	13672.914622	1.414138
14	4.8540	5.0661	9.6380	5.1734	13672.383525	13673.278707	0.895182
15	4.9510	5.1054	9.6540	4.9721	13672.617466	13672.987345	0.369879
16	4.9824	5.0091	9.6358	5.1012	13672.657458	13672.891603	0.234145
17	5.0170	5.0421	9.6456	4.9956	13672.747190	13672.843931	0.096741
18	5.0461	4.9899	9.6366	5.0493	13672.796413	13672.857676	0.061263
19	5.0079	5.0192	9.6398	5.0486	13672.815490	13672.840684	0.025194
20	5.0506	5.0113	9.6419	5.0492	13672.821410	13672.838706	0.017296
21	5.0379	5.0151	9.6416	5.0153	13672.824962	13672.837473	0.012511
22	5.0168	5.0280	9.6426	5.0209	13672.828209	13672.834848	0.006639
23	5.0245	5.0143	9.6402	5.0348	13672.831810	13672.835991	0.004181
24	5.0266	5.0204	9.6417	5.0212	13672.832710	13672.834359	0.001649
25	5.0187	5.0215	9.6413	5.0300	13672.833840	13672.834648	0.000808
26	5.0212	5.0237	9.6420	5.0226	13672.834037	13672.834135	0.000098

revealed the superiority of the presented method with respect to other reported methods. The application of Benders decomposition divides the original CHPED optimization into a master problem and subproblems. The subproblem generates Benders cuts corresponding to heat limit violations at provided power added to the master problem for re-calculating the heat generation. The key feature of the proposed approach is that it solves CHPED problem with non-convex constraints on cogeneration feasible operation region. In addition, the solution obtained from Benders decomposition algorithm is feasible at each iteration. It means that the algorithm provides usable solution, even if it is terminated after a finite number of iterations and before it has converged to the global solution. As a result, the approach based on Benders decomposition is an efficient and practical algorithm to solve the non-convex CHPED problem.

Appendix A.

A.1. Example 1

The evolution of the proposed method and the obtained result for Example 1 are tabulated in Tables A.1 and A.2 respectively. In Table A.1, λ_2, λ_3 and λ_4 represent the dual variable associated with three equality constraints ($h_i = h_i^{master}$ for $i = 2, 3, 4$).

A.2. Example 2

The evolution of the proposed method and the obtained result for Example 2 are tabulated in Tables A.3–A.8 respectively. In this

Table A.4
Power and heat economic dispatch for the case 1 of Example 2.

