Consensus of First-order Agents with Neighbors' Delayed Information

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***Abstract*—This paper is concerned with the consensus problem for general first-order multi-agent systems over undirected topology. Under the assumption that the topology is connected and every agent receives neighbors information with time-varying communication delay, allowable delay bound depending on the agent dynamics, topology structure and the control gain is obtained. In particular, in case of first-order integrator systems, any large yet bound delay is tolerant for consensus. Finally, the effectiveness of the theoretical result is illustrated through numerical example.**

Index Terms-Consensus, multi-agent systems, communication delay, undirected topology.

I. Introduction

Recently, more and more researchers have begun to pay attention on the consensus of multi-agent systems because of its wide range of applications ([1]-[3]). Consensus means to design appropriate algorithm or protocol to guarantee that all the agents can reach an agreement on certain quantities of interest. In the past few years, numerous researches have devoted to the study of consensus problem. In [4], consensus for single-integrator multi-agent systems was studied. Applying the nonnegative matrix theory, exponential consensus was investigated in [5] for generic multi-agent systems. In [6], consensus of switched multi-agent systems including both the continuous-time and discrete-time subsystems was addressed. When the network communication was affected by measurement or channel noise, [7] considered the strong consensus of the discrete-time multi-agent systems.

Considering the finiteness of axonal signal transmission and switching speeds, communication delay is unavoidable in the process of signal transmission. In the real network, time delay also induces unexpected dynamic behaviors, such as instability and oscillation and oscillation [8]. Therefore, it is necessary to study the effect of delay on consensus. So far, there are many work to address this issue. For the first-order integrator multi-agent systems, [9] provided an necessary and sufficient condition on the upper bound of the constant communication delay. By adopting a part of agent's own historical input information in the protocol design, consensus condition depending on the dynamic structure, time delay and network topology was obtained in [10]. Under jointly connected topologies, [11] researched the consensus of second-order multi-agent systems with time-delay. Suppose that the agent dynamics are at most critically unstable, low gain methods are used in [12, 13] to weaken the effect of delay on consensus. If communication delay only influences neighbors' transmitted information, [14] established consensus condition for the first-order integrator multi-agent systems. Reference [15] addressed the consensus problem in discrete-time multi-agent systems with time-invariant delays. For any fixed control gain chosen from stable ranges, consensus achievement of agents with both input and communication delays was studied in [16].

Provided the network is undirected and the time-varying communication delay only affects neighbors' transmitted information, we study the consensus of generic first-order multi-agent systems. Thus, the system considered is more general than those in [9] and [14]. The delay being unknown leads that the delay information is unavailable in the protocol design as in [10, 12]. Additionally, the technique in [16] can not work when the communication delay is time-varying. To derive consensus, we design protocol without the precise information of the communication delay. By jointly investigating the structure of agent dynamics, network topology and time delay, consensus conditions are obtained. Especially, it is proved that time-delay is allowed to be any large yet bounded if the agent dynamic is the first-order integrator system.

This paper is organized as follow. In section II, relevant graph theory are introduced. Section III shows the problem statement. In section IV, consensus results are provided. Numerical example is given in section V and concluding remarks are finally shown in section VI.

Before closing this section, some notations are introduced. We use $\mathbf{R}$ to denote the set of real number. Symbol $diag\{a_1, a_2, \ldots, a_N\}$ is a diagonal matrix with elements $a_1, a_2, \ldots, a_N$. Let $\mathbf{1}_N$ be an N dimensional column vector with all ones and $\|\cdot\|$ be the Euclidean norm. For a positive scalar $\tau > 0$, denote $\mathscr{C}_\tau$ the Banach space of continuous vector function mapping the interval [-τ, 0] into $\mathbf{R}$.

