Dynamic Simulation of a Three-Phase Induction Motor Using Matlab Simulink

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Abstract **– The theory of reference frames has been effectively used as an efficient approach to analyze the performance of the induction electrical machines. This paper presents a step by step Simulink implementation of an induction machine using dq0 axis transformations of the stator and rotor variables in the arbitrary reference frame. For this purpose, the relevant equations are stated at the beginning, and then a generalized model of a three-phase induction motor is developed and implemented in an easy to follow way. The obtained simulated results provide clear evidence that the reference frame theory is indeed an attractive algorithm to demonstrate the steady-state behavior of the induction machines.**

I. INTRODUCTION

The voltage and torque equations that describe the dynamic behavior of an induction motor are time-varying. It is successfully used to solve such differential equations and it may involve some complexity. A change of variables can be used to reduce the complexity of these equations by eliminating all time-varying inductances, due to electric circuits in relative motion, from the voltage equations of the machine [1, 2, 3, 4]. By this approach, a poly phase winding can be reduced to a set of two phase windings (q-d) with their magnetic axes formed in quadrature. In other words, the stator and rotor variables (voltages, currents and flux linkages) of an induction machine are transferred to a reference frame, which may rotate at any angular velocity or remain stationary. Such a frame of reference is commonly known in the generalized machines analysis as arbitrary reference frame [5, 6, 7].

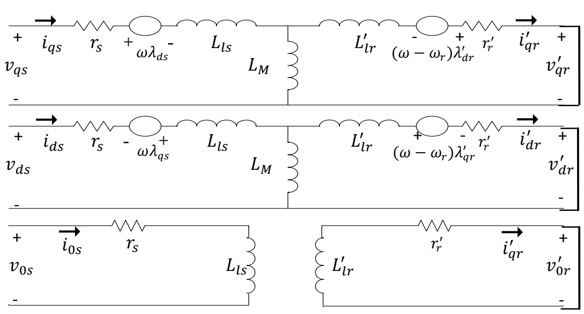

Figure 1 The dq0 equivalent circuit of an induction motor

The dynamic analysis of the symmetrical induction machines in the arbitrary reference frame has been intensively used as a standard simulation approach from which any particular mode of operation may then be developed. Matlab/Simulink has an advantage over other machine simulators in modeling the induction machine using dq0 axis transformation [8, 9]. It can be a powerful technique in implementing the machine equations as they are transferred to a particular reference frame. Thus, every single equation among the model equations can be easily implemented in one block so that all the machine variables can be made available for control and verification purposes. In this paper, Matlab/Simulink is used to simulate the dynamic performance of an induction motor model whose stator and rotor variables are referred to an arbitrary reference frame [1, 6, 8, 10]. The provided machine model is simulated in a way that makes it easy for the reader to follow and understand the implementation process since it gives full details about Simulink structure of each of the model equations. The equivalent circuit of the induction machine in the arbitrary reference frame is shown in figure 1 below [11, 12].

II. INDUCTION MOTOR MODEL

Driving the model equations can be generated from the dq0 equivalent circuit of the induction machine shown in figure 1. The flux linkages equations associated with this circuit can be found as follows:

$$\frac{d\psi_{qs}}{d_t} = \omega_b\left[v_{qs} - \frac{\omega_e}{\omega_b}\psi_{ds} + \frac{R_s}{X_{ls}}\left(\psi_{mq} - \psi_{qs}\right)\right] \quad (1)$$

$$\frac{d\psi_{ds}}{d_t} = \omega_b\left[v_{ds} + \frac{\omega_e}{\omega_b}\psi_{qs} + \frac{R_s}{X_{ls}}\left(\psi_{md} - \psi_{ds}\right)\right] \quad (2)$$

$$\frac{d\psi_{qr}}{d_t} = \omega_b\left[v_{qr} - \frac{(\omega_e-\omega_r)}{\omega_b}\psi_{dr} + \frac{R_r}{X_{lr}}\left(\psi_{mq} - \psi_{qr}\right)\right] \quad (3)$$

$$\frac{d\psi_{dr}}{d_t} = \omega_b\left[v_{dr} + \frac{(\omega_e-\omega_r)}{\omega_b}\psi_{qr} + \frac{R_r}{X_{lr}}\left(\psi_{md} - \psi_{dr}\right)\right] \quad (4)$$

