Three-Phase to Seven-Phase Power Converting Transformer

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***Abstract*—Multiphase (more than three-phase electric power) electric drive system is the focus of a significant research in the last decade. Multiphase power transmission system is also investigated in the literature because multiphase transformers are needed at the input of rectifiers. In the multiphase power transmission and multiphase rectifier systems, the number of phases investigated is a multiple of three. However, the variable speed multiphase drive system considered in the literature are mostly of five, seven, nine, eleven, twelve, and fifteen phases. Such multiphase drive systems are invariably supplied from power electronic converters. In contrast, this paper proposes technique to obtain seven-phase output from three-phase supply system using special and novel transformer connections. Thus, with the proposed technique, a pure seven-phase sine-wave voltage/current is obtained, which can be used for motor testing purposes. In addition, a seven-phase power transmission and rectifier system may benefit from the proposed connection scheme. Complete design and testing of the proposed solution is presented. Analytical analysis, simulation, and experimental verifications are presented in the paper.**

***Index Terms*—Converting transformer, multiphase drive systems, multiphase system, multiphase transmission, three-to-seven phase.**

I. Introduction

THE advantages of multiphase systems compared to three-phase systems have brought about researchers' interest. The applicability of multiphase systems is explored in electric power generation [1]–[5], [34], [35], transmission [6]–[12], and utilization [13]–[24], [32], [33], [37]–[39]. The research on six-phase transmission systems was initiated due to rising cost of right of way for transmission corridors, environmental issues, and various strict licensing laws. Six-phase transmission lines can provide the same power capacity with a lower line voltage and smaller towers as compared to a standard double circuit three-phase line [8]. The dimension of the six-phase smaller towers may also lead to the reduction of magnetic fields and electromagnetic interference [9]. The research on multiphase generators has recently started and few references are available [1]–[5], [34], [35]. The present work on multiphase power generation investigates an asymmetrical six-phase (two set of stator windings with 30° phase displacement) induction generator configuration as a solution for the use in renewable energy systems [5]. Ward and Harer [40] proposed multiphase motor drives, but the research on it was slow in its release. The research on multiphase drive systems has been significantly developed since the beginning of this century due to advancement in semiconductor devices and digital signal processors technologies. Detailed reviews on state-of-the-art multiphase drive research are available in [15]–[18] and [36]. It is to be emphasized here that ac/dc/ac converters generally supply the multiphase motors. Thus, the focus of the current research on multiphase electric drives is limited to the modeling and controlling of the power converters [19]–[24], [32], [33], [37]–[39]. Little effort is being made to develop static transformation system to change the phase number from three-to-n-phase (where $n > 3$ and odd). An exception is [25], where a new type of transformer is presented, which is three-to-five-phase system. In [41] and [42], the authors presented an interesting solution for three-to-five-phase conversion. At the end of [41], the authors briefly mention the seven-phase system; however, no study or analysis was done on three-to-seven-phase transformer. Accordingly, this paper is based on the same principle as that of [25]. The analysis and design, however, are completely different. In our approach, in contrast to the system of [25], the phase angle between two consecutive phases is not an integer number.

Multiphase, especially 6- and 12-phase, systems are found to produce less amplitude of ripples with higher frequency in ac–dc rectifier system [31]. Thus, 6- and 12-phase transformers are designed to feed a multipulse rectifier system and the technology is matured. Recently, 24- and 36-phase transformer systems were proposed for supplying a multipulse rectifier system [26]–[29]. The reason of adopting a 6-, 12-, or 24-phase system is that these numbers are multiples of three and designing such system is simple and straightforward. However, increasing the number of phases certainly affects the complexity of the system. No such design is available for odd number of phases, such as 7, 11, etc., as far as is known to the authors.

The usual practice for analysis is to test the designed motor for a number of operating conditions with pure sinusoidal supply [30]. Normally, no-load test, blocked rotor, and load tests are performed on a motor to determine its parameters. Although the supply used for multiphase motor drives obtained from multiphase inverters could have more current ripples, there are control

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methods available to lower the current distortion below 1%, based on application and requirement [23]. The machine parameters obtained using a PWM inverter may not provide the correct value. Thus, a pure sinusoidal supply system is required to feed the motor for better analysis. Accordingly, this paper proposes a special transformer connection scheme to obtain a balanced three-to-seven-phase supply with sinusoidal waveforms. The expected application areas of the proposed transformer are the electric power transmission system, power electronic converters (ac–dc and ac–ac), and the multiphase electric drive system.

The fixed three-phase voltage and fixed frequency available in grid power supply can be transformed to fixed voltage and fixed frequency seven-phase output supply. Furthermore, the output magnitude may be made variable by inserting a three-phase autotransformer at the input side.

In this paper, the input and output supply can be arranged in the following manners:

1) Input star, output star.
2) Input star, output heptagon.
3) Input delta, output star.
4) Input delta, output heptagon.

Since input is a three-phase system the windings are connected in usual manner. The output/secondary side star connection is discussed in the following sections. The heptagon output connection may be derived following a similar approach. Thus, only star output connection is discussed in the following section and other connections are omitted.

II. Winding Arrangement for Seven-Phase Star Output

Three separate iron cores are designed with each of them carrying one primary and four secondary coils, except in one core where five secondary coils are wound. Six terminals of primaries are connected in an appropriate manner resulting in star and/or delta connections, and the 26 terminals of secondaries are connected in a different fashion resulting in a star or heptagon output. The connection scheme of secondary windings to obtain star output is illustrated in Figs. 1 and 2 and the corresponding phasor diagram is illustrated in Fig. 3. The construction of output phases with requisite phase angles of 360/7 = 51.43° between each phase is obtained using appropriate turn ratios and the governing phasor equation is illustrated in (1c). The turn ratios are different in each phase as shown in Fig. 1. The choice of turn ratio is the key in creating the requisite phase displacement in the output phases. The turn ratios between different phases are given in Table I.

