IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received April 17, 2021, accepted May 5, 2021, date of publication May 11, 2021, date of current version May 19, 2021.

*Digital Object Identifier 10.1109/ACCESS.2021.3079161*

# Crystal Structure Algorithm (CryStAl): A Metaheuristic Optimization Method

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This work was supported in part by the University of Tabriz under Grant 1615, and in part by the University of Liverpool.

**ABSTRACT** Metaheuristics are computational procedures that intelligently lead the search process through the efficient exploration of the search space associated with an optimization problem. With the progressive outburst of problems with large data sets in various fields, there is an ongoing quest for enhancing existing metaheuristic algorithms as well as developing new ones with greater accuracy and efficiency. In general, a powerful and efficient metaheuristic algorithm is based on a rich inspiration source, implemented effectively through a precise mathematical model. Aiming to develop a highly efficient, nature-inspired optimization algorithm, here we propose a novel metaheuristic called Crystal Structure Algorithm (CryStAl). This method is chiefly inspired by the principles underlying the formation of crystal structures from the addition of the basis to the lattice points, which is a natural phenomenon that can be seen in the symmetric arrangement of constituents (i.e. atoms, molecules, or ions) in crystalline minerals such as quartz. A total number of 239 mathematical functions which are categorized into four different groups are utilized to evaluate the overall performance of the proposed method. To validate the results of this novel algorithm, 12 different classical and modern metaheuristic algorithms are selected from the literature. The minimum, mean, and standard deviation values alongside the number of function evaluations for CryStAl and the other metaheuristics for a specific tolerance are calculated and presented accordingly. The obtained results, further supported by a complete statistical analysis, demonstrated that the proposed algorithm is capable of providing very competitive results, outperforming the other metaheuristics in most cases.

**INDEX TERMS** Crystal Structure Algorithm (CryStAl), lattice, function, metaheuristic, optimization, statistical analysis.

## I. INTRODUCTION

Many design problems in nature can be considered as optimization problems that demand appropriate optimization techniques and methods to be dealt with. Nowadays, design problems have become extremely complex for which classical optimization methods based on mathematical principles are incapable of providing satisfactory results in a reasonable period of time. Gradient-based methods, which utilize the gradient of the objective function for the configuration of the optimization problem, are a type of these mathematical methods. Over the past few decades, exploring the deficiencies of classical optimization methods and introducing new efficient optimization algorithms have been of great interest. Based on recent technological advances, there is a growing interest in introducing new optimization methods with enhanced efficiency, accuracy, and increased speed rate for tackling difficult optimization problems. Besides, some other concerns in dealing with some specific issues such as the local optima issues alongside the smoothness and convexity of the search spaces have been of great importance for a long period of time.

The presented concerns about the classical optimization algorithms have led optimization experts to a new methodology in solving different optimization problems called ''Metaheuristic''. Glover [1] firstly proposed this term in 1986 which is comprised of the main word, i.e. heuristics, and a prefix, i.e. meta, which both have Greek origins. The

The associate editor coordinating the review of this manuscript and approving it for publication was Emanuele Crisostomi.

term "heuristic" comes from *heuriskein* which is an old Greek word meaning "to discover", while "meta" means "beyond the normal or natural limits of something". Metaheuristics are solution techniques that implement higher-level strategies into search processes in order to guide an optimization process to perform a powerful search into the search space with some special capabilities such as avoiding local optima.

As presented by Sörensen [2], the history of utilizing metaheuristics as the solution methods for dealing with real-world problems can generally be categorized into five distinct periods. In the first period which is named the "pre-theoretical" period (until 1940), there was not any formal presentation of heuristics and metaheuristics methods. Despite that, these methods had been used for solving some simple optimization problems in this period. In the second period which is from 1940 to 1980 and known as the "early" period, some studies were conducted on heuristics which was the first formal introduction and discussion in this field. In the third period which is called the "method-centric" period (1980 to 2000), multiple metaheuristics were proposed and developed for specific applications which extended the field of heuristics and metaheuristics. In the fourth period, which is from 2000 until now and known as the "framework-centric" period, the methodology of utilizing metaheuristics as frameworks alongside methods has been successfully presented with considerable growth of intuition in this field. In the fifth or last period which is named the "scientific" or "future" period, the design and introduction of new metaheuristics will turn into a matter of science rather than art. A summary of the abovementioned historical periods is presented in Table 1.