Iteration (ν)	p_1	p_2	p_3	p_4	h_2	h_3	h_4	h_5	Cost ($)
1	135.0000	47.8078	21.4672	95.7250	73.2083	39.6251	15.4583	21.7083	13764.854714
2	132.6237	57.3763	20.0000	90.0000	90.0000	0.0000	0.0000	60.0000	13899.872528
3	135.0000	49.4183	18.2826	97.2991	83.1303	6.8697	0.0000	60.0000	13763.716798
4	135.0000	40.2855	19.7145	105.0000	70.7818	19.2182	0.0000	60.0000	13696.142708
5	123.2744	41.7256	45.0000	90.0000	49.5046	55.0000	0.0000	45.4954	14195.527965
6	135.0000	41.1665	29.7869	94.0466	57.7652	48.4801	0.0000	43.7547	13767.437645
7	135.0000	41.1092	18.8908	105.0000	63.6816	43.8103	0.0000	42.5081	13676.897760
8	135.0000	45.9810	14.0190	105.0000	80.1630	30.8018	0.0000	39.0352	13680.862637
9	135.0000	40.1586	19.8414	105.0000	72.6574	33.0345	0.0000	44.3081	13673.984938
10	135.0000	41.7906	18.2094	105.0000	76.3819	43.5183	0.0000	30.0998	13677.463278
11	135.0000	41.8216	18.1784	105.0000	70.0946	42.0901	0.0000	37.8153	13673.832827
12	135.0000	42.6721	17.3279	105.0000	77.3066	34.9904	0.0000	37.7030	13674.210062
13	135.0000	40.5775	19.4225	105.0000	75.0219	35.9808	0.0000	38.9973	13672.914622
14	135.0000	41.0972	18.9028	105.0000	70.4067	37.9815	0.0000	41.6118	13673.278707
15	135.0000	41.1818	18.8182	105.0000	72.1846	38.8524	0.0000	38.9630	13672.987345
16	135.0000	40.7457	19.2543	105.0000	72.8553	36.4832	0.0000	40.6615	13672.891603
17	135.0000	40.8564	19.1436	105.0000	73.4734	37.2545	0.0000	39.2721	13672.843931
18	135.0000	40.6096	19.3904	105.0000	74.0625	35.9590	0.0000	39.9785	13672.857676
19	135.0000	40.7670	19.2330	105.0000	73.3237	36.7069	0.0000	39.9694	13672.840684
20	135.0000	40.8153	19.1847	105.0000	73.3759	36.7219	0.0000	39.9022	13672.838706
21	135.0000	40.7238	19.2762	105.0000	73.8885	36.5796	0.0000	39.5319	13672.837473
22	135.0000	40.7967	19.2033	105.0000	73.4821	36.9124	0.0000	39.6055	13672.834848
23	135.0000	40.7321	19.2679	105.0000	73.6381	36.5737	0.0000	39.7882	13672.835991
24	135.0000	40.7563	19.2437	105.0000	73.6711	36.7201	0.0000	39.6088	13672.834359
25	135.0000	40.7672	19.2328	105.0000	73.5235	36.7516	0.0000	39.7249	13672.834648
26	135.0000	40.7687	19.2313	105.0000	73.5957	36.7759	0.0000	39.6284	13672.834135

Table A.5
Evolution of the BD algorithm for the Case 2 of Example 2.

Iteration (ν)	λ_2	λ_3	λ_4	λ_5	C_{down}	C_{up}	Error
1	3.5925	2.4882	6.0850	5.6557	$-\infty$	12157.382752	∞
2	48.9572	92.5555	3.7000	2.0109	11958.276650	14475.766130	2517.489480
3	51.1720	−2.9915	3.7001	3.4870	12037.026005	14147.931311	2110.905306
4	25.9542	37.0504	3.8593	4.9736	12053.702446	12480.744834	427.042388
5	3.8826	39.8832	4.3127	6.5708	12091.882393	12305.820687	213.938294
6	18.9475	28.1994	4.7410	6.3409	12091.950448	12168.220139	76.269691
7	25.5543	1.0604	3.9130	6.0670	12098.285262	12470.879294	372.594032
8	18.7389	2.1961	4.7781	6.5709	12101.709266	12164.974279	63.265013
9	4.2591	26.2943	4.9581	6.5709	12102.798552	12130.855162	28.056610
10	17.1853	2.5299	5.4466	5.5130	12104.504607	12118.472704	13.968097
11	4.3019	2.4209	5.1276	6.2647	12106.057762	12125.814946	19.757184
12	4.3525	25.2909	5.5949	5.2248	12106.952563	12124.029700	17.077137
13	4.3604	2.5633	5.6319	5.2548	12109.374438	12119.272270	9.897832
14	4.3708	25.2582	5.3475	5.7316	12114.891332	12117.448439	2.557107
15	4.3708	2.5079	5.3500	5.7336	12115.224917	12117.736904	2.511987
16	4.3858	24.9494	5.4776	5.4769	12116.015119	12116.653479	0.638360
17	4.3857	2.5437	5.4781	5.4775	12116.099258	12116.736957	0.637699
18	17.0267	2.5611	5.5438	5.3460	12116.590036	12116.894988	0.304952
19	17.0778	2.5498	5.4979	5.4341	12116.595907	12116.673631	0.077724
20	17.1042	2.5440	5.4743	5.4795	12116.598936	12116.620047	0.021111
21	4.3872	2.5407	5.4608	5.5054	12116.600648	12116.609507	0.008859
22	4.3877	2.5427	5.4687	5.4904	12116.600687	12116.602904	0.002217
23	4.3880	2.5437	5.4729	5.4824	12116.600685	12116.601231	0.000546

Table A.6
Power and heat economic dispatch for case 2 of Example 2.