The input phases are designated with letters "X," "Y," and "Z" and the output are designated with letters "a," "b," "c," "d," "e," "f," and "g." The mathematical basis for this connection is the basic addition of real and imaginary parts of the vectors. For example, the solution for (1a) gives the turn ratio of phase "b," (V_b taken as unity)

$$V_x\left[\cos\left(\frac{2\pi}{7}\right)+j\sin\left(\frac{2\pi}{7}\right)\right] - V_z\left[\cos\left(\frac{\pi}{21}\right)-j\sin\left(\frac{\pi}{21}\right)\right]=1. \quad (1a)$$

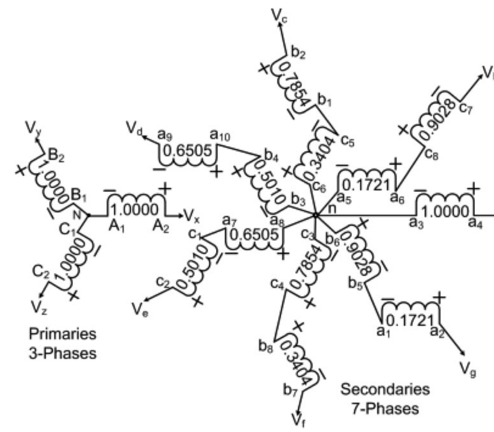


Fig. 1. Proposed transformer winding connection (star).

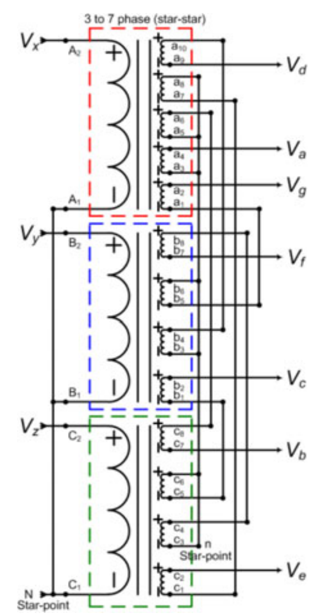


Fig. 2. Proposed transformer winding arrangements (star-star).

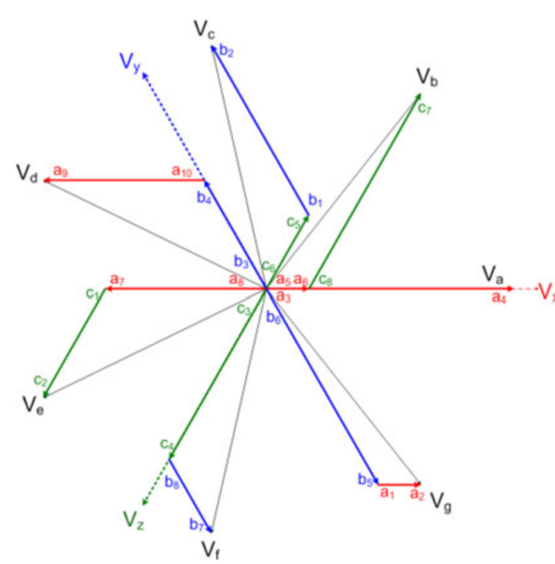


Fig. 3. Phasor diagram of the proposed transformer connection (star-star).

TABLE I
TURN RATIO SECONDARY TURNS (N_2) TO PRIMARY ($A_1 A_2$) TURNS (N_1)

Name of winding	Turn Ratio N_2/N_1	Name of winding	Turn Ratio N_2/N_1	Name of winding	Turn Ratio N_2/N_1
a_1a_2	0.1721	b_1b_2	0.7854	c_1c_2	0.5010
a_3a_4	1.0000	b_3b_4	0.5010	c_3c_4	0.7854
a_5a_6	0.1721	b_5b_6	0.9028	c_5c_6	0.3404
a_7a_8	0.6505	b_7b_8	0.3404	c_7c_8	0.9028
a_9a_{10}	0.6505				

Equating real and imaginary parts and solving for V_x and V_z, we get

$$|V_x| = \left|\frac{\sin(\pi/21)}{\sin(\pi/3)}\right| = 0.1721$$
$$|V_z| = \left|-\frac{\sin(2\pi/7)}{\sin(\pi/3)}\right| = 0.9028. \tag{1b}$$

Equation (1c) is the result of solutions of equations like (1a) for other phases.

Therefore, by simply summing the voltages of two different coils, one output phase is created. It is important to note that the phase "a" output is generated from only one coil namely "a_3a_4" in contrast to other phases which utilizes two coils. Thus, the voltage rating of "a_3a_4" coil should be kept to that of rated phase voltage to obtain balanced and equal voltages

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \\ V_g \end{bmatrix} = \frac{1}{\sin(\pi/3)} \times \begin{bmatrix} \sin\left(\frac{\pi}{3}\right) & 0 & 0 \\ \sin\left(\frac{\pi}{21}\right) & 0 & -\sin\left(\frac{2\pi}{7}\right) \\ 0 & \sin\left(\frac{5\pi}{21}\right) & -\sin\left(\frac{2\pi}{21}\right) \\ -\sin\left(\frac{4\pi}{21}\right) & \sin\left(\frac{\pi}{7}\right) & 0 \\ -\sin\left(\frac{4\pi}{21}\right) & 0 & \sin\left(\frac{\pi}{7}\right) \\ 0 & -\sin\left(\frac{2\pi}{21}\right) & \sin\left(\frac{5\pi}{21}\right) \\ \sin\left(\frac{\pi}{21}\right) & -\sin\left(\frac{2\pi}{7}\right) & 0 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}. \tag{1c}$$

Where the three-phase voltages (line-to-neutral) are defined as

$$V_j = V_{\max} \sin\left(\omega t - n\frac{\pi}{3}\right),$$
$$j = x, y, z, \text{ and } n = 0, 2, 4, \text{ respectively,} \tag{2}$$
$$V_k = V_{\max} \sin\left(\omega t - n\frac{\pi}{7}\right),\ k = a, b, c, d, e, f, g,$$
$$\text{and } n = 0, 2, 4, 6, 8, 10, 12, \text{ respectively.} \tag{3}$$

Using (1c), a seven-phase output can be created from a three-phase input supply.