**TABLE 1. Summary of the historical periods of metaheuristics evolution.**

| No. | Name | Duration | Details |
|---|---|---|---|
| 1 | Pre-theoretical | Until 1940 | No formal presentation with limited applications. |
| 2 | Early | 1940 to 1980 | Heuristics were formally introduced and discussed. |
| 3 | Method-centric | 1980 to 2000 | Multiple metaheuristics were proposed and developed for specific applications. |
| 4 | Framework-centric | 2000 to now | The methodologies of utilizing metaheuristics as frameworks alongside various methods have been successfully presented. |
| 5 | Scientific or future | Future | The design and introduction of new metaheuristics will turn into a matter of science rather than art. |

Considering the development of various metaheuristic algorithms, four classifications can be made in terms of their inspiration. The first category is entitled "evolutionary algorithms" including the Memetic Algorithm (MA) [3], Genetic Algorithm (GA) [4], Differential Evolution (DE) [5], and the Evolution Strategies (ES) [6], which were developed based on the biological evolution and reproduction. The second category contains swarm intelligence-based algorithms that were formed based on the cooperative behavior of decentralized and self-organized natural or artificial systems. The Particle Swarm Optimization (PSO) [7], Ant Colony Optimization (ACO) [8], Artificial Bee Colony (ABC) [9], Cat Swarm Optimization (CSA) [10], Firefly Algorithm (FA) [11], Krill Herd (KH) algorithm [12], and Slap Swarm Algorithm (SSA) [13] are some of the well-known methods in this category. The third category consists of algorithms motivated by physical laws. The Simulated Annealing (SA) [14], Magnetic Optimization Algorithm (MOA) [15], Gravitational Search Algorithm (GSA) [16], Charged System Search (CSS) algorithm [17], Ray Optimization Algorithm (ROA) [18], Colliding Bodies Optimization (CBO) [19], Multiverse Algorithm (MVO) [20], and the Sine Cosine Algorithm (SCA) [21] are some methods belonging to this category. Beyond these methods, some other metaheuristic algorithms were presented based on the lifestyle of humans and animals (the fourth category) such as the Harmony Search (HS) [22], Teaching–learning-based Optimization (TLBO) [23], Creativity-Oriented Optimization Algorithm (COOA) [24], Human Behavior-Based Optimization (HBBO) [25], and the Gaining Sharing Knowledge-based algorithm (GSK) [26]. In addition to these standard algorithms, some other challenges in developing, upgrading, or hybridizing standard algorithms have also been achieved [27]–[38]. A summary of these metaheuristic algorithms is presented in Table 2.

**TABLE 2. Summary of the classification of the metaheuristic algorithms.**

| Classification | Algorithm | Year of Proposal |
|---|---|---|
| Evolution | Memetic Algorithm (MA) [3] | 1989 |
| | Genetic Algorithm (GA) [4] | 1992 |
| | Differential Evolution (DE) [5] | 1997 |
| | Evolution Strategies (ES) [6] | 2002 |
| Swarm intelligence | Particle Swarm Optimization (PSO) [7] | 1995 |
| | Ant Colony Optimization (ACO) [8] | 1996 |
| | Artificial Bee Colony (ABC) [9] | 2006 |
| | Cat swarm Optimization (CSA) [10] | 2006 |
| | Firefly Algorithm (FA) [11] | 2010 |
| | Krill Herd (KH) algorithm [12] | 2012 |
| | Slap Swarm Algorithm (SSA) [13] | 2017 |
| Physical laws | Simulated Annealing (SA) [14] | 1983 |
| | Magnetic Optimization Algorithm (MOA) [15] | 2008 |
| | Gravitational Search Algorithm (GSA) [16] | 2009 |
| | Charged System Search (CSS) algorithm [17] | 2010 |
| | Ray Optimization Algorithm (ROA) [18] | 2012 |
| | Colliding bodies Optimization (CBO) [19] | 2014 |
| | Multi-verse Algorithm (MVO) [20] | 2016 |
| | Sine Cosine Algorithm (SCA) [21] | 2016 |
| Lifestyle | Harmony Search (HS) [22] | 2001 |
| | Teaching–learning-based optimization (TLBO) [23] | 2011 |
| | Creativity-Oriented Optimization Algorithm (COOA) [24] | 2015 |
| | Human Behavior-Based Optimization (HBBO) [25] | 2017 |
| | Gaining Sharing Knowledge-based algorithm (GSK) [26] | 2019 |