Where

$$\psi_{mq} = X_{ml}\left[\frac{\psi_{qs}}{X_{ls}} + \frac{\psi_{qr}}{X_{lr}}\right] \quad (5)$$

$$\psi_{md} = X_{ml}\left[\frac{\psi_{ds}}{X_{ls}} + \frac{\psi_{dr}}{X_{lr}}\right] \quad (6)$$

$$X_{ml} = 1/\left(\frac{1}{X_m} + \frac{1}{X_{ls}} + \frac{1}{X_{lr}}\right) \quad (7)$$

Then substituting the values of the flux linkages to find the currents;

$$i_{qs} = \frac{1}{X_{ls}}\left(\psi_{qs} - \psi_{mq}\right) \quad (8)$$

$$i_{ds} = \frac{1}{X_{ls}}\left(\psi_{ds} - \psi_{md}\right) \quad (9)$$

$$i_{qr} = \frac{1}{X_{lr}}\left(\psi_{qr} - \psi_{mq}\right) \quad (10)$$

$$i_{dr} = \frac{1}{X_{lr}}\left(\psi_{dr} - \psi_{md}\right) \quad (11)$$

Based on the above equations, the torque and rotor speed can be determined as follows:

$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)\frac{1}{\omega_b}\left(\psi_{ds}i_{qs} - \psi_{qs}i_{ds}\right) \quad (12)$$

$$\omega_r = \int \frac{P}{2J}\left(T_e - T_L\right) \quad (13)$$

Where P: number of poles; J: moment of inertia (Kg/m^2). For squirrel cage induction motor, the rotor voltages V_{qr} and V_{dr} in the flux equations are set to zero since the rotor cage bars are shorted. After driving the torque and speed equations in term of d-q flux linkages and currents of the stator, the d-q axis transformation should now be applied to the machine input (stator) voltages [1, 13, 14].

The three-phase stator voltages of an induction machine under balanced conditions can be expressed as:

$$V_a = \sqrt{2}\, V_{rms} \sin(\omega t) \quad \text{.............................} \quad (14)$$

$$V_b = \sqrt{2}\, V_{rms} \sin\left(\omega t - \frac{2\pi}{3}\right) \quad \text{.......................} \quad (15)$$

$$V_c = \sqrt{2}\, V_{rms} \sin\left(\omega t + \frac{2\pi}{3}\right) \quad \text{.......................} \quad (16)$$

These three-phase voltages are transferred to a synchronously rotating reference frame in only two phases (d-q axis transformation). This can be done using the following two equations.

$$\begin{bmatrix} V_\alpha \\ \\ V_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & 1/2 & -1/2 \\ \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \text{................} (17)$$

Then, the direct and quadrature axes voltages are

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \quad \text{.........................} (18)$$

The instantaneous values of the stator and rotor currents in three-phase system are ultimately calculated using the following transformation:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad \text{..........................} (19)$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & 0 \\ -1/2 & -\sqrt{3}/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad \text{......................} (20)$$

III MATLAB/SIMULINK IMPLEMENTATION

In this section, the three phase induction machine model is simulated by using the Matlab/Simulink. The Model is implemented using the same set of equations provided above in sections II. Figure 2 depicts the complete Simulink scheme of the described induction machine model.

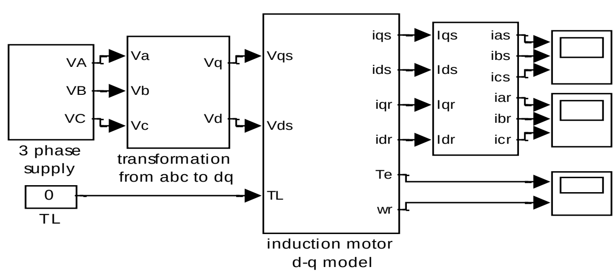

Figure 2 the 3-phase induction motor Matlab/Simulink model

In this model the simulation starts with generating a three-phase stator voltages according to the equations (14, 15, 16), and then transforming these balanced voltages to two phase voltages referred to the synchronously rotating frame using Clarke and Park transformation as in equations (17, 18). After that the d-q flux linkage and current equations were implemented as to be demonstrated below. Figure 3 illustrates the internal structure of the induction machine d-q model by which the flux linkages, currents, torque and the rotor angular speed are calculated.