A general expression for an "n" phase system is derived and shown in (4)

$$V_r = [(-1)^a V_x \sin(\theta) + (-1)^b V_y \sin(\phi) + (-1)^c V_z \sin(\gamma)] \tag{4}$$

where r = phase number = 1,2,3,. . ...,n;

$$V_x = 0 \text{ when } \left(\frac{\pi}{3} \le \frac{2(r-1)\pi}{n} \le \frac{2\pi}{3}\right)$$
$$\text{or } \left(\frac{4\pi}{3} \le \frac{2(r-1)\pi}{n} \le \frac{5\pi}{3}\right) \tag{4a}$$

where n = number of phases in the system;

$$V_y = 0 \text{ when } \left(0 \le \frac{2(r-1)\pi}{n} \le \frac{\pi}{3}\right)$$
$$\text{or } \left(\pi \le \frac{2(r-1)\pi}{n} \le \frac{4\pi}{3}\right) \tag{4b}$$
$$V_z = 0 \text{ when } \left(\frac{2\pi}{3} \le \frac{2(r-1)\pi}{n} \le \pi\right)$$
$$\text{or } \left(\frac{5\pi}{3} \le \frac{2(r-1)\pi}{n} \le 2\pi\right) \tag{4c}$$
$$a = \begin{cases} 1, \text{ when } \left(\frac{2\pi}{3} < \frac{2(r-1)\pi}{n} < \frac{4\pi}{3}\right) \text{ (small arc)} \\ 2, \text{ when } \left(\frac{5\pi}{3} < \frac{2(r-1)\pi}{n} < \frac{\pi}{3}\right) \text{ (small arc)} \end{cases} \tag{4d}$$
$$b = \begin{cases} 1, \text{ when } \left(\frac{4\pi}{3} < \frac{2(r-1)\pi}{n} < 2\pi\right) \text{ (small arc)} \\ 2, \text{ when } \left(\frac{\pi}{3} < \frac{2(r-1)\pi}{n} < \pi\right) \text{ (small arc)} \end{cases} \tag{4e}$$
$$c = \begin{cases} 1, \text{ when } \left(0 < \frac{2(r-1)\pi}{n} < \frac{2\pi}{3}\right) \text{ (small arc)} \\ 2, \text{ when } \left(\pi < \frac{2(r-1)\pi}{n} < \frac{5\pi}{3}\right) \text{ (small arc)} \end{cases} \tag{4f}$$

$$\theta=\begin{cases}\left(\dfrac{\pi}{3}-\dfrac{2(r-1)\pi}{n}\right), & \text{when}\left(0<\dfrac{2(r-1)\pi}{n}<\dfrac{\pi}{3}\right)\\ \left(\dfrac{2(r-1)\pi}{n}-\dfrac{5\pi}{3}\right), & \text{when}\left(\dfrac{5\pi}{3}<\dfrac{2(r-1)\pi}{n}<2\pi\right)\\ \left(\dfrac{2(r-1)\pi}{n}-\dfrac{2\pi}{3}\right), & \text{when}\left(\dfrac{2\pi}{3}<\dfrac{2(r-1)\pi}{n}<\pi\right)\\ \left(\dfrac{4\pi}{3}-\dfrac{2(r-1)\pi}{n}\right), & \text{when}\left(\pi<\dfrac{2(r-1)\pi}{n}<\dfrac{4\pi}{3}\right)\end{cases} \tag{4g}$$

$$\phi=\begin{cases}\left(\dfrac{2\pi}{3}-\dfrac{2(r-1)\pi}{n}\right), & \text{when}\left(\dfrac{\pi}{3}<\dfrac{2(r-1)\pi}{n}<\dfrac{2\pi}{3}\right)\\ \left(\dfrac{2(r-1)\pi}{n}-\dfrac{2\pi}{3}\right), & \text{when}\left(\dfrac{2\pi}{3}<\dfrac{2(r-1)\pi}{n}<\pi\right)\\ \left(\dfrac{2(r-1)\pi}{n}-\dfrac{4\pi}{3}\right), & \text{when}\left(\dfrac{4\pi}{3}<\dfrac{2(r-1)\pi}{n}<\dfrac{5\pi}{3}\right)\\ \left(2\pi-\dfrac{2(r-1)\pi}{n}\right), & \text{when}\left(\dfrac{5\pi}{3}<\dfrac{2(r-1)\pi}{n}<2\pi\right)\end{cases} \tag{4h}$$

$$\gamma=\begin{cases}\left(\dfrac{2(r-1)\pi}{n}\right), & \text{when}\left(0<\dfrac{2(r-1)\pi}{n}<\dfrac{\pi}{3}\right)\\ \left(\dfrac{2\pi}{3}-\dfrac{2(r-1)\pi}{3}\right), & \text{when}\left(\dfrac{\pi}{3}<\dfrac{2(r-1)\pi}{n}<\dfrac{2\pi}{3}\right)\\ \left(\dfrac{2(r-1)\pi}{n}-\pi\right), & \text{when}\left(\pi<\dfrac{2(r-1)\pi}{n}<\dfrac{4\pi}{3}\right)\\ \left(\dfrac{5\pi}{3}-\dfrac{2(r-1)\pi}{n}\right), & \text{when}\left(\dfrac{4\pi}{3}<\dfrac{2(r-1)\pi}{n}<\dfrac{5\pi}{3}\right).\end{cases} \tag{4i}$$