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II. ALGEBRAIC GRAPH THEORY

In this paper, we model the information exchange among agents by an undirected graph. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a diagraph with the set of vertices $\mathcal{E} = \{1, 2, \ldots, N\}$, the set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, and the weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N \times N}$ is symmetric. In $\mathcal{G}$, the i-th vertex represents the i-th agent. Assume $a_{ij} > 0$ if and only if $(i, j) \in \mathcal{E}$, i.e., there is a communication link among agent i and j. There is a sequence satisfying $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$ with $(i_{j-1}, i_j) \in \mathcal{E}$ for $j \in \{2, 3, \ldots, N\}$. Undirected $\mathcal{G}$ is connected if any two distinct agents of $\mathcal{G}$ can be connected via a path that follows the edges of $\mathcal{G}$. For agent i, the degree is defined as $d_i \triangleq \sum_{j=1}^{N} a_{ij}$. Diagonal matrix $D = diag\{d_1, d_2, \ldots, d_N\}$ is used to denote the degree matrix of diagraph $\mathcal{G}$. If undirected graph $\mathcal{G}$ is connected, it is known from [7] that matrix $[D^{-1}\mathcal{A}]$ is irreducible and there is a unique vector $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_N]^T$ with $\alpha_i \geq 0$ and $\sum_{i=1}^{N} \alpha_i = 1$ such that $\alpha^T [D^{-1}\mathcal{A}] = \alpha^T$. When the diagraph $\mathcal{G}$ is connected, from [16], we known $[D^{-1}\mathcal{A}]$ is diagonalizable and has a simple eigenvalue $\mu_1 = 1$, and other eigenvalues are real with modulus no bigger than 1, i.e., $-1 \leq \mu_2 \leq \ldots \leq \mu_N < 1$.

III. PROBLEM STATEMENT

Consider the multi-agent systems consisting of N agents indexed by $1, 2, \ldots, N$, respectively. The dynamics of each agent can be expressed as follows:

$$\dot{x}_i(t) = a x_i(t) + b u_i(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i(t) \in \mathbf{R}$ and $u_i(t) \in \mathbf{R}$ are the state and input of agent i; $a \in \mathbf{R}$, $b \in \mathbf{R}$ with $b \neq 0$ are constant coefficients. It is assumed that the initial value $x_i(0)$ is known initial value.

The relationships among above N agents are described by a weighted undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V}, \mathcal{E}$ and $\mathcal{A}$ are defined in Section 2.

It should be noted that the communication delay is unavoidable in the network. Let $\tau(t) : [0, \infty) \rightarrow \mathbf{R}^+$ denote the time-varying communication delay affecting the neighbors' transmitted information. Making use of the relative state information, we propose the following consensus protocol

$$u_i(t) = \frac{k}{b d_i} \sum_{j=1}^{N} a_{ij} [x_j(t - \tau(t)) - x_i(t)], \tag{2}$$

where $k \in \mathbf{R}$ is a fixed control gain and $d_i = \sum_{j=1}^{N} a_{ij}$ is the degree of agent i.

To make the agent dynamics (1) operational under protocol (2), we additionally set the initial value $x_i(s) = 0$ for any $s < 0$. The definition of consensus is show as follows.

Definition For a given diagraph $\mathcal{G}$, the multi-agent systems (1) achieve consensus under protocol (2) if for any initial values, $\lim_{t \rightarrow \infty} \|x_j(t) - x_i(t)\| = 0$, $\forall i, j = 1, 2, \ldots, N$.

In view of above definition, when $a < 0$ in (1), it is obvious that $u_i(t) \equiv 0$ can solve the consensus problem. To make the problem interesting, we assume $a \geq 0$ in (1). The objective of this paper is to provide conditions such that the multi-agent systems (1) achieve consensus under protocol (2).

IV. MAIN RESULTS

Before presenting the main results, the following technical lemmas are first shown.

Lemma 1: [17] For any scalars γ_1 and γ_2 with $\gamma_2 \geq \gamma_1$, vector function $\omega : [\gamma_1, \gamma_2] \rightarrow \mathbf{R}^n$ such that the integrations in the following are well-defined, then

$$\begin{aligned} &\int_{\gamma_1}^{\gamma_2} \omega^T(\beta) d\beta P \int_{\gamma_1}^{\gamma_2} \omega(\beta) d\beta \\ \leq\ & (\gamma_2 - \gamma_1) \int_{\gamma_1}^{\gamma_2} \omega^T(\beta) P \omega(\beta) d\beta. \end{aligned}$$

The following lemma can be obtained from the Razumikhin Stability Theorem in [18]