Iteration (v)	p_1	p_2	p_3	p_4	h_2	h_3	h_4	h_5	Cost ($)
1	135.0000	41.0519	11.0313	62.9168	59.4583	35.8750	31.7083	47.9584	12157.382752
2	77.8713	92.1287	45.0000	35.0000	120.0000	55.0000	0.0000	0.0000	14475.766130
3	89.7944	110.2000	15.0056	35.0000	135.6000	19.9775	0.0000	19.4225	14147.931311
4	135.0000	62.6997	13.3181	38.9822	94.5954	41.4220	0.0000	38.9826	12480.744834
5	135.0000	40.4138	24.2694	50.3168	68.8855	46.1154	0.0000	59.9991	12305.820687
6	135.0000	43.0419	10.9349	61.0232	77.6260	40.4006	0.0000	56.9734	12168.220139
7	135.0000	61.6596	13.0170	40.3234	93.6976	27.9321	0.0000	53.3703	12470.879294
8	135.0000	42.6083	10.6036	61.7881	77.2516	37.5857	0.1627	60.0000	12164.974279
9	135.0000	40.1181	10.1234	64.7585	73.2543	40.0529	1.6928	60.0000	12130.855162
10	135.0000	40.1643	10.0170	64.8187	75.1418	39.9319	13.8466	46.0797	12118.472704
11	135.0000	40.0778	10.1964	64.7258	73.8501	39.2147	5.9636	55.9716	12125.814946
12	135.0000	40.0491	10.7114	64.2395	74.2745	40.3049	18.1327	42.2879	12124.029700
13	135.0000	40.0477	10.1076	64.8447	74.2959	39.5696	18.4515	42.6830	12119.272270
14	135.0000	40.0110	10.0041	64.9849	74.8378	40.0018	11.2039	48.9565	12117.448439
15	135.0000	40.0109	10.0277	64.9614	74.8382	39.8896	11.2889	48.9833	12117.736904
16	135.0000	40.0032	10.0001	64.9967	74.9521	40.0000	14.4432	45.6047	12116.653479
17	135.0000	40.0033	10.0071	64.9896	74.9521	39.9716	14.4632	45.6131	12116.736957
18	135.0000	40.0125	10.0000	64.9875	75.0108	40.0000	16.1069	43.8823	12116.894988
19	135.0000	40.0055	10.0000	64.9945	75.0047	40.0000	14.9535	45.0418	12116.673631
20	135.0000	40.0019	10.0000	64.9981	75.0016	40.0000	14.3585	45.6399	12116.620047
21	135.0000	40.0000	10.0000	65.0000	74.9998	40.0000	14.0197	45.9805	12116.609507
22	135.0000	40.0000	10.0000	65.0000	74.9999	40.0000	14.2177	45.7824	12116.602904
23	135.0000	40.0000	10.0000	65.0000	75.0000	40.0000	14.3226	45.6774	12116.601231

Table A.7
Evolution of the BD algorithm for the Case 3 of Example 2.

Iteration (v)	λ_2	λ_3	λ_4	λ_5	C_{down}	C_{up}	Error
1	45.1656	79.0897	50.0429	6.5709	$-\infty$	11845.820273	∞
2	46.4403	−2.8110	44.3912	6.5709	11733.058486	11761.161986	28.103500
3	46.4664	75.4195	43.8690	6.5709	11743.377418	11771.470912	28.093494
4	45.9524	75.2038	47.4010	6.5709	11752.441354	11759.687826	7.246472
5	46.2358	75.0447	45.6345	6.5709	11753.725183	11758.561768	4.836585
6	46.1027	−2.8297	46.5294	6.5709	11754.856602	11759.533445	4.676843
7	46.0959	75.1068	46.5198	6.5709	11755.050433	11758.314589	3.264156
8	46.1670	−2.8245	46.0782	6.5709	11755.848888	11758.068518	2.219630
9	46.1668	75.0649	46.0781	6.5709	11756.192062	11758.110963	1.918901
10	46.1318	−2.8264	46.2994	6.5709	11756.244368	11758.077014	1.832646
11	46.1494	−2.8254	46.1888	6.5709	11757.554308	11758.063785	0.509477
12	46.1406	75.0763	46.2440	6.5709	11757.846693	11758.074679	0.227986
13	46.1538	75.0695	46.1611	6.5709	11757.905126	11758.065095	0.159969
14	46.1582	−2.8249	46.1335	6.5709	11757.966102	11758.064357	0.098255
15	46.2472	75.0224	45.5720	6.5709	11758.064272	11758.064293	0.000021