Since a transformer works as a two-port network, the reverse connection is also possible, i.e., if a seven-phase supply is given at the input the output can be three phase. This is especially important if electric power is generated using a seven-phase alternator and the supply to the grid is given as three phase. To obtain three-phase outputs from a seven-phase input supply, following relations hold good

$$\begin{bmatrix}V_x\\V_y\\V_z\end{bmatrix}=\frac{1}{\sin(\pi/7)}\begin{bmatrix}\sin\left(\frac{\pi}{7}\right) & 0 & 0\\ 0 & 0 & -\sin\left(\frac{2\pi}{21}\right)\\ 0 & \sin\left(\frac{\pi}{21}\right) & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \sin\left(\frac{\pi}{21}\right)\\ 0 & -\sin\left(\frac{2\pi}{21}\right) & 0\end{bmatrix}^t * \begin{bmatrix}V_a\\V_b\\V_c\\V_d\\V_e\\V_f\\V_g\end{bmatrix}. \tag{5}$$

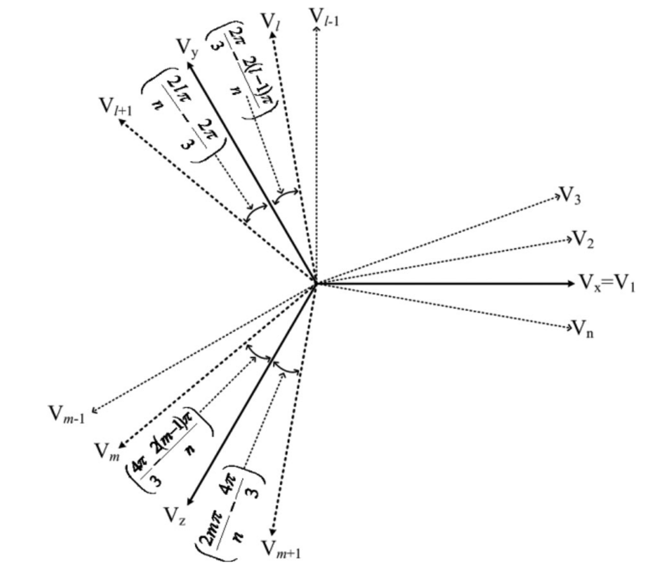


Fig. 4. General Phasor diagram for three-phase system from "n" phase system.

Due to redundancy, three more combinations make it possible to obtain a three-phase supply from a seven-phase input as given in the following equations:

$$\begin{bmatrix}V_x\\V_y\\V_z\end{bmatrix}=\frac{1}{\sin(2\pi/7)}\begin{bmatrix}\sin\left(\frac{2\pi}{7}\right) & 0 & 0\\ 0 & 0 & 0\\ 0 & \sin\left(\frac{4\pi}{21}\right) & 0\\ 0 & \sin\left(\frac{2\pi}{21}\right) & 0\\ 0 & 0 & \sin\left(\frac{2\pi}{21}\right)\\ 0 & 0 & \sin\left(\frac{4\pi}{21}\right)\\ 0 & 0 & 0\end{bmatrix}^t * \begin{bmatrix}V_a\\V_b\\V_c\\V_d\\V_e\\V_f\\V_g\end{bmatrix} \tag{6}$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \frac{1}{\sin(3\pi/7)} \begin{bmatrix} \sin\left(\frac{3\pi}{7}\right) & -\sin\left(\frac{2\pi}{21}\right) & -\sin\left(\frac{2\pi}{21}\right) \\ 0 & 0 & 0 \\ 0 & \sin\left(\frac{\pi}{3}\right) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sin\left(\frac{\pi}{3}\right) \\ 0 & 0 & 0 \end{bmatrix}^t * \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \\ V_g \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \frac{1}{\sin(4\pi/7)} \begin{bmatrix} \sin\left(\frac{4\pi}{7}\right) & 0 & 0 \\ 0 & \sin\left(\frac{4\pi}{21}\right) & 0 \\ 0 & 0 & 0 \\ 0 & \sin\left(\frac{8\pi}{21}\right) & 0 \\ 0 & 0 & \sin\left(\frac{8\pi}{21}\right) \\ 0 & 0 & 0 \\ 0 & 0 & \sin\left(\frac{4\pi}{21}\right) \end{bmatrix}^t * \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \\ V_g \end{bmatrix}. \quad (8)$$

Similarly, from Fig. 4, we derived one of the general expressions for three-phase system from "*n*" phase system in (9).

$$\text{For simplicity, we assume } V_x = V_1\text{; and} \quad (9a)$$

$n =$ number of phases in the system.

$V_x, V_y, V_1, V_2, V_3, \ldots, V_l, \ldots V_m \ldots$ are phasors. Then,

$$V_y = \frac{1}{\sin(2\pi/n)} \times \left[\sin\left(\frac{2l\pi}{n} - \frac{2\pi}{3}\right) V_l + \sin\left(\frac{2\pi}{3} - \frac{2(l-1)\pi}{n}\right) V_{l+1}\right] \quad (9b)$$

where $l = 2, 3, \ldots.,$ and $\left(\frac{2l\pi}{n}\right) > \left(\frac{2\pi}{3}\right) \geq \left(\frac{2(l-1)\pi}{n}\right)$

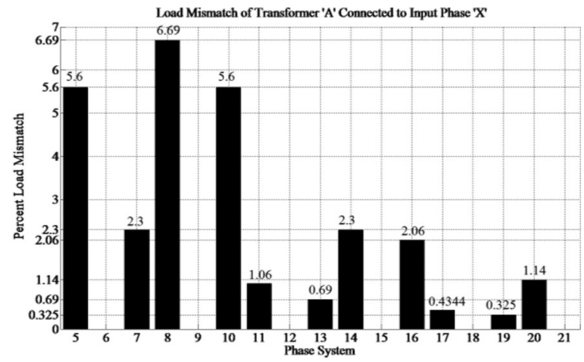


Fig. 5. Load mismatch in different output system of Transformer "A" Connected to input Phase "X."

$$V_z = \frac{1}{\sin(2\pi/n)} \times \left[\sin\left(\frac{2m\pi}{n} - \frac{2\pi}{3}\right) V_m + \sin\left(\frac{2\pi}{3} - \frac{2(m-1)\pi}{n}\right) V_{m+1}\right] \quad (9c)$$

where $\left(\frac{2m\pi}{n}\right) > \left(\frac{2\pi}{3}\right) \geq \left(\frac{2(m-1)\pi}{n}\right)$ and $m > l$.