In this paper, a novel metaheuristic optimization method called Crystal Structure Algorithm (CryStAl) is proposed which is inspired by the principles underlying the formation of crystal structures from the addition of the basis to the lattice points. A total number of 239 mathematical functions which are categorized into four different groups are utilized to evaluate the overall performance of the proposed method. To validate the results of CryStAl, 12 different classical and modern metaheuristic algorithms are selected from the literature. The minimum, mean, and standard deviation values alongside the number of function evaluations for CryStAl and the other metaheuristics for a specific tolerance are calculated and presented accordingly.

In general, the efficiency of novel metaheuristic algorithms in producing improved solutions to well-known optimization problems has been a significant research challenge for algorithm developers in recent decades. Considering the source of inspiration and the mathematical model as the two foundations of metaheuristic algorithms, this mission can generally be accomplished by utilizing solid mathematical models developed based on suitable inspirational concepts.

In this regard, this paper proposes CryStAl as a metaheuristic algorithm conceptualized based on the principles underlying the formation of crystal structures as a well-known physical paradigm in nature. This method is implemented using a fully-detailed mathematical model comprised of the details of crystalline configurations which have been established by crystallographers over the past few centuries. By developing a metaheuristic based on such a rich inspiration source followed by a precise mathematical model, we have shown that excellent results in dealing with different optimization problems can be achieved.

It should be also noted that the proposed approach, i.e. CryStAl, is a parameter-free metaheuristic algorithm in which there is no internal parameter to be determined throughout the optimization procedure. In other words, a notable feature of this algorithm is its parameter-free framework in which the exploitation and exploration phases of optimization are adjusted through the main loop of the algorithm. Besides, the position updating process of candidate solutions in this method is conducted in four separate phases in which the local and global searches of the entire search space are satisfied in a more precise way that results in excellent responses.

A summary of this paper is as follows. In section 2, the inspirational background of the proposed algorithm alongside the mathematical model of the new optimization algorithm is presented. In section 3, some mathematical functions with different characteristics are presented for further utilization in evaluating the proposed metaheuristic algorithm along with some other alternative approaches. In section 4, the selected alternative metaheuristic algorithms for comparative purposes are presented in detail. In section 5, the results of CryStAl alongside the other metaheuristics in dealing with mathematical test functions are presented. In section 6, a comprehensive statistical analysis is conducted to compare the results of the new algorithm with the other metaheuristic approaches. In section 7, the main findings of this paper including the conclusions alongside some suggestions for future challenges are presented accordingly.

## II. CRYSTAL STRUCTURE ALGORITHM (CryStAl)

### A. INSPIRATION

Solid minerals the constituent components (molecules, atoms, or ions) of which are regularly and repeatedly arranged in three spatial directions or have a crystallographic order are called crystals. Crystalline solids are highly diverse and can have isotropic or anisotropic properties. The word crystal has Greek roots and means "frozen by cold". They believed that if water was kept at very low temperatures for some time, it would become stable at high temperatures. "Crystal" is also an Arabic word derived from the Greek word "berlis" meaning emerald [39]. A representative example of a typical crystal is depicted in Fig. 1a.