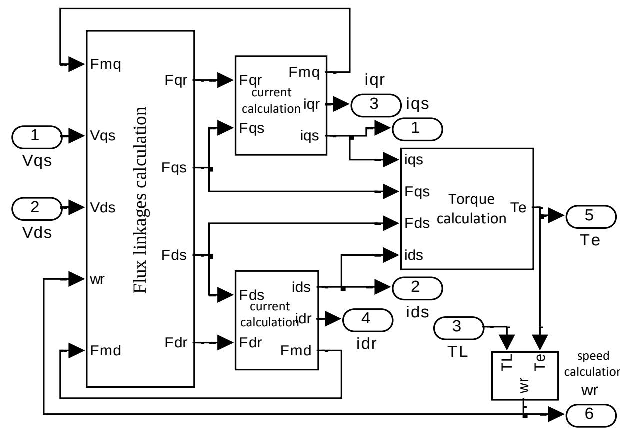

Figure 3 the internal structure of the 3-phase induction motor d-q model

The Matlab/Simulink model to find the flux linkage ψ_{qs}, ψ_{ds}, ψ_{qr}, ψ_{dr} as stated in equations (1)-(4) is shown in figure 4.

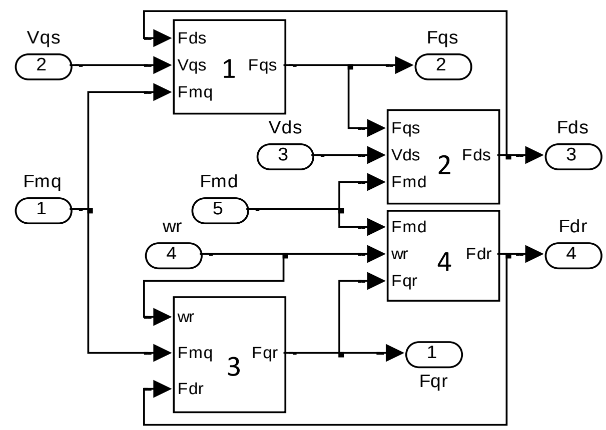

Figure 4 the internal structure of the block to calculate the flux linkages

Figures.5 show the Simulink blocks used to calculate the currents i_{qs}, i_{ds}, i_{qr}, i_{dr} according to the equations (8) – (11), also ψ_{mq}, ψ_{md} in equations (5),(6). Figures 6 & 7 show the implementation of torque T_e and angular speed ω_r as expressed in equations (12), (13) respectively.

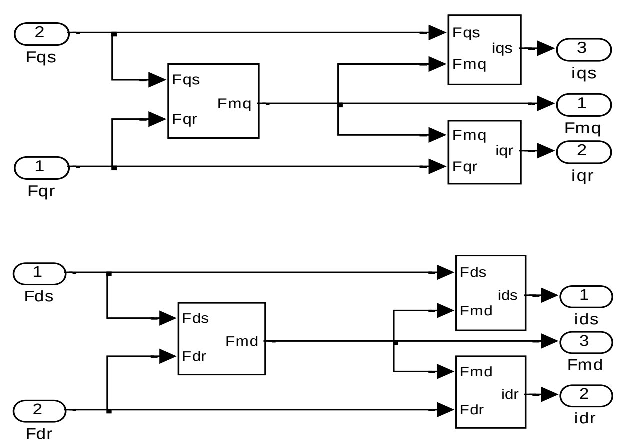

Figure 5 the internal structure of the block to calculate the currents i_{ds}, i_{dr}, i_{qs}, i_{qr}, and the fluxes ψ_{mq}, ψ_{md} .