Lemma 2: Consider a retarded functional differential equation

$$\begin{cases} \dot{x}(t) = f(t, x_t), x(t) \in \mathbf{R}, t \geq t_0, \\ x_{t_0}(\theta) = \varphi(\theta), \forall \theta \in [-\tau, 0], \end{cases}$$

where $x_t = x(t + \theta), \forall \theta \in [-\tau, 0]$. The function $f : \mathbf{R} \times \mathscr{C}_\tau$ is such that the image by f of $\mathbf{R} \times$(a bounded subset of $\mathscr{C}_\tau$) is a bounded subset of $\mathbf{R}$ and the functions $u, v, w, p : \mathbf{R}^+ \rightarrow \mathbf{R}^+$ are continuous, nondecreasing and positive for all $s > 0, u(0) = v(0) = 0$, v is strictly increasing and $p(s) > s$. Suppose that there is a continuous function $V : \mathbf{R} \rightarrow \mathbf{R}$, if for all $x \in \mathbf{R}$ the following conditions are satisfied,

1) $u(\|x\|) \leq V(x) \leq v(\|x\|)$;
2) $\dot{V}(x) \leq -w(\|x\|)$, if $V(x(t + \theta)) < p(V(x(t))), \forall \theta \in [-\tau, 0]$;
3) $\lim_{s \rightarrow \infty} u(s) = \infty$.

Then, the solution $x(t) \equiv 0$ is globally asymptotically stable.

Now, we are in position to give the main results of this paper. We first provide the consensus result of multi-agent systems (1) with $a > 0$ under protocol (2).

Theorem 1: Assume $a > 0$ and undirected graph $\mathcal{G}$ is connected. Then, for $k > \max\{\frac{a}{1-\mu_i}, i = 2, \ldots, N\}$ in (2), the multi-agent systems (1) achieve consensus if the time-varying communication delay satisfies

$$0 \leq \tau(t) \leq \overline{\tau} < \min_{i=2,3,\ldots,N} \frac{k(1 - \mu_i) - a}{2k^2 |\mu_i|},$$

Proof: From the condition that undirected graph $\mathcal{G}$ is connected, it follows that all diagonal elements of matrix D are positive, i,e., $d_i > 0$ for $i = 1, \ldots, N$. Inserting protocol (2) into the agent dynamics (1) concludes

$$\dot{x}_i(t) = a x_i(t) + k \sum_{j=1}^{N} \frac{a_{ij}}{d_i} [x_j(t - \tau(t)) - x_i(t)],$$

which implies

$$\dot{x}_i(t) = [a - k] x_i(t) + k \sum_{j=1}^{N} \frac{a_{ij}}{d_i} x_j(t - \tau(t)).$$

Denote $x(t) \triangleq [x_1(t), x_2(t), \ldots, x_N(t)]^T$. Then,

$$\dot{x}(t) = [a - k] x(t) + k [D^{-1}\mathcal{A}] x(t - \tau(t)) \tag{3}$$

is in force.

Denote $\overline{x}(t) \triangleq \sum_{i=1}^{N} \alpha_i x_i(t) = \alpha^T x(t)$ be the weighed average state of all agents at time t, where $\alpha = [\alpha_1, \ldots, \alpha_N]^T$ satisfying $\sum_{i=1}^{N} \alpha_i = 1$ and $\alpha_i \geq 0$ for $i = 1, \ldots, N$ is the left eigenvector of $[D^{-1}\mathcal{A}]$ corresponding to $\mu_1 = 1$. In view of $\alpha^T[D^{-1}\mathcal{A}] = \alpha^T$, it follows

$$\dot{\overline{x}}(t) = [a-k]\overline{x}(t) + k\overline{x}(t-\tau(t)).$$

Considering that $\mathbf{1}_N$ is the right eigenvector of $[D^{-1}\mathcal{A}]$ corresponding to $\mu_1 = 1$, it follows

$$\mathbf{1}_N\dot{\overline{x}}(t) = [a-k]\mathbf{1}_N\overline{x}(t) + k[D^{-1}\mathcal{A}]\mathbf{1}_N\overline{x}(t-\tau(t)). \quad (4)$$

Let $\delta(t) = x(t) - \mathbf{1}_N\overline{x}(t)$, combining equation (3) and (4) follows

$$\dot{\delta}(t) = [a-k]\delta(t) + k[D^{-1}\mathcal{A}]\delta(t-\tau(t)). \quad (5)$$