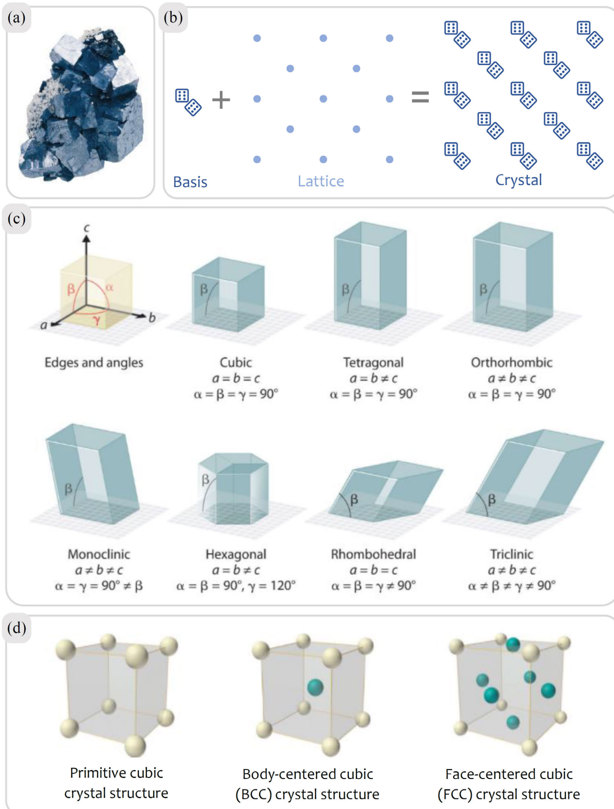


**FIGURE 1. (a) An example of a natural crystal called Galena. (b) Definition of a crystal as a basis added to a lattice. (c) Various lattice configuration options. (d) Three common varieties of the cubic crystal system (Parts a, c, and d are adapted from [39]).**

The earliest references to the regular arrangement of particles that make up crystals can be found in the works of Johannes Kepler in 1619 and Robert Hooke in 1665. Sometime later in 1690, Christine Hogens studied the optical properties of calcite crystals and hypothesized that the crystals were made of very small particles with a definite shape. Since then, different physical and chemical formulations for crystals have been proposed and investigated experimentally [39]. Furthermore, crystals and their rich symmetries have inspired the conception and design of many man-made structures, mechanisms, and artworks [40]–[80].

The underlying component of a crystal is a "lattice" which represents a periodic array of points in predefined spaces, though it is not capable of defining the specific locations of atoms in the material. On the other hand, the location of atoms in the structure of crystals is determined by the "basis" associated with each lattice point. Hence, crystals are determined by the combination of these two elements, i.e. the basis and the lattice, as illustrated in Fig 1b.

Since the lattice determines only the overall shape of the crystal, different geometrical shapes can be composed considering the fact that infinite geometrical shapes are found in nature; however, here we consider some of the most well-known regular shapes, as represented in Fig. 1c.

For the basis, different configurations of atoms in the lattice can be considered in which the location of atoms can be in the corner points alongside other irregular patterns. In Fig. 1d, this aspect is represented in a simple cubic crystal system.

As a mathematical representation of these aspects should be defined for numerical investigations, the Bravais model [39] is considered in this paper for defining crystal configurations. In this model, a periodic crystal structure is defined by considering infinite lattice shape in which any lattice pint is described by the location of their lattice point with a vector as follows:

$$r = \sum n_i a_i, \tag{1}$$

where $n_i$ is an integer, $a_i$ is the shortest vector along the principal crystallographic directions, and $i$ is the number of crystal corners.