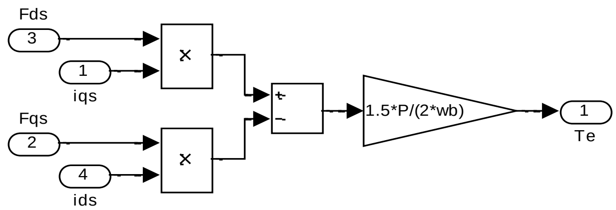

Figure. 6 the implementation of the torque equation Te (12)

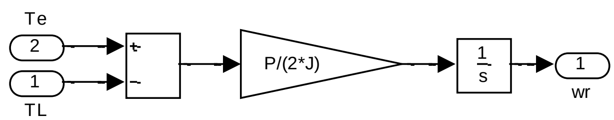

Figure. 7 the implementation of the angular speed equation ω_r (13)

Figure.8 shows the internal structure of the blocks (1- 4) in figure 4 in which the equations (1)-(4) are implemented in Matlab/Simulink format.

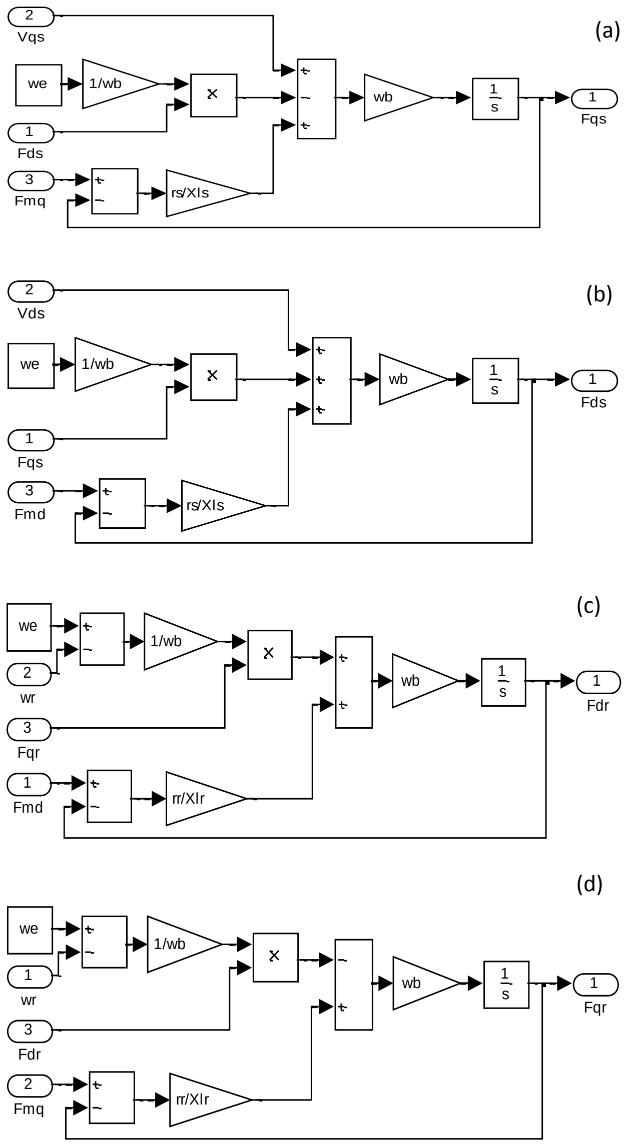

Figure 8 the implementation of the equation (1)-(4)

Figure.9 presents the implementation of the flux linkages ψ_{mq}, ψ_{md} found in figure. 5. Also, figure.10 depicts how the currents i_{qs}, i_{ds}, i_{qr}, i_{dr} are constructed.