Since $\|x_j(t) - x_i(t)\| \leq \|x_j(t) - \overline{x}(t)\| + \|x_i(t) - \overline{x}(t)\|$ and $x_i(t) - \overline{x}(t) = \frac{\sum_{j=1}^{N}[x_j(t)-x_i(t)]}{N}$, it is easy to know the consensus is achieved if and only if system (5) is asymptotically stable. From the fact that $[D^{-1}\mathcal{A}]$ is diagonalizable, construct invertible matrix Φ to transform $[D^{-1}\mathcal{A}]$ into a diagonal form, i.e., $\Phi^{-1}[D^{-1}\mathcal{A}]\Phi = diag\{\mu_1, \mu_2, \ldots, \mu_N\}$, where the first row of Φ^{-1} is vector α^T. Denote $\tilde{\delta}(t) \triangleq \Phi^{-1}\delta(t) = [\tilde{\delta}_1(t), \ldots, \tilde{\delta}_N(t)]^T$. Thus $\tilde{\delta}_1(t) = \alpha^T\delta(t) \equiv 0$, and the consensus problem is equivalent to the simultaneous stability of error system

$$\dot{\tilde{\delta}}_i(t) = [a-k]\tilde{\delta}_i(t) + k\mu_i\tilde{\delta}_i(t-\tau(t)) \quad (6)$$

for $i = 2, \ldots, N$. Let $\Delta\tilde{\delta}_i(t) \triangleq \tilde{\delta}_i(t) - \tilde{\delta}_i(t-\tau(t))$, $-k[1-\mu_i] \triangleq \sigma_i$ and $k\mu_i = f_i$, then there holds

$$\dot{\tilde{\delta}}_i(t) = [a+\sigma_i]\tilde{\delta}_i(t) - f_i\Delta\tilde{\delta}_i(t). \quad (7)$$

If $\mu_i = 0$, i.e., $f_i = 0$, it is trivial to yield $\lim_{t\to\infty}\tilde{\delta}_i(t) = 0$ from the condition that $k > \max\{\frac{a}{1-\mu_i}, i = 2, \ldots, N\}$. Additionally, from $tr[D^{-1}\mathcal{A}] = 0$, there exists eigenvalues $\mu_i \neq 0$. Without loss of generality, we assume $\mu_i \neq 0$ for $i = 2, \ldots, N$.

Take function $V(\tilde{\delta}_i(t)) = \frac{1}{2}\tilde{\delta}_i^2(t)$, then along the trajectory of (7) follows

$$\begin{aligned}\dot{V}(\tilde{\delta}_i(t)) &= \frac{1}{2}\dot{\tilde{\delta}}_i(t)\tilde{\delta}_i(t) + \frac{1}{2}\tilde{\delta}_i(t)\dot{\tilde{\delta}}_i(t) \\ &= \frac{1}{2}[(a+\sigma_i)\tilde{\delta}_i(t) - f_i\Delta\tilde{\delta}_i(t)]\tilde{\delta}_i(t) \\ &\quad + \frac{1}{2}\tilde{\delta}_i(t)[(a+\sigma_i)\tilde{\delta}_i(t) - f_i\Delta\tilde{\delta}_i(t)] \\ &= [a+\sigma_i+\frac{1}{2}c_i]\tilde{\delta}_i^2(t) - \frac{1}{2}c_i\tilde{\delta}_i^2(t) \\ &\quad -\frac{1}{2}f_i\Delta\tilde{\delta}_i(t)\tilde{\delta}_i(t) - \frac{1}{2}f_i\tilde{\delta}_i(t)\Delta\tilde{\delta}_i(t),\end{aligned}$$

where c_i is a constant to be designed. Then, making use of $\tilde{\delta}_i(t) = \Delta\tilde{\delta}_i(t) + \tilde{\delta}_i(t-\tau(t))$, we have

$$\begin{aligned}&\dot{V}(\tilde{\delta}_i(t)) \\ = \; & [a+\sigma_i+\frac{1}{2}c_i]\tilde{\delta}_i^2(t) - [\frac{1}{2}c_i + f_i]\Delta\tilde{\delta}_i^2(t) \\ & -\frac{1}{2}c_i\tilde{\delta}_i^2(t-\tau(t)) - [\frac{1}{2}c_i + \frac{1}{2}f_i]\Delta\tilde{\delta}_i(t)\tilde{\delta}_i(t-\tau(t)) \\ & -[\frac{1}{2}c_i + \frac{1}{2}f_i]\tilde{\delta}_i(t-\tau(t))\Delta\tilde{\delta}_i(t) \\ \leq \; & [a+\sigma_i+\frac{1}{2}c_i]\tilde{\delta}_i^2(t) + \frac{1}{2}[l_i - c_i]\tilde{\delta}_i^2(t-\tau(t)) \\ & +\frac{1}{2}[\frac{(c_i+f_i)^2}{l_i} - c_i - 2f_i]\Delta\tilde{\delta}_i^2(t),\end{aligned}$$