III. Load Sharing of Secondary Windings

Let $V_1 * I_1 = S_1$, where V_1 and I_1 are input phase voltage and current, respectively, and S_1 is average per phase input voltampere (VA).

Also, let $V_2 * I_2 = S_2$, where V_2 and I_2 are output phase voltage and current, respectively, and S_2 is per phase output VA. After neglecting the losses, we have: $3\ S_1 = 7\ S_2$. For transformer A: VA of winding $a_1 a_2$

$$S_{a1a2} = \frac{3S_1}{7\sin(\pi/3)} \cos\left(\frac{2\pi}{7}\right) \sin\left(\frac{\pi}{21}\right) \quad (10)$$

where $(2\pi/7)$ is the angle between input V_x and output V_g, in which winding $a_1 a_2$ is connected, and $\sin(\pi/21)/\sin(\pi/3)$ is the turn ratio of secondary winding $a_1 a_2$ to primary winding $A_1 A_2$. The VA relationship for transformer A is shown as follows:

$$\begin{bmatrix} S_{a_1a_2} \\ S_{a_3a_4} \\ S_{a_5a_6} \\ S_{a_7a_8} \\ S_{a_9a_{10}} \end{bmatrix} = \frac{3S_1}{7\sin(\pi/3)} \begin{bmatrix} \sin\left(\frac{\pi}{21}\right)\cos\left(\frac{2\pi}{7}\right) \\ \sin\left(\frac{\pi}{3}\right) \\ \sin\left(\frac{\pi}{21}\right)\cos\left(\frac{-2\pi}{7}\right) \\ -\sin\left(\frac{4\pi}{21}\right)\cos\left(\frac{6\pi}{7}\right) \\ -\sin\left(\frac{4\pi}{21}\right)\cos\left(\frac{8\pi}{7}\right) \end{bmatrix}. \quad (11)$$

Negative signs indicate opposite polarity of connection for that particular winding.

The sum of VA of all secondary windings of transformer A is equal to $1.023 * S_1$. This means that the rating of transformer A should be 2.3% more than average VA rating.

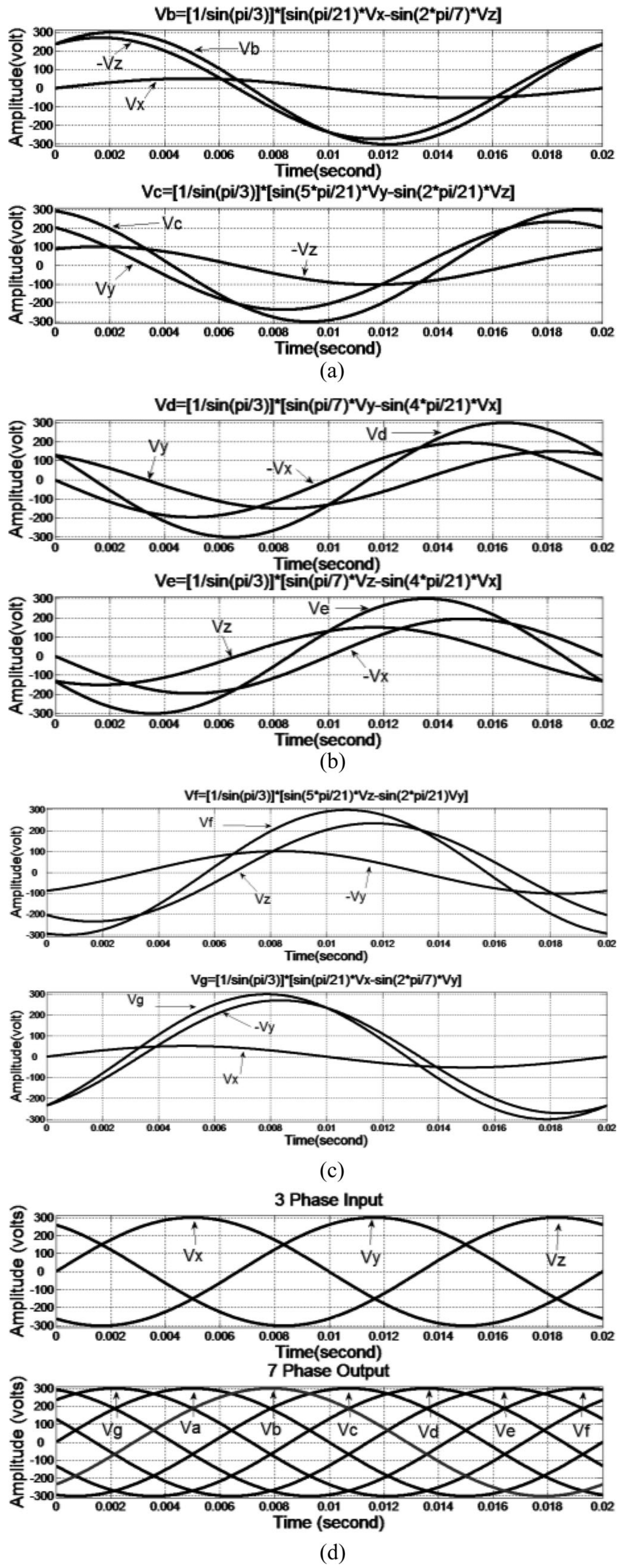


Fig. 6. (a) Input V_x, V_y, and V_z-phases and output V_b and V_c phase voltage waveforms. (b) Input V_x, V_y, and V_z-phases and output V_d and V_e phase voltage waveforms. (c) Input V_x, V_z, and V_x-phases and output V_f and V_g phase voltage waveforms. (d) Simulated three-phase input and seven-phase output voltages.