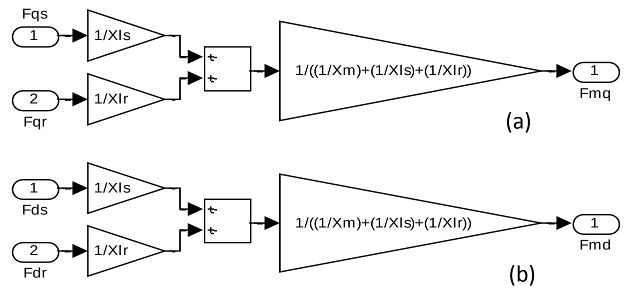

Figure 9 the calculation of the flux linkage ψ_{mq}, ψ_{md}

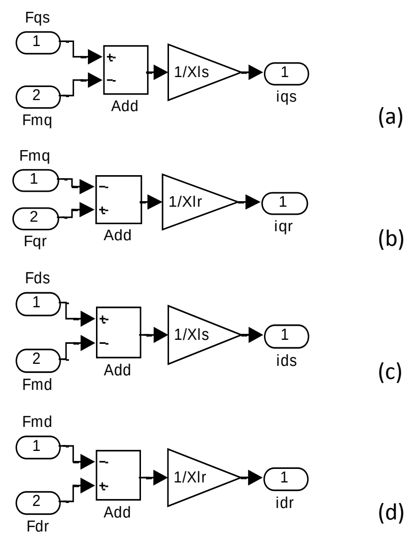

Figure 10 the implementation of the d-q current equations

IV. MATLAB/SIMULINK RESULTS

Two induction motors; 3hp and 2250 hp were tested in this simulated model. The results of the simulation are given for the first induction motor with the following specifications:

```
Hp = 3        VL = 220      f = 60
Rs = 0.435    Xls = 0.754   P = 4
Rr = 0.816    Xlr = 0.754   J = 0.089
Xm = 26.13    rpm = 1710
```

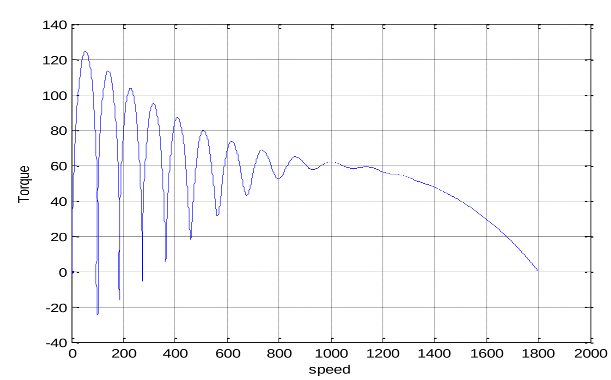

Figure 11 Torque speed characteristics for the 3 hp induction motor

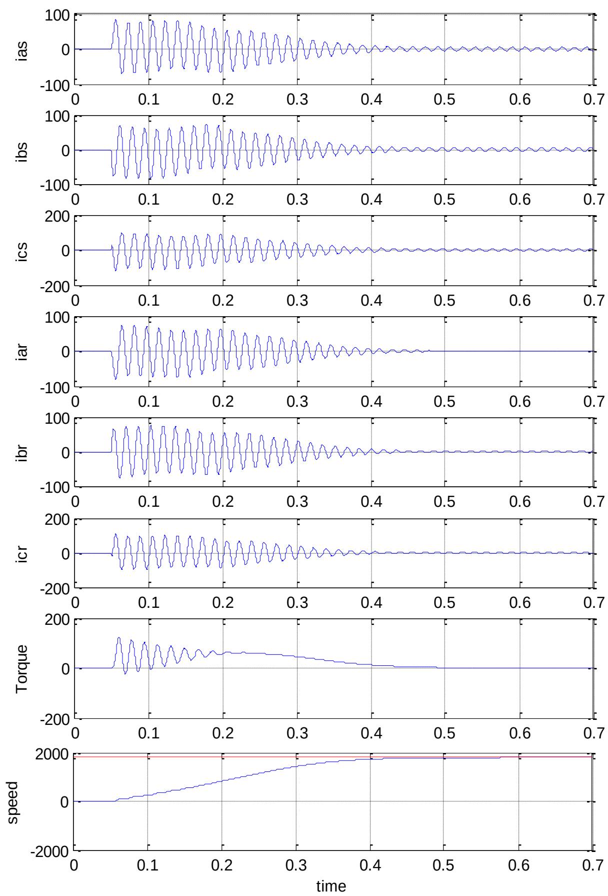

Figure 12 Machine variables during free acceleration of a 3-hp induction motor

The results of the simulation are also given for the other induction motor with the following specifications:

```
Hp = 2250      VL = 2300     f = 60
Rs = 0.029    Xls = 0.226    P = 4
Rr = 0.022    Xlr = 0.226    J = 63.87
Xm = 13.04    rpm = 1786
```