where $l_i > 0$ is an arbitrary constant. Let $l_i = c_i > 0$, then

$$\frac{(c_i+f_i)^2}{l_i} - c_i - 2f_i = \frac{f_i^2}{c_i}$$

and

$$\dot{V}(\tilde{\delta}_i(t)) \leq [a+\sigma_i+\frac{1}{2}c_i]\tilde{\delta}_i^2(t) + \frac{1}{2}\times\frac{f_i^2}{c_i}\Delta\tilde{\delta}_i^2(t).$$

Integrating both sides of equation (6) from $t-\tau(t)$ to t follows

$$\begin{aligned}\Delta\tilde{\delta}_i(t) &= \int_{t-\tau(t)}^{t}\dot{\tilde{\delta}}_i(s)ds \\ &= \int_{t-\tau(t)}^{t}[(a-k)\tilde{\delta}_i(s) + k\mu_i\tilde{\delta}_i(s-\tau(s))]ds.\end{aligned}$$

Then, employing Lemma 1 concludes

$$\begin{aligned}&\Delta\tilde{\delta}_i^2(t) \\ \leq \; & \tau(t)\int_{t-\tau(t)}^{t}[(a-k)\tilde{\delta}_i(s) + k\mu_i\tilde{\delta}_i(s-\tau(s))]^2ds \\ \leq \; & 2\tau(t)\int_{t-\tau(t)}^{t}[(a-k)^2\tilde{\delta}_i^2(s) + k^2\mu_i^2\tilde{\delta}_i^2(s-\tau(s))]^2ds.\end{aligned}$$

Suppose $V(\tilde{\delta}_i(t+\theta)) < \rho V(\tilde{\delta}_i(t))$ for any $\theta \in [-\overline{\tau}, 0]$, where $\rho > 1$ is a constant to be determined. Thus, $\Delta\tilde{\delta}_i^2(t) \leq 2\rho\tau^2(t)[(a-k)^2 + k^2\mu_i^2]\tilde{\delta}_i^2(t)$ and

$$\dot{V}(\tilde{\delta}_i(t)) \leq \{a+\sigma_i+\frac{c_i}{2}+\frac{f_i^2}{c_i}\rho\tau^2(t)[(a-k)^2+k^2\mu_i^2]\}\tilde{\delta}_i^2(t).$$

Considering $0 \leq \tau(t) < \overline{\tau}$, if the following relation holds

$$a+\sigma_i+\frac{c_i}{2}+\frac{f_i^2}{c_i}\overline{\tau}^2[(a-k)^2+k^2\mu_i^2] < 0 \quad (8)$$

for $i = 2, \ldots, N$. Design

$$\rho = \min_{i=2,\ldots,N}\frac{f_i^2\overline{\tau}^2[(a-k)^2+k^2\mu_i^2] - \frac{1}{2}c_i^2 - (a+\sigma_i)c_i}{2f_i^2\overline{\tau}^2[(a-k)^2+k^2\mu_i^2]},$$

then there holds

$$\begin{aligned}&a+\sigma_i+\frac{c_i}{2}+\frac{f_i^2}{c_i}\rho\tau^2(t)[(a-k)^2+k^2\mu_i^2] \\ \leq \; & \frac{1}{2}\{a+\sigma_i+\frac{c_i}{2}+\frac{f_i^2}{c_i}\overline{\tau}^2[(a-k)^2+k^2\mu_i^2]\} \\ < \; & 0.\end{aligned}$$

From Lemma 2 and equation (8), consensus is derived if

$$\overline{\tau}^2 < \min_{i=2,\ldots,N} \frac{-\frac{1}{2}c_i^2 - (a+\sigma_i)c_i}{f_i^2[(a-k)^2 + k^2\mu_i^2]}.$$

Design $c_i = -(a+\sigma_i) = -[a - k(1-\mu_i)] > 0$. Then, using $\sigma_i = -k(1-\mu_i)$ and $f_i = k\mu_i$, the allowable delay bound for consensus is

$$\overline{\tau}^2 < \min_{i=2,\ldots,N} \frac{[k(1-\mu_i) - a]^2}{2k^2\mu_i^2[(a-k)^2 + k^2\mu_i^2]}.$$

i.e.,

$$\overline{\tau} < \min_{i=2,\ldots,N} \frac{[k(1-\mu_i) - a]}{k|\mu_i|\sqrt{2[(a-k)^2 + k^2\mu_i^2]}}.$$