### B. MATHEMATICAL MODEL

In this section, the mathematical model of CryStAl is presented in which the basic concepts of crystals are utilized with necessary modifications. In this model, each candidate solution of the optimization algorithm is considered as a single crystal in the space. For iterative purposes, a number of crystals are randomly determined for initialization.

$$Cr = \begin{bmatrix} Cr_1 \\ Cr_2 \\ \vdots \\ Cr_i \\ \vdots \\ Cr_n \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \cdots & x_1^j & \cdots & x_1^d \\ x_2^1 & x_2^2 & \cdots & x_2^j & \cdots & x_2^d \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_i^1 & x_i^2 & \cdots & x_i^j & \cdots & x_i^d \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^1 & x_n^2 & \cdots & x_n^j & \cdots & x_n^d \end{bmatrix}, \quad \begin{cases} i = 1, 2, \ldots, n \\ j = 1, 2, \ldots, d \end{cases} \tag{2}$$

where $n$ is the number of crystals (i.e., candidate solutions) and $d$ is the dimension of the problem. The initial positions of these crystals are randomly determined in the search space as follows:

$$x_i^j(0) = x_{i,\min}^j + \xi(x_{i,\max}^j - x_{i,\min}^j), \quad \begin{cases} i = 1, 2, \ldots, n \\ j = 1, 2, \ldots, d \end{cases} \tag{3}$$

where $x_i^j(0)$ determines the initial position of the crystals; $x_{i,\min}^j$ and $x_{i,\max}^j$ are the minimum and maximum allowable values, respectively, for the $j^{\text{th}}$ decision variable of the $i^{\text{th}}$ candidate solution; and $\xi$ is a random number in the interval [0,1].

Based on the concept of 'basis' in crystallography, all the crystals at the corners are considered as the *main crystals*, $Cr_{main}$, determined randomly by considering the initially-created crystals (candidate solutions). It should be noted that the random selection process for each step is determined by omitting the current $Cr$. The crystal with the *best* configuration is determined as $Cr_b$ while the mean values of randomly-selected crystals are denoted by $F_c$.

To update the positions of the candidate solutions in the search space, basic lattice principles are considered in which four kinds of updating process are determined as follows:

**(i) Simple cubicle:**

$$Cr_{new} = Cr_{old} + rCr_{main}, \tag{4}$$

**(ii) Cubicle with the best crystals:**

$$Cr_{new} = Cr_{old} + r_1 Cr_{main} + r_2 Cr_b, \tag{5}$$

**(iii) Cubicle with the mean crystals:**

$$Cr_{new} = Cr_{old} + r_1 Cr_{main} + r_2 F_c, \tag{6}$$

**(iv) Cubicle with the best and mean crystals:**

$$Cr_{new} = Cr_{old} + r_1 Cr_{main} + r_2 Cr_b + r_3 F_c, \tag{7}$$

where, in the four equations above, $Cr_{new}$ is the new position, $Cr_{old}$ is the old position, and $r$, $r_1$, $r_2$ and $r_3$ are random numbers.

It should be mentioned that exploration and exploitation, as two critical features of metaheuristics, have been considered in this algorithm through (4) to (7) in which local and global searches are conducted simultaneously. In order to deal with the solution variables $x_i^j$ violating the boundary conditions of the variables, a mathematical flag is defined in which for the $x_i^j$ outside the variables range, the flag orders a boundary change for the violating variables. The terminating criterion is considered based on the maximum number of iterations in which the optimization process is terminated after a fixed number of iterations. The pseudo-code of the algorithm is presented in Fig. 2.

## III. MATHEMATICAL TEST FUNCTIONS

In this section, a number of mathematical functions are selected to be utilized as test functions for the performance evaluation of the proposed algorithm. A total number of 239 mathematical functions are tested which are categorized into four different groups based on their specific

```
procedure Crystal Structure Algorithm (CryStAl)
        Create random values for initial positions (x_i^j) of initial
crystals (Cr_i)
        Evaluate fitness values for each crystal
        while (t < maximum number of iterations)
            for i=1: number of initial crystals
                Create Cr_main
                Create new crystals by Eq. 4
                Create Cr_b
                Create new crystals by Eq. 5
                Create F_c
                Create new crystals by Eq. 6
                Create new crystals by Eq. 7
                    if new crystals violate boundary conditions
                        Control the position constraints for new crystals and
amend them
                    end if
                Evaluate the fitness values for new crystals
                Update Global Best (GB) if a better solution is found
            end for
            t = t+1
        end while
        Return GB
end procedure
```

**FIGURE 2.** The pseudo-code of the Crystal Structure Algorithm (CryStAl).

characteristics. These functions have been derived from various references [41]–[45] in which different mathematical functions with different characteristics had been reviewed and presented for utilization in the validation of novel metaheuristic algorithms.