Table A.8
Power and heat economic dispatch for case 3 of Example 2.

Iteration (v)	p_1	p_2	p_3	p_4	h_2	h_3	h_4	h_5	Cost ($)
1	35.2902	57.3763	15.835	51.4985	90.0000	42.5007	27.4993	60.0000	11845.823646
2	43.9692	66.7283	10.0065	39.2960	98.0731	39.9742	21.9527	60.0000	11761.161986
3	44.1639	66.9277	10.8361	38.0723	98.2452	40.3583	21.3965	60.0000	11771.470912
4	40.8029	63.1401	10.0570	46.0000	94.9756	40.0244	25.0000	60.0000	11759.687826
5	42.6812	65.2271	10.0276	42.0641	96.7772	40.0119	23.2109	60.0000	11758.561768
6	41.7131	64.2261	10.0071	44.0537	95.9131	39.9716	24.1153	60.0000	11759.533445
7	41.7538	64.1950	10.0138	44.0374	95.8862	40.0059	24.1079	60.0000	11758.314589
8	42.2261	64.7194	10.0000	43.0545	96.3389	40.0000	23.6611	60.0000	11758.068518
9	42.2246	64.7179	10.0035	43.0540	96.3376	40.0015	23.6609	60.0000	11758.110963
10	41.9927	64.4598	10.0000	43.5475	96.1148	40.0000	23.8852	60.0000	11758.077014
11	42.1095	64.5896	10.0000	43.3009	96.2269	40.0000	23.7731	60.0000	11758.063785
12	42.0509	64.5245	10.0005	43.4241	96.1707	40.0002	23.8291	60.0000	11758.074679
13	42.1385	64.6219	10.0001	43.2395	96.2548	40.0000	23.7452	60.0000	11758.065095
14	42.1678	64.6545	10.0000	43.1777	96.2829	40.0000	23.7171	60.0000	11758.064357
15	42.1454	64.6296	10.0000	43.2250	96.2614	40.0000	23.7386	60.0000	11758.064293

tables, λ_2, λ_3, λ_3 and λ_5 represent the dual variable associated with four equality constraints ($h_i = h_i^{master}$ for $i = 2, 3, 4, 5$).

Appendix B.

B.1. Illustration of the iterative procedure of BD on Example 1

To clarify how the BD works, two iterations of proposed algorithm on Example 1 are presented. If variables $\boldsymbol{h}$ are considered to be complicating variables, the CHPED problem is solved using the Benders decomposition algorithm.

The objective function of the CHPED problem is

$$min\ Cost = \sum_{i=1}^{4} cost_i$$

where

$$\begin{aligned} cost_1 &= 50p_1 \\ cost_2 &= 2650 + 14.5p_2 + 0.0345p_2^2 + 4.2h_2 + 0.03h_2^2 + 0.031p_2h_2 \\ cost_3 &= 1250 + 36p_3 + 0.0435p_3^2 + 0.6h_3 + 0.027h_3^2 + 0.011p_3h_3 \\ cost_4 &= 23.4h_4 \end{aligned}$$

subjected to the equality and inequality constraints ($b(\boldsymbol{p})$, $c(\boldsymbol{h})$ and $d(\boldsymbol{h}, \boldsymbol{p})$):