Based on the following relations

$$-k < a - k < k[1-\mu_i] - k \leq 2k - k = k,$$

and $|\mu_i| \leq 1$ for $i = 2, \ldots, N$, the consensus is achieved if

$$\overline{\tau} < \min_{i=2,\ldots,N} \frac{[k(1-\mu_i) - a]}{2k^2|\mu_i|}.$$

The proof is complete. □

For the special case of $a = 0$, the agent dynamics (1) become the first-order integrator system, and the consensus under protocol (2) can be guaranteed for any large yet bounded communication delay. This result is shown below.

Corollary 1: Given a connected undirected graph $\mathcal{G}$ and the following first-order integrator agent dynamics

$$\dot{x}_i(t) = u_i(t), \quad i = 1, 2, \ldots, N. \tag{9}$$

Then, for any large yet bounded time-varying communication delay $0 \leq \tau(t) \leq \overline{\tau}$, the consensus is asymptotically derived by selecting the gain in protocol (2) as $0 < k < \frac{1-\mu_2}{2\overline{\tau}}$, where $\mu_2 = \max\{\mu_i, i = 2, \ldots, N\}$.

Proof: When $a = 0$, it yields from Theorem 1 that the acquirement for control gain is $k > \max\{\frac{a}{1-\mu_i}, i = 2, \ldots, N\} = 0$, and the allowable delay bound for consensus becomes

$$\overline{\tau} < \min_{i=2,\ldots,N} \frac{k(1-\mu_i)}{2k^2|\mu_i|} = \frac{(1-\mu_i)}{2k|\mu_i|}.$$

From $|\mu_i| \leq 1$ and $\mu_2 = \max\{\mu_i, i = 2, \ldots, N\}$, the delay condition is satisfied if $\overline{\tau} < \frac{1-\mu_2}{2k}$. Thus, for any large yet bounded communication delay $0 \leq \tau(t) \leq \overline{\tau}$, the consensus can be achieved by taking $0 < k < \frac{1-\mu_2}{2\overline{\tau}}$ and the proof is complete. □

V. NUMERICAL EXAMPLE

In this section, an example is carried out to verify the theoretical result obtained in the previous section.

Consider a network with 3 agents, and the dynamics is described as

$$\dot{x}_i(t) = 1.5x_i(t) + 2u_i(t), \quad i = 1, 2, 3.$$

Clearly, above system is unstable with pole $\frac{1}{2}$. The topology among the agents is described by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$. The adjacency matrix and degree matrix are

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

It is easy to know that above network is connected and all eigenvalues of matrix $[D^{-1}\mathcal{A}]$ are $\mu_1 = 1, \mu_2 = 0$ and $\mu_3 = -1$. Based on Theorem 1, take $k = 2$. In addition, the allowable delay bound for consensus is computed as $\overline{\tau} < 0.332$. Take time-varying delay $\tau(t) = 0.3sin^2(t)$ in protocol (2). Then, for the initial values $x(0) = [-6; -4; 2]$, Fig. 1 shows that the error states $x_i(t) - \overline{x}(t), i = 1, 2, 3$ converge to zero asymptotically, where $\overline{x}(t) = \frac{1}{2}x_1(t) + \frac{1}{4}x_2(t) + \frac{1}{4}x_3(t)$.

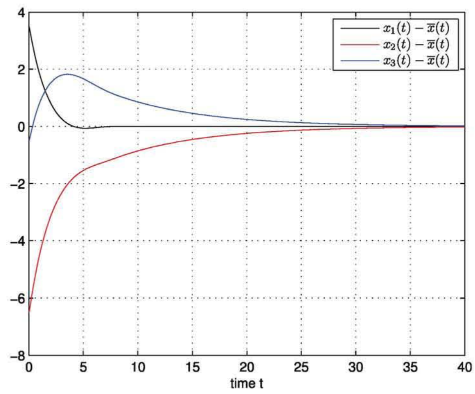

Fig. 1 Error States

VI. CONCLUSION

In this paper, we consider the consensus problem with neighbors' delayed information. Under the assumption that the delay is time-varying and the network is undirected, consensus conditions are provided for unstable agent dynamics. It is worth mentioning that the proposed results are expected to the directed topology case and the vector case.

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