In the first group, 117 mathematical functions are presented which have minimum and maximum dimensions of 2 and 10, respectively. Among these functions, which are named $F_1$ to $F_{117}$, the first 90 functions have 2 dimensions whereas the other 27 functions have dimensions of 3 to 10. In this paper, these functions are called the 'two-dimensional (2D)' test functions and are presented in Table 3. The second group of mathematical functions consists of 58 test functions in which the dimensions of functions are variable due to their specific formulations and are called the '$N$-dimensional ($N$D)' test functions. A maximum number of dimensions of 50 is considered in dealing with the functions of this group, called the 50-dimensional (50D) test functions, which are named $F_{118}$ to $F_{175}$ and presented in Table 4. For the third group, the mathematical functions of the second group are considered with the maximum dimension of 100 and are called the 100-dimensional (100D) test functions; these functions, named $F_{175}$ to $F_{233}$, are presented in Table 5. For the fourth group, three composite and three hybrid mathematical functions are considered which are named $F_{233}$ to $F_{239}$, presented in Table 6. In these tables, C, NC, D, ND, S, NS, Sc, NSc, U, and M denote Continuous, Non-Continuous, Differentiable, Non-Differentiable, Separable, Non-Separable, Scalable, Non-Scalable, Unimodal, and Multi-modal, respectively. Furthermore, $R$, $D$, and $Min$. represent the variables range, variables dimension, and the global minimum of the functions.

Based on the fact that a larger number of mathematical functions (239 functions) are considered in this paper, the 3D plots for some of these functions are presented in the following. The 3D plots for some of the 2D functions are shown in Fig. 3, while those of the 50D and 100D functions are depicted in Figs. 4 and 5, respectively. The complete mathematical formulations of these test functions are presented in Refs. [81]–[85].

## IV. ALTERNATIVE METAHEURISTICS FOR COMPARISON

In order to evaluate the overall performance of the proposed algorithm, some different optimization algorithms are utilized as alternative approaches to provide a valid comparative study. The utilized metaheuristics for this purpose are the ABC, ACO, BA, FA, GA, HS, MFO, MVO, PSO, SA, SCA, and SSA. Based on the fact that some of the selected optimization algorithms are recently proposed or developed for special purposes, the most recent and improved versions of these algorithms are used in this paper. Knowing that the internal parameters of the optimization algorithms have the most vital role in their convergence performance, a parameter summary of the selected algorithms is presented in Table 7. The values of these parameters have been determined using the reference-based parameter identification process in which the internal parameters of these algorithms are selected based on relevant previously published research papers.

In many metaheuristic algorithms, some specific parameters are utilized for tuning the exploration and exploitation rates during the optimization process which are often problem-dependent parameters and so they should be tuned for each specific optimization problem. The mentioned parameters for the alternative algorithms in Table 7 were derived from the latest and most successful configurations of these algorithms available in the literature which resulted in acceptable optimum results in most of the previously considered optimization problems.

Knowing that such algorithms are potentially vulnerable to entrapment in local optima or even having convergence problems, we have proposed CryStAl as a simple algorithm without any internal or external parameters to be tuned. This characteristic can be considered as the major advantage of this algorithm over competing algorithms. In fact, as mentioned earlier in this section, CryStAl considers exploration and exploitation through (4) to (7) where local and global searches are performed simultaneously.