Similarly, the VA relationships for transformers B and C are shown in (12) and (13)

$$\begin{bmatrix} S_{b_1 b_2} \\ S_{b_3 b_4} \\ S_{b_5 b_6} \\ S_{b_7 b_8} \end{bmatrix} = \frac{3S_1}{7\sin(\pi/3)} \begin{bmatrix} \sin\left(\frac{5\pi}{21}\right)\cos\left(\frac{2\pi}{21}\right) \\ \sin\left(\frac{\pi}{7}\right)\cos\left(\frac{4\pi}{21}\right) \\ -\sin\left(\frac{2\pi}{7}\right)\cos\left(-\frac{20\pi}{21}\right) \\ -\sin\left(\frac{2\pi}{21}\right)\cos\left(\frac{16\pi}{21}\right) \end{bmatrix}. \tag{12}$$

The sum of VA of all secondary windings of transformer B is equal to $0.9885 * S_1$, which means rating of transformer B can be 1.15% less than the average VA rating

$$\begin{bmatrix} S_{c_1 c_2} \\ S_{c_3 c_4} \\ S_{c_5 c_6} \\ S_{c_7 c_8} \end{bmatrix} = \frac{3S_1}{7\sin(\pi/3)} \begin{bmatrix} \sin\left(\frac{\pi}{7}\right)\cos\left(\frac{4\pi}{21}\right) \\ \sin\left(\frac{5\pi}{21}\right)\cos\left(\frac{2\pi}{21}\right) \\ -\sin\left(\frac{2\pi}{21}\right)\cos\left(\frac{16\pi}{21}\right) \\ -\sin\left(\frac{2\pi}{7}\right)\cos\left(-\frac{20\pi}{21}\right) \end{bmatrix}. \tag{13}$$

The sum of VA of all secondary windings of transformer C is equal to $0.9885 * S_1$. This means the rating of transformer C can be 1.15% less than the average VA rating.

The sum of VA of all three transformers

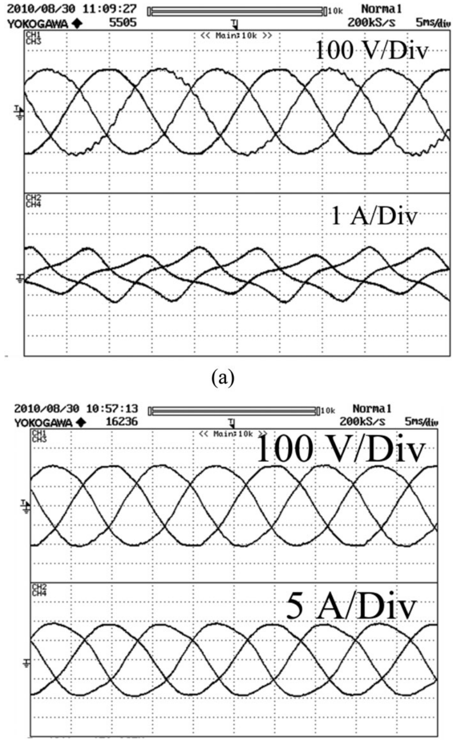
$$= 1.023 * S_1 + 0.9885 * S_1 + 0.9885 * S_1 = 3 * S_1.$$

After careful study of load mismatch, it was found that all systems which are multiple of "3," i.e., 6, 9, 12, etc., have zero mismatch whereas 5-phase and 10-phase systems have a mismatch of 5.6%, and 7-phase, 14-phase systems have mismatch of 2.3%, but a 4-phase system has highest mismatch of 50%. As phase system number increases, especially in prime numbered systems, the mismatch decreases such as in 19-phase system the mismatch is 0.325%. The load mismatch in Transformer "A," which is connected to input phase "X," is shown in Fig. 5.

When there is a phase shift between phase "X" and V_a only, then output will not have equal voltage and phase difference. A phase shift of 9° was given in input "X" phase (lag), then it was found that voltage V_b, and V_e increased by 2% and 4.4%, whereas that of V_d and V_g decreased by 4.7% and 2.2%, respectively. The total variation in voltages and currents found were only 0.5%. Hence, the VA variation shall be negligible.

IV. Simulation Results

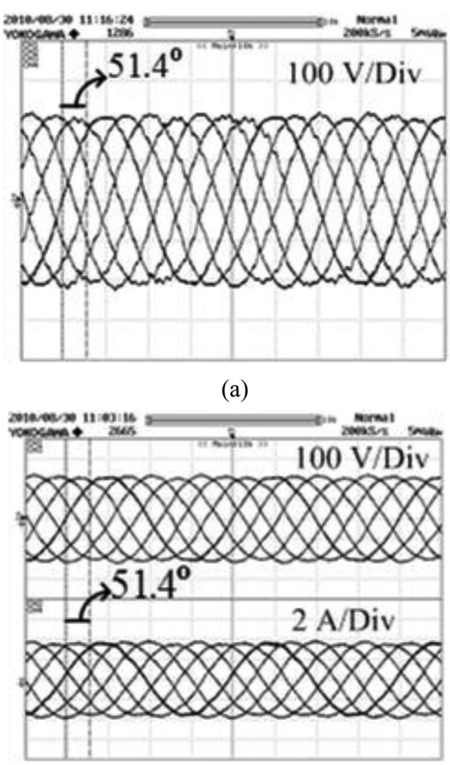
The designed transformer is at first simulated using "SimPowerSystem" block sets of the MATLAB/Simulink software. The inbuilt transformer blocks are used to simulate the conceptual design. The appropriate turn ratios are set in the dialog box and the simulation is run. Turn ratios are shown in Table I. The resulting input and output voltage waveforms are illustrated in Fig. 6. It is seen that the output is a balanced seven-phase supply for a balanced three-phase input. The output will



(a)

(b)

Fig. 7. (a) Input three-phase voltage and current waveforms (no-load) of the designed transformer primary. (b) Input three-phase voltage and current waveforms (loaded) of the designed transformer primary.