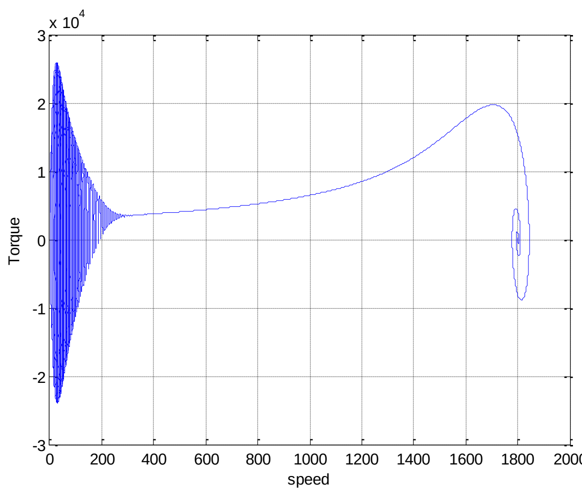

Figure 13 Torque speed characteristics for the 2250 hp induction motor

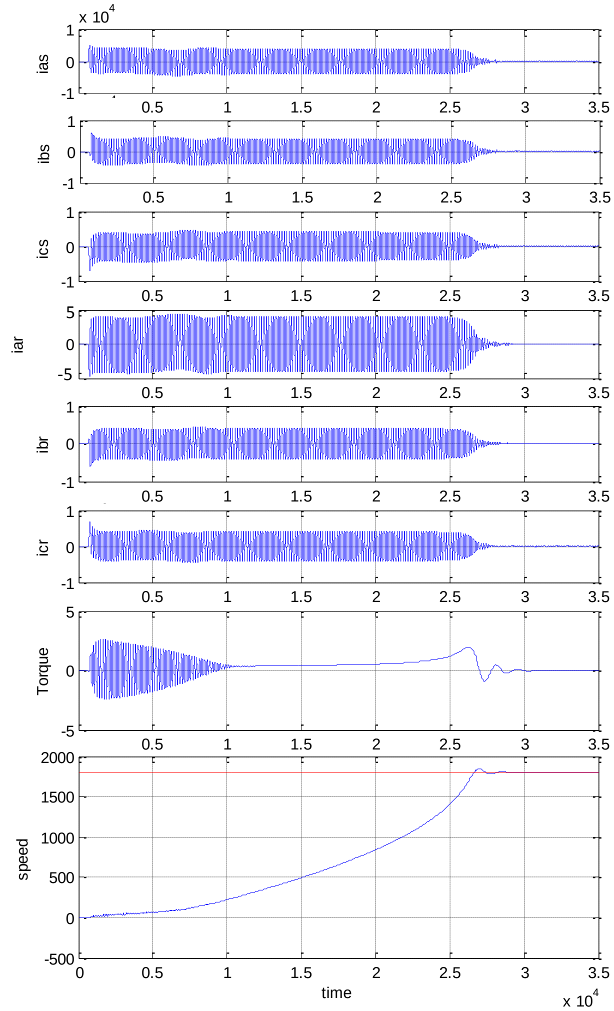

Figure 14 Machine variables during free acceleration of a 2250-hp induction motor

Finally, the machine parameters should be defined to the simulated machine system in order to complete the simulation process. There are many ways to input the required data. The method used here is the graphical user interface (GUI). The machine parameters are entered through the convenient graphical user interface (GUI) available in Matlab/Simulink, where you can right click with your mouse and choose (edit mask) then choose parameters to be added. Figure 15 shows the GUI of the induction machine d-q model shown earlier in figure 2.

After achieving the Matlab/Simulink implementation of the described machine model using the Matlab/Simulink, a Matlab code program was assigned to the same model using the same set of equations. The code provided similar results to those obtained by Matlab/Simulink. However, it was found that Matlab/Simulink is more convenient in terms of simplicity in construction and control algorithms which may be set forth for this model. The code compilation steps can be stated by the flow chart shown below in figure 16.