## V. NUMERICAL RESULTS

In this section, the obtained results of the optimization run for CryStAl alongside the alternative metaheuristic approaches in dealing with the mathematical test functions are presented. The optimization problem is formulated with the maximum population size taken as 50 and the maximum number of Function Evaluations (FEs) selected to be 150000 for all of the metaheuristics. The maximum number of iterations in

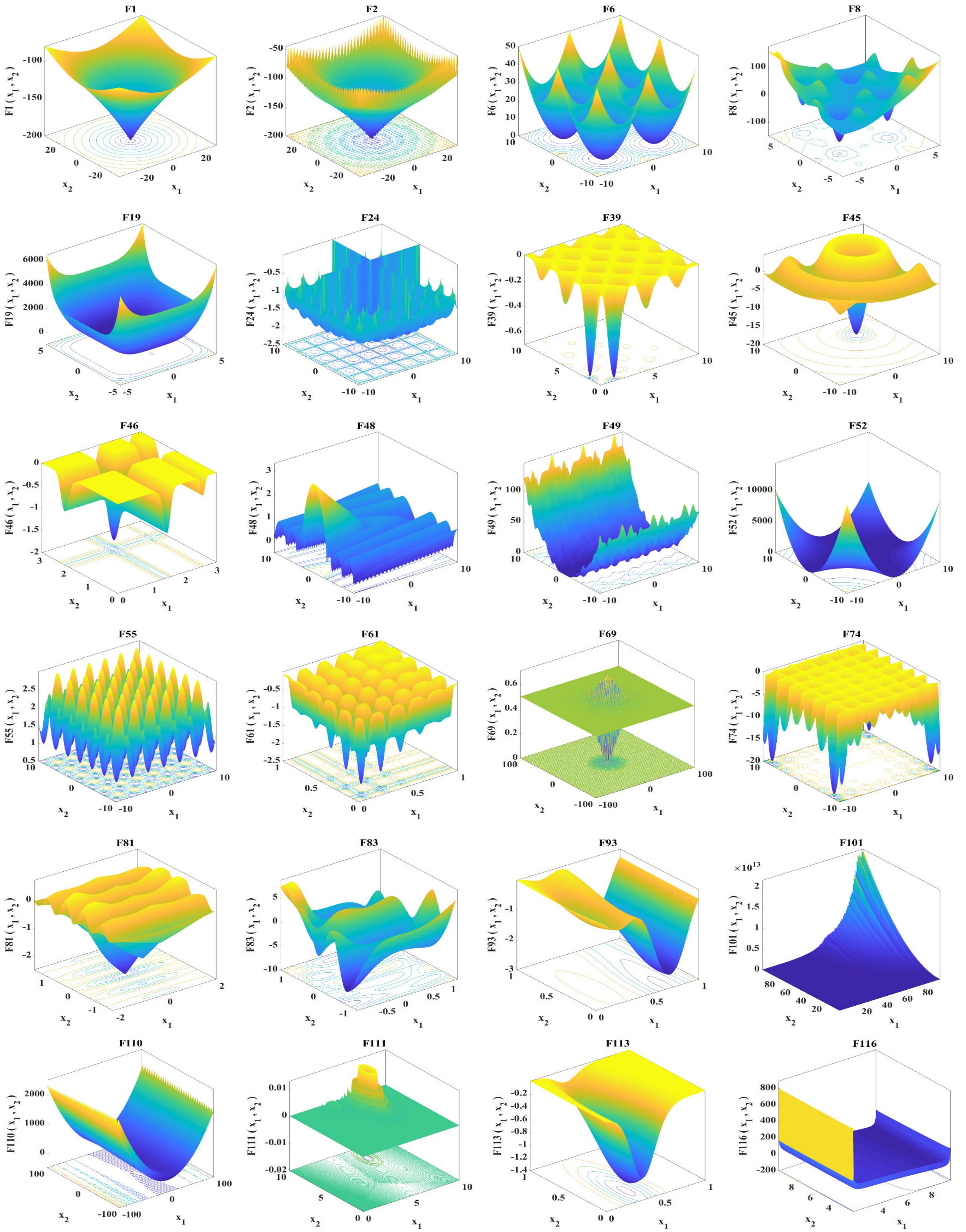


**FIGURE 3.** The 3D plots of the 2D mathematical functions.