(a)

(b)

Fig. 8. (a) Seven-phase output voltage waveform (No-load, showing phase difference of 51.4286°) of the designed transformer secondary. (b) Seven-phase output voltage and current waveforms (loaded, showing phase difference of 51.4286°) of the designed transformer secondary.

be unbalanced if the input is unbalanced. The unbalancing study is out of the scope of this paper and will be dealt separately and reported in the future. Individual output phases are also shown along with their respective input voltages. The phase V_a is not shown because $V_a = V_x$, i.e., the input and the output phases are same.

Fig. 6 shows the reconstruction of seven-phase output waveform from a three-phase input waveform. The three-phase output from a seven-phase input supply can also be obtained in similar fashion.

V. Experimental Results

This section elaborates on the experimental setup and the results obtained using the designed three-to-seven-phase transformation system. The designed transformation system has 1:1 input:output ratio, hence the output voltage is equal to the input voltage. Nevertheless, this ratio can be altered to suit the step-up or step-down requirements. This can be achieved by simply multiplying the gain factor in the turn ratios. No-load and load tests are performed on the three-to-seven-phase transformer, and the load test is performed by connecting seven-phase *RL* load. The value is kept at $R = 300\ \Omega$ and $L = 48$ mH. The resulting waveforms of the primary side (three phase) and secondary side (seven phase) are depicted in Figs. 7 and 8, respectively.

In the present scheme for experimental purposes, three single-phase autotransformers are used to supply input phases of the transformer connections. The output voltages can be adjusted by simply varying the taps of the autotransformer. For balanced output, the input must have balanced voltages. Any unbalancing in the input is directly reflected in the output phases. Under no-load conditions, 150 V_{rms} is applied at the primary side. The input side voltage and current waveforms, under no-load and loaded steady-state conditions, are recorded and shown in Fig. 7. Under no-load conditions, a nearly 1.1 A (peak) current is drawn and the magnetizing current waveform is evident. The input voltage and currents under loaded conditions are 150 V and 6.36 A (rms) or (1.8Div.$*$5 A/Div.$=$) 9 A peak. Corresponding no-load and loaded condition voltage and current waveforms for the secondary side (seven phase) are presented in Fig. 8. The loaded current in the secondary side is nearly 2.55 A (r.m.s.) or (1.8Div.$*$2 A/Div.$=$) 3.6 A peak. Here, input and output voltages are the same, and the ratio of input to output currents is (9/3.6) $=$ 2.5, which agrees with theoretical input to output current ratio (7/3) $=$ 2.33. The input and output voltage waveforms show the successful implementation of the designed transformer. The waveform shows some distortion due to slight distortions in the input itself, which can be seen in Fig. 7 (top trace).

Fig. 9 shows the Lissajous pattern of input voltage and current in open circuit mode, and from there we get

$$\begin{aligned}\mathrm{Core\,Loss} &= V_{\mathrm{rms}} * I_{\mathrm{rms}} \\ &= 200.417 * (3.09609\,\mathrm{mV}/100\,\mathrm{mV}) = 6.2\,\mathrm{W}\end{aligned}$$

where the current probe sensitivity was at 100 mV/A. This measurement is done by a CRO, which can integrate and give the rms value of the wave shape. This is almost in agreement

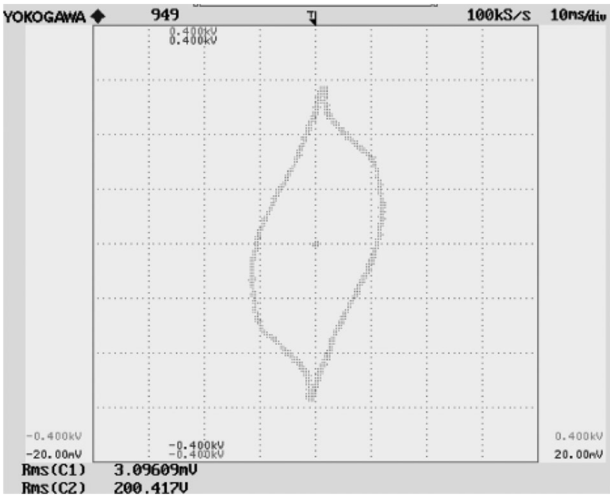


Fig. 9. Lissajous pattern of input voltage verses input current (measured with current probe of 100 mV = 1 A) showing core loss.

with the open circuit test where it was 5.5 W. This 0.7-W error may be due to $I_{\rm rms}$ which is the phasor sum of loss, as well as the magnetizing components of the transformer and tolerance of instruments.

VI. Conclusion

This paper proposes a new transformer connection scheme to transform the three-phase grid power to a seven-phase output supply. The connection scheme and the phasor diagram, along with the turn ratios, are illustrated. The successful implementation of the proposed connection scheme is elaborated upon using both simulation and experimentation. It is expected that the proposed connection scheme can be used in drives and other multiphase applications, e.g., ac–ac and dc–ac power conversion systems.

Thermal study shall be presented in our future paper.

Appendix

1) Derivation of (10)

Let us assume that

$$|V_x| = |V_y| = |V_z| = |V_a| = |V_b| = |V_c| = |V_d| \\ = |V_e| = |V_f| = |V_g|. \qquad \text{(A.1)}$$

And, neglecting losses, we have $3{*}S_1 = 7{*}S_2$.