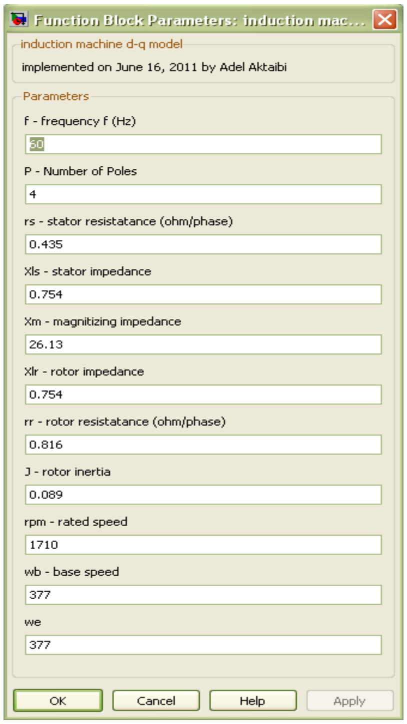

Figure 15 the graphical user interface (GUI) used to define the input data to the simulated induction motor.

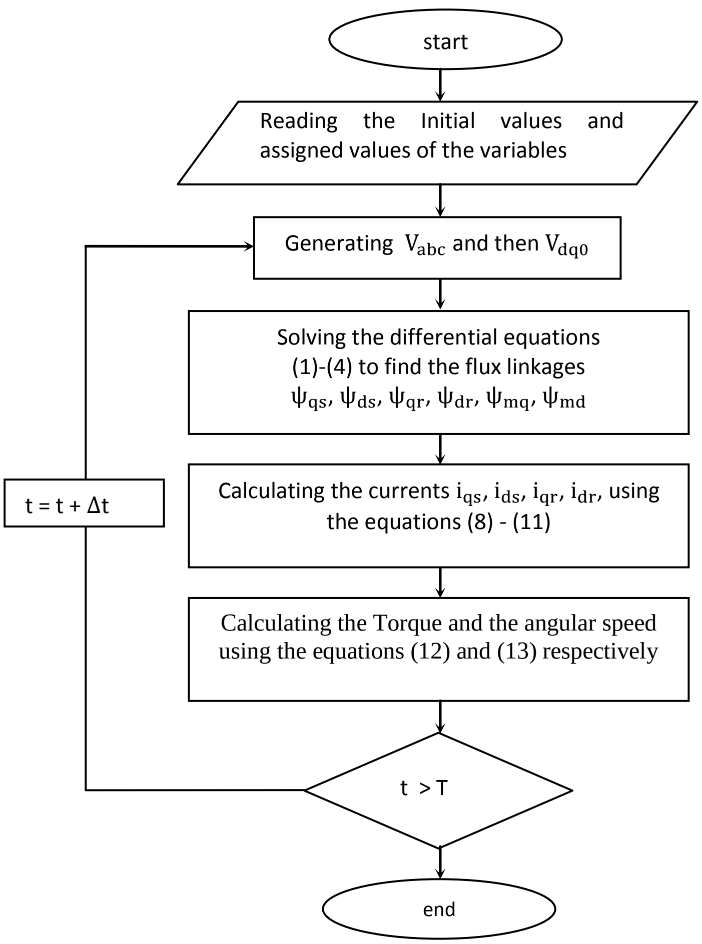

Figure 16 the flow of the induction motor simulation model

V. CONCLUSIONS

In this paper, an implementation and dynamic modeling of a three-phase induction motor using Matlab/Simulink are presented in a step-by-step manner. The model was tested by two different ratings of a small and large induction motors. The two simulated machines have given a satisfactory response in terms of the torque and speed characteristics. Also, the model was evaluated by Matlab m-file coding program. Both methods have given the same results for the same specifications of the three phase induction motors used in this simulation. This concludes that the Matlab/Simulink is a reliable and sophisticated way to analyze and predict the behavior of induction motors using the theory of reference frames.

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