$$\begin{aligned} &b1:\ p_1 + p_2 + p_3 = 200 \\ &b2:\ 0 - p_1 \leqslant 0 \\ &b3:\ p_1 - 150 \leqslant 0 \\ &b4:\ 44 - p_3 \leqslant 0\ if\ h_2 \leqslant 15.9 \\ &b5:\ p_3 - 125.8 \leqslant 0 \\ &c1:\ h_2 + h_3 + h_4 = 115 \\ &c2:\ 0 - h_2 \leqslant 0 \\ &c3:\ 0 - h_3 \leqslant 0 \\ &c4:\ 0 - h_4 \leqslant 0 \\ &c5:\ h_4 - 2695.2 \leqslant 0 \\ &d1:\ 1.781914894\ h_2 - p_2 - 105.7446809 \leqslant 0 \\ &d2:\ 0.177777778\ h_2 + p_2 - 247.0 \leqslant 0 \\ &d3:\ -0.169847328\ h_2 - p_2 + 98.8 \leqslant 0 \\ &d4:\ 1.158415842\ h_3 - p_3 - 46.88118818 \leqslant 0 \\ &d5:\ 0.151162791\ h_3 + p_3 - 130.6976744 \leqslant 0 \\ &d6:\ -0.067681895\ h_3 - p_3 + 45.07614213 \leqslant 0\ if\ h_2 \geqslant 15.9 \end{aligned}$$

Note that the feasible operation region of unit 3 in this case (second cogeneration unit) is a non-convex polygon and cannot be expressed in the form $g_i(x) \leqslant 0$ and therefore inequality constraints $b4$ and $d6$ presented in the conditional form.

The solution algorithm proceeds as follows.

Step 0: Initialization. The iteration counter is initialized to $v = 1$. The initial values for the complicating variables $\boldsymbol{h}$ are found such that $0 \leqslant h_2 \leqslant 180, 0 \leqslant h_3 \leqslant 135.6, 0 \leqslant h_4 \leqslant 2695.2$ and $h_2 + h_3 + h_4 = 115$. These values are obtained simply by using linear programming techniques as follows: $h_2^{(1)} = 34.5737, h_3^{(1)} = 32.1871$ and $h_4^{(1)} = 48.2392$ .The lower bound of the objective function is set to $C_{down}^{(1)} = -\infty$.

Step 1: Subproblem solution. The subproblem below is solved.

$$\begin{aligned} \min \quad & Cost = 50p_1 + 2650 + 14.5p_2 + 0.0345p_2^2 + 4.2h_2 + 0.03h_2^2 \\ & \quad + 0.031p_2h_2 + 1250 + 36p_3 + 0.0435p_3^2 + 0.6h_3 \\ & \quad + 0.027h_3^2 + 0.011p_3h_3 + 23.4h_4 \\ \text{s.t.} \quad & 0 \leqslant p_1 \leqslant 150 \\ & 92.9277 \leqslant p_2 \leqslant 240.8536 \\ & 42.8977 \leqslant p_3 \leqslant 125.8 \\ & p_1 + p_2 + p_3 = 200 \\ & h_2 = 34.5737 :\ \lambda_2 \\ & h_3 = 32.1871 :\ \lambda_3 \\ & h_4 = 48.2392 :\ \lambda_4 \end{aligned}$$

whose solution is $p_1^{(1)} = 0, p_2^{(1)} = 157.1023, p_3^{(1)} = 42.8977, \lambda_2^{(1)} = 11.1446, \lambda_3^{(1)} = 1.8845$ and $\lambda_4^{(1)} = 23.4$ with an objective function value $Cost$ = 10194.5694. The upper bound of the objective function optimal value is $C_{up}^{(1)} = 10194.5694$.

Step 2: Convergence check. The expression $|C_{up}^{(1)} - C_{down}^{(1)}| = \infty$ is not small enough, therefore, the procedure continues in Step 3.