Where, S_1 is per phase input VA and, S_2 is per phase output VA. We have from Fig. A.1. that $\bar{V}_g = \bar{V}_{a1a2} + \bar{V}_{b6b5}$. The angle between "$V_x$ and V_g" is $(-2\pi/7)$ and between "$-V_y$ and V_g" is $(\pi/21)$ and between "V_y and V_g" is $(-20\pi/21)$

$$\therefore |V_g| = |V_{a1a2}|\cos(-2\pi/7) + |V_{b5b6}|\cos(\pi/21). \qquad \text{(A.2)}$$

We have

$$|V_{a1a2}| = \frac{\sin(\pi/21)}{\sin(\pi/3)}|V_x| \text{ and } |V_{b6b5}| = \frac{\sin(2\pi/7)}{\sin(\pi/3)}|V_y|$$

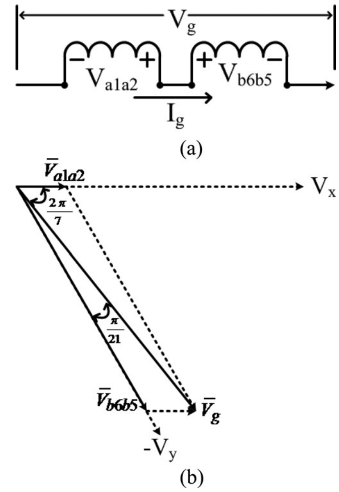


Fig. A.1. (a) Current through the windings of phase voltage V_g. (b) Phasor diagram of winding voltages.

$$\therefore |V_g| = \frac{\sin(\pi/21)}{\sin(\pi/3)}|V_x|\cos\left(-\frac{2\pi}{7}\right) + \frac{\sin(2\pi/7)}{\sin(\pi/3)}|V_y|\cos(\pi/21). \qquad \text{(A.3)}$$

Now per phase output $\text{VA} = S_2 = S_{a1a2} + S_{b5b6} = |V_g|\,|I_g|$

$$|V_g|\,|I_g| = \frac{\sin(\pi/21)\cos(2\pi/7)}{\sin(\pi/3)}|V_x|\,|I_g| + \frac{\sin(2\pi/7)\cos(\pi/21)}{\sin(\pi/3)}|V_y|\,|I_g| \qquad \text{(A.4)}$$

$$\because \cos\left(-\frac{2\pi}{7}\right) = \cos\left(\frac{2\pi}{7}\right)$$

$$|V_g|\,|I_g| = \frac{\sin(\pi/21)\cos(2\pi/7)}{\sin(\pi/3)}|V_x|\,|I_g| + \frac{\sin(2\pi/7)\cos(\pi/21)}{\sin(\pi/3)}|V_y|\,|I_g| \qquad \text{(A.5)}$$

$$= \frac{\sin(\pi/21)\cos(2\pi/7)}{\sin(\pi/3)}|V_g|\,|I_g| + \frac{\sin(2\pi/7)\cos(\pi/21)}{\sin(\pi/3)}|V_g|\,|I_g|$$

$$= \frac{\sin(\pi/21)\cos(2\pi/7)}{\sin(\pi/3)}S_2 + \frac{\sin(2\pi/7)\cos(\pi/21)}{\sin(\pi/3)}S_2$$

$$= \frac{\sin(\pi/21)\cos(2\pi/7)}{\sin(\pi/3)}\frac{3S_1}{7} + \frac{\sin(2\pi/7)\cos(\pi/21)}{\sin(\pi/3)}\frac{3S_1}{7}$$

$$= S_{a1a2} + S_{b5b6}.$$

$$\text{Hence, } S_{a1a2} = \frac{3S_1}{7}\frac{\sin(\pi/21)\cos(2\pi/7)}{\sin(\pi/3)}\text{ VA} \qquad \text{(A.6)}$$

$$\text{and } S_{b5b6} = \frac{3S_1}{7}\frac{\sin(2\pi/7)\cos(\pi/21)}{\sin(\pi/3)} = \frac{3S_1}{7}\frac{-\sin(2\pi/7)\cos(-20\pi/21)}{\sin(\pi/3)}. \qquad \text{(A.7)}$$

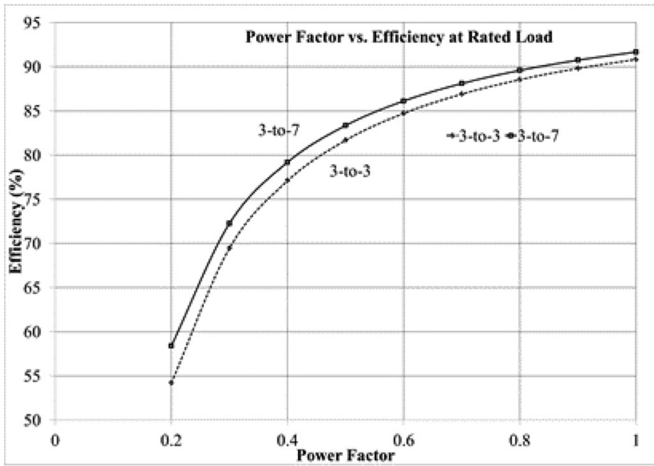


Fig. A.2. Comparison of efficiency of three-to-three and three-to-seven.

TABLE A.1
RESISTANCE OF INPUT AND OUTPUT PHASES

	Phases	Resistance (Ω)	
		R_{dc}	Rac
INPUT	X	0.8	0.88
	Y	0.8	0.88
	Z	0.816	0.8976
OUTPUT	A	1.29032	1.419352
	B	1.59566	1.755226
	C	1.44444	1.58884
	D	1.6708	1.83788
	E	1.64103	1.805133
	F	1.48019	1.628209
	G	1.53986	1.693846

2) Efficiency Comparison Between Three-to-Three and Three-to-Seven Transformers

The dc resistance of each winding was measured and it is converted into ac resistance considering the skin and proximity effects. The skin effect is not significant because the diameter of primary (2.032 mm) and secondary (1.422 mm) winding wires is lesser than the skin depth (10.75 mm at room temperature or 8.77 mm at 100 °C) at 50 Hz. Here, we have considered ratio $(R_{\rm ac}/R_{\rm dc}) = 1.1$.

The resistances of secondary outputs are more than the primary inputs because the diameter of the primary wire is greater than the secondary winding wires and the number of turns of secondary windings is higher than the primary. However, the copper loss at rated load in the three-to-three-phase transformer is higher than the three-to-seven-phase transformer because the current in the secondary windings in three-to-seven-phase transformer is reduced by a factor of (3/7) assuming voltage ratio of primary to secondary is one. Hence, the efficiency of a three-to-seven transformer is higher than the three-to-three transformer. The power factor versus efficiency is plotted and shown in Fig. A.2. The measured resistances are given in Table A.1.

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