Simulation of a Permanent Magnet Synchronous Motor using Matlab-Simulink

Aishwarya Apte[1], Rahee Walambe[2], Vrunda Joshi[3], Kirti Rathod [4] and Jaywant Kolhe [5]

***Abstract*—In the recent past, the use of permanent magnet synchronous motors (PMSMs) has increased considerably owing to their inherent advantages. The high performance speed and/or position control of a PMS motor requires an accurate knowledge of rotor shaft position and angular speed in order to synchronize the phase excitation pulses to the rotor position. The vector control method is used for dynamic speed control. In this paper mathematical model of PMSM was developed using PARK's transformation. The conventional vector control of drive is implemented using SVPWM. In order to test and validate the control scheme, the model is developed in Matlab - Simulink and is tested for various operating conditions. MATLAB version R10 is used.**

Keywords: PMSM Model, Park's Transformation, Modified Parks Transformation, SVPWM, Vector Control

I. INTRODUCTION

Recently, the development and easier deployment of permanent magnet materials has driven an increased use of permanent magnet synchronous motors (PMSMs) in high performance variable speed motors for many industrial applications [1][2].Inherent advantages of using a PMSM drive include: a high ratio of torque to weight, high power factor, faster response, rugged construction, easy maintenance, ease of control, high efficiency.The PMSMs are employed in the industrial field, especially in the small and medium power drives. The cost increment is due to the presence of precious permanent magnets in the inductor system. The PM motors are widely adopted in the high performance drives, where the particular performances justify the costs, higher than other motors with the same torque but realized with a different technology. PMSM are used in high-accuracy direct-drive applications mainly due to their advantages. Compared to conventional DC motors, they have no brushes or mechanical commutators, which eliminate the problems due to mechanical wear of the moving parts. In addition, the better heat dissipation characteristic and ability to operate at high speed make them a better option. The PMSM and Brushless DC (BLDC) motor are very similar. The difference being that in BLDC the back emf is varying trapezoidally while in PMSM the waveform of back emf is sinusoidal. The focus of this research is mainly in two areas:

1) Development and implementation of the SVPWM based standard Vector control drive for a PMSM.
2) Extension of this (with-sensor) vector control drive to the sensor-less vector control where the position and/or speed sensors are replaced by some standard sensor-less methodology.

A ready-to-use PMSM-drive block is available in Simulink(R10).It is based on the modified Park's transformation [1][2][3][4][5]. However, we required a model based on Park's transformation[2] .Since most of the literature [5][6][7][8] uses the standard Park's model based on [2]. Also, we did not see any specific advantage of using a modified Park's transformation for the PMSM mathematical model. It is important to note that the later versions of Simulink (R12, R13) provides an option to use either the modified Parks transformations or Parks transformation based PMSM. Retrospectively, this and the aforementioned difficulties led us to develop our own model.For the first part of the research objectives, the standard vector control strategy was required. The ready-to-use Simulink block of PMSM drive works on the current hysteresis based control which was not required. The SVPWM for generating the inverter pulses require the transformation to $\alpha-\beta$ reference frame. For the sensorless vector control drive, we required the electrical quantities in $\alpha-\beta$ reference frame which were not easily available in the ready-to-use model since the current hysteresis control do not provide the electrical quantities in $\alpha-\beta$ reference frame.Also for futher development in sensorless algorithms, we needed the PMSM model in Per Unit system which was not avaiable directly in Simulink. Due to the above mentioned difficulties, we developed our own Matlab-Simulink model of the whole PMSM speed control drive. Our contribution lies in development of a vector control based constant speed drive for a PMSM in Matlab-Simulink. All the standard assumptions and mathematical relationships are followed.

The paper is organized as follows. Section 2 gives construction principle and classification of PMSM. Section 3 discusses vector control of PMSM. Section 4 deals with mathematical model of PMSM. Section 5 discusses simulation results. Section 6 discusses results and conclusion.

II. PMSM CONSTRUCTION AND OPERATION

The permanent magnet synchronous motor is a rotating electric machine where the construction of the stator is

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similar to that of an induction motor and the conventional synchronous motor. The rotor is a permanent magnet rotor.

A. Structure and operating principle

The stator and the rotor both are cylindrical and laminated iron material and they are separated by an air gap[1]. The magnets are mounted in the rotor and their differential magnetic permeability is very similar to that of the air. Therefore, according to their placement in the rotor and to the design of the latter, it is possible to obtain rotor structures isotropic and anisotropic from the magnetic point of the view. The first can be obtained by placing the magnets on the external surface of the rotor, the latter is obtained with the magnets buried inside the rotor. Generally the PM synchronous machines can be classified as follow:
SPM - Surface-mounted Permanent Magnet;
IPM - Interior Permanent Magnet
In machine with an isotropic rotor electromechanical conversion is actuated according to principle of electrodynamic systems, as for the DC motor. Conversion is based on interaction between the flux generated by stator currents and the magnetic fields created by permanent magnets. In SPM motors the conductors subjected to the forces are placed in the stator, while the magnets are on the rotor. In anisotropic machine the electromechanical conversion follows double principle of the electrodynamic and reluctance systems. The torque generated by this motor is given by the sum of the contributions of these two mechanisms. The winding can be fed by an external three-phase source through the terminals of each phase.The control of these motors is however rather complex since motor dynamics are nonlinear and multivariable and critical parameters such as load torque are uncertain. A major drawback of PMSM is the need for rotor position sensor, such as a high resolution encoder or resolver, for proper control of the inverter switches. This information is required to maintain stator flux at the angle with respect to the rotor field generated by the rotor magnet. This is known as electronic commutation. To resolve this problem, the stator current must be commuted in synchronism with the rotor position. This is achieved with rotor position information taken from position sensor The whole PMSM fed by a PWM inverter constitutes a non linear system which is relatively difficult to control by the linear commands. Moreover the PWM inverter transistors only need discrete command signals. Consequently it would be more convenient to use a non linear command system based on the two level regulation technique for the PMSM. In recent years many models have been made in the study of synchronous static machine converter. A typical approach has been based on the PMSM vector control. The PMSM vector command needs a precise knowledge of the rotor position which ensures the machine self driving. This knowledge can be directly obtained by a position sensor or indirectly by a speed sensor.

III. VECTOR CONTROL SCHEME OF PMSM

Vector control is also known as decoupling or field orientated control. The flux is controlled by the field current alone and this current may be termed as, flux-producing current. Keeping the field current constant at any time instant and hence the flux constant, the torque is controlled independently by the armature current alone, and then, this armature current may be seen as the torque-producing current. Controlling the field and armature current magnitudes as they are dc variables, the flux and electromagnetic torque are controlled precisely in a separately excited dc motor drive. The problem of dynamic control can be handled if such separate and independent controls of flux and torque are obtained. The key to it, lies in finding an equivalent flux-producing current and torque-producing current (i.e., the armature current) in ac machines leading to the control of the flux and torque channels in them[1][6][7][8][9].

The control scheme of PMSM is presented in Figure 1. The field-oriented controller is based on a current-controlled

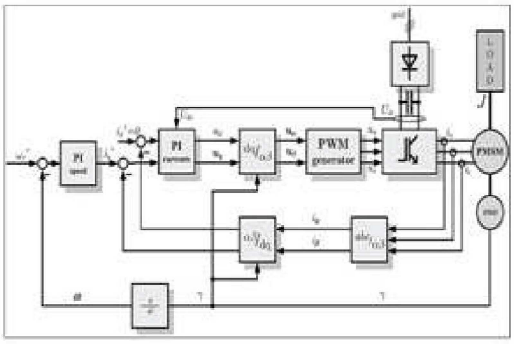

Fig. 1. Vector Control Scheme of PMSM

voltage source inverter structure . The current control loops are arranged in the 2-phase synchronously rotating rotor reference frame d-q aligned with rotor flux (also rotor position θ), while the rotor position and speed detection operates in the 2- phase stationary reference frame $\alpha - \beta$. To produce electromagnetic torque, in general, a rotor flux and a stator mmf has to be present that are stationary with respect to each other but having a nonzero phase shift between them. In a PMSM, the necessary rotor flux is present due to rotor PMs. Currents in the stator windings generate the stator mmf. The zero relative speed between the stator mmf and the rotor flux is achieved if the stator mmf is revolving at the same speed as the rotor flux, i.e., rotor speed and also in the same direction. The revolving stator mmf is the result of injecting a set of polyphase currents phase shifted from each other by the same amount of phase shift between the polyphase windings. For example, a three-phase machine with three windings shifted in space by electrical 120^0 from each other and injected with currents shifted by the same amount of electrical 120^0 between them produces a rotating magnetic field constant in magnitude and travelling at the angular frequency of the currents.

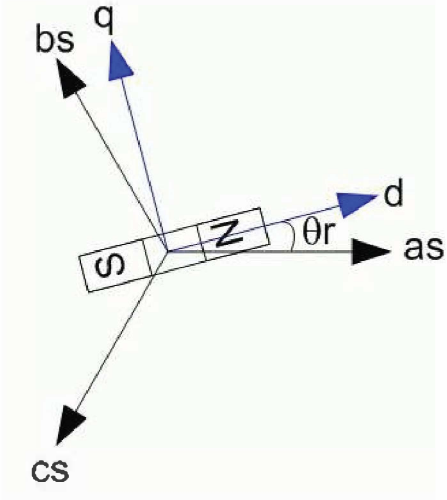

Fig. 2. Reference frames and assumed current directions in original Park's Transformation

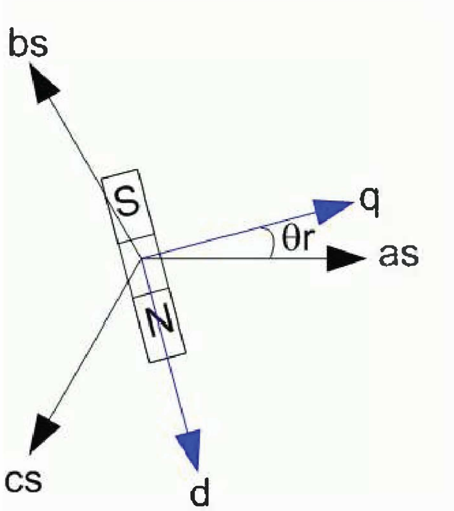

Fig. 3. Reference frames and current directions for modified Park's Transform

IV. MATHEMATICAL MODEL OF A PMSM

The mathematical model is derived based on the following assumptions:

- The stator windings are balanced with sinusoidally distributed magnetomotive force (mmf).
- The inductance versus rotor position is sinusoidal.
- The saturation and parameter changes are neglected.
- The three-phase machine is assumed to have balanced windings and balanced inputs.

Assuming that each of the three-phase windings has T_1 turns per phase, and equal current magnitudes, the two-phase windings will have $3T_1/2$ turns per phase for mmf equality. The d- and q-axes mmfs are found by resolving the mmfs of the three phases along the d- and q-axes[1],[2].The common term, i.e., the number of turns in the winding, is cancelled on either side of the equations leaving the current equal. The q-axis here is assumed to be lagging behind the a-axis by θ_r. The relationship between dqo and abc currents is given by different types of transformation matrices.

A. Transformations used in PMSM Modelling

1) Park's Transformation (Original): The transformation originally proposed by Park uses the reference axes as shown in Figure 2. In this, θ is the angle between d-axis of 2-phase reference frames and a-axis of 3-phase reference frames. The q axis leads the d-axis by 90^0. The assumed current directions are as marked above. The Park's transformation (i.e. transformation from 3-ph to 3-ph is given as :

$$V_{dq0} = T * V_{abc} \tag{1}$$

where,

$$[T] = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ -\sin\theta_r & -\sin(\theta_r - \frac{2\pi}{3}) & -\sin(\theta_r + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

and Inverse Park's Transformation is:

$$V_{abc} = T^{-1} * V_{dq0} \tag{2}$$

where,

$$T^{-1} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r & 1 \\ \cos(\theta_r - \frac{2\pi}{3}) & -\sin(\theta_r - \frac{2\pi}{3}) & 1 \\ \cos(\theta_r + \frac{2\pi}{3}) & -\sin(\theta_r + \frac{2\pi}{3}) & 1 \end{bmatrix}$$

There is another transformations which are used by some authors [5,6], which is termed as modified Park's Transformation.

2) Modified Parks' Transformation: In this, θ is the angle between a axis of 3-phase reference frames and q-axis of 2-phase reference frames[1]. The q axis leads the d-axis by 90^0. The assumed current directions are as marked in Figure 3. It is important to note that the modified Park's Transformation eliminate the negative signs from the transformation matrix. Also, the trigonometric projections on q and d axes differ from the original Park's transform. The modified Park transformation [6] is more convenient for vector control because the maximum phase induction occurs at $\theta = 0$. Modified Park's Transformation:

$$V_{qd0} = T * V_{abc} \tag{3}$$

where,

$$[T] = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin\theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Inverse of Modified Park's Transform:

$$V_{abc} = T^{-1} * V_{qd0} \tag{4}$$

where,

$$T^{-1} = \begin{bmatrix} \cos\theta_r & \sin\theta_r & 1 \\ \cos(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r - \frac{2\pi}{3}) & 1 \\ \cos(\theta_r + \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) & 1 \end{bmatrix}$$

B. Equivalent Circuits

The equivalent circuit for the d-axis and q-axis of the PMSM can be derived from the stator equations. The equivalent circuits are useful in system studies, particularly with regard to faults. Based on the equivalent circuits, the expressions for voltage V_d and V_q can be obtained in terms of i_d and i_q as below:

$$v_d = R_s * i_d + L_d * \frac{di_d}{dt} - \omega_r L_q i_q \quad (5)$$

$$v_q = R_s * i_q + L_q \frac{di_q}{dt} + \omega_r L_d i_d + \omega_r \lambda_f \quad (6)$$

C. Expression for Electromagnetic Torque T_e

The electromagnetic torque is the most important output variable that determines the mechanical dynamics of the machine such as the rotor position and speed. Therefore, its importance cannot be overstated in all the simulation studies. It is derived from the machine matrix equation by looking at the input power and its various components such as resistive losses, mechanical power, and the rate of change of stored magnetic energy.

$$T_e = \frac{3}{2} * \frac{P}{2} [\lambda_f + (L_q - L_d) i_d] \, i_q (N.m) \quad (7)$$

The Mechanical model consisting of the swing equation is given as:

$$T_e = J * \frac{d\omega_r}{dt} + B * \omega_r + T_m \quad (8)$$

The relationships between rotor position and rotor angular velocity is given as:

$$\frac{d\theta_r}{dt} = \omega_r \quad (9)$$

Equations 5-9 listed above give the complete Mathematical model of the PMSM. This model is developed in Simulink and is tested. Next section demonstrates the Simulink mod (Figure 4 and 5) and the subsequent results obtained.

V. MATLAB-SIMULINK SIMULATIONS

We developed the PMSM dynamic model using bo methods of transformations listed above, viz; Original Park and Modified Parks[6]-[10].SVPWM technique is used f pulse generation[11].For simulations, we have used Park transformation and results are presented in the next section. simulations, the PU model of PMSM drive based on [1] used. The Motor Parameters used are listed in Table 1.

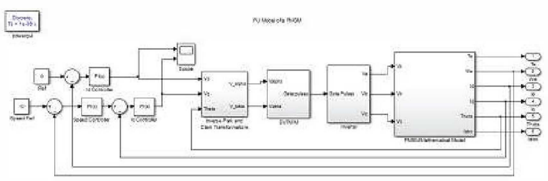

Fig. 4. Simulink model of a PMSM Drive

No	Parameter	Value
1	Poles	4
2	D-axis Inductance (Ldn)	0.2155
3	Q-axis Inductance (Lqn)	0.2155
4	Resistance (Rn)	0.0875
5	Inverter DC Link Voltage	1.5674
6	Rated Torque (TLn)	1
7	Reference Speed	0.7268
8	Inertia (Jn)	0.0151
9	Frictional Constant (Bn)	0.0378
10	Flux Linkages (Lambdan)	1
11	Stator Frequency(Ws)	376.9911
12	Base Speed(Wb)	518.5934 rps
13	Base Power	890W
14	Base current	4.65A
15	Base Torque	2.43 Nm

TABLE I
MOTOR PARAMETERS(PU)

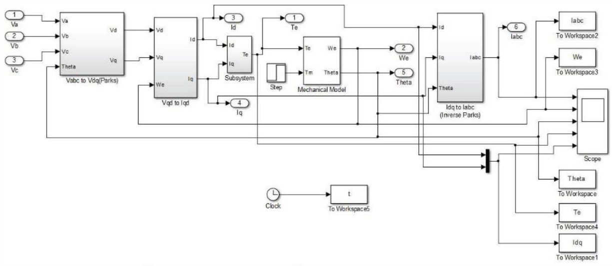

Fig. 5. Simulink model of a PMSM Drive

It is important to note that the rotor is surface mounted and hence $L_d = L_q$. For simulation study and verification of the model, the motor model is converteted into per unit and motor is started with no load and is loaded at 0.2s with different values of load torque. The results obtained are shown in Figure 6 - 8.

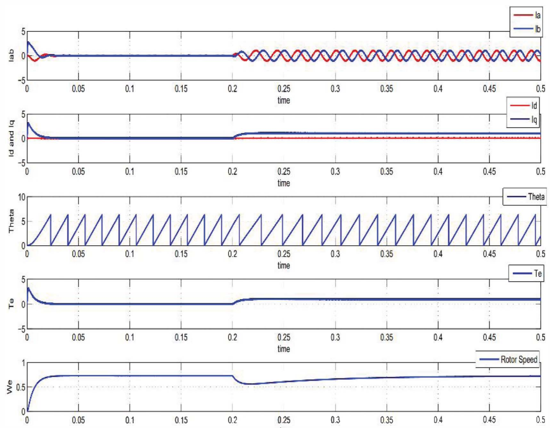

Fig. 6. Parameter Plots with TLoad = 2.43N-m given at 0.2s.

VI. RESULTS AND CONCLUSION

Angular velocity (ω_e), torque (T_e), d-q axes currents (i_d and i_q), 3-ph currents (I_{abc}) and angular position (θ) are plotted. Different load torque is applied at 0.2 seconds and above parameters are plotted. Results are presented in Figures 6-8. This paper demonstrates the development of a

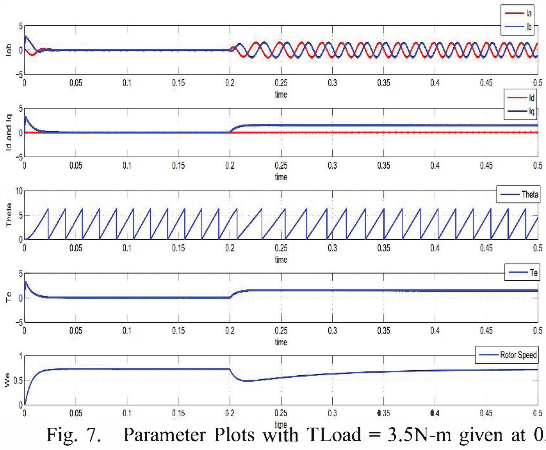

Fig. 7. Parameter Plots with TLoad = 3.5N-m given at 0.2s.

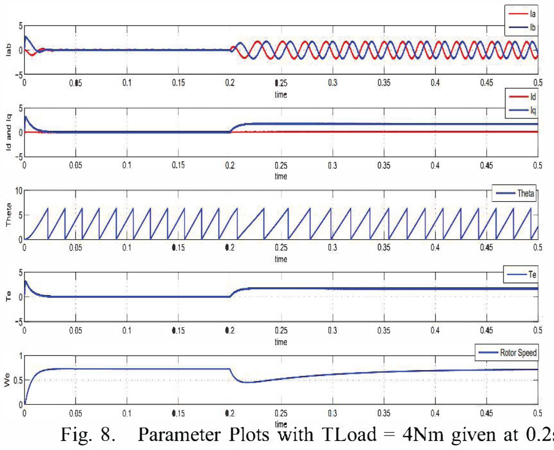

Fig. 8. Parameter Plots with TLoad = 4Nm given at 0.2s.

dynamic model of a PMSM. The main contribution includes the development of each and every block of a PMSM from first principle. Also Park's transformation and modified Park transformation are studied and their subsequent simulink models are developed. The complete vector control scheme is implemented by the authors and is tested with both these types of transformation. The plots obtained (Figures 6-8) demonstrate that the PMSM Model and its subsequent Matlab-Simulink implementation gives satisfactorily results. The model was tested with both modified Park's and Original Parks Transformation methods. Both techniques of transformations give similar results. This model can be used for further development.The model developed follows the standard mathematical relationships. All the control equations governing the dynamic model are kept intact. This model gives expected results in simulation. The same scheme when implemented in DSP based hardware, gives good speed control in both sensor-less and sensor operations.

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