Step 3: Master problem solution. The iteration counter is updated, $v = 1 + 1 = 2$. The master problem below is solved.

$$\begin{aligned} &\text{minimize} \quad \alpha \\ &\quad \text{s.t.} \\ &\qquad \alpha \geqslant C_{up}^{(1)} + \lambda_2^{(1)}(h_2 - h_2^{(1)}) + \lambda_3^{(1)}(h_3 - h_3^{(1)}) + \lambda_4^{(1)}(h_4 - h_4^{(1)}) \\ &\qquad 0 \leqslant h_2 \leqslant 180 \\ &\qquad 0 \leqslant h_3 \leqslant 135.6 \\ &\qquad 0 \leqslant h_4 \leqslant 2695.2 \\ &\qquad h_2 + h_3 + h_4 = 115 \end{aligned}$$

The solution of this problem is $h_2^{(2)} = 0, h_3^{(2)} = 115, h_4^{(2)} = 0$ and $\alpha^{(2)}$ = 8836.5194. The lower bound of the objective function optimal value is $C_{down}^{(2)} = \alpha^{(2)} = 8836.5194$. The procedure continues in Step 1.

Step 1: Subproblem solution. The subproblem below is solved.

$$\begin{aligned} \min \quad & Cost = 50p_1 + 2650 + 14.5p_2 + 0.0345p_2^2 + 4.2h_2 \\ & \quad + 0.03h_2^2 + 0.031p_2h_2 + 1250 + 36p_3 + 0.0435p_3^2 \\ & \quad + 0.6h_3 + 0.027h_3^2 + 0.011p_3h_3 + 23.4h_4 \\ & \text{s.t.} 0 \leqslant p_1 \leqslant 150 \\ & 98.8 \leqslant p_2 \leqslant 247 \\ & 86.3366 \leqslant p_3 \leqslant 113.314 \\ & p_1 + p_2 + p_3 = 200 \\ & h_2 = 0 : \lambda_2 \\ & h_3 = 115 : \lambda_3 \\ & h_4 = 0 : \lambda_4 \end{aligned}$$

whose solution is $p_1^{(2)} = 0,\ p_2^{(2)} = 113.6634,\ p_3^{(2)} = 86.3366,\ \lambda_2^{(2)} = 7.7236,\ \lambda_3^{(2)} = 33.7470$ and $\lambda_4^{(2)} = 23.3999$ with an objective function value $Cost$ = 9961.4960. The upper bound of the objective function optimal value is $C_{up}^{(2)} = 9961.4960$.

Step 2: Convergence check. The expression $|C_{up}^{(2)} - C_{down}^{(2)}| = 1124.98$ is not small enough, therefore, the procedure continues in Step 3.

Step 3: Master problem solution. The iteration counter is updated, $v = 2 + 1 = 3$. The master problem below is solved.

$$\begin{aligned} &\text{minimize} \quad \alpha \\ &\quad \text{s.t.} \\ &\qquad \alpha \geqslant C_{up}^{(1)} + \lambda_2^{(1)}(h_2 - h_2^{(1)}) + \lambda_3^{(1)}(h_3 - h_3^{(1)}) + \lambda_4^{(1)}(h_4 - h_4^{(1)}) \\ &\qquad \alpha \geqslant C_{up}^{(2)} + \lambda_2^{(2)}(h_2 - h_2^{(2)}) + \lambda_3^{(2)}(h_3 - h_3^{(2)}) + \lambda_4^{(2)}(h_4 - h_4^{(2)}) \\ &\qquad 0 \leqslant h_2 \leqslant 180 \\ &\qquad 0 \leqslant h_3 \leqslant 135.6 \\ &\qquad 0 \leqslant h_4 \leqslant 2695.2 \\ &\qquad h_2 + h_3 + h_4 = 115 \end{aligned}$$

The solution of this problem is $h_2^{(3)} = 31.8839, h_3^{(3)} = 83.1161, h_4^{(3)} = 0$ and $\alpha^{(3)}$ = 9131.7683. The lower bound of the objective function optimal value is $C_{down}^{(3)} = \alpha^{(3)} = 9131.7683$. The procedure continues in Step 1.

The iterates generated by the BD algorithm converge to the optimal solution. Detailed result of proposed algorithm on Example 1 is given in Tables A.1 and A.2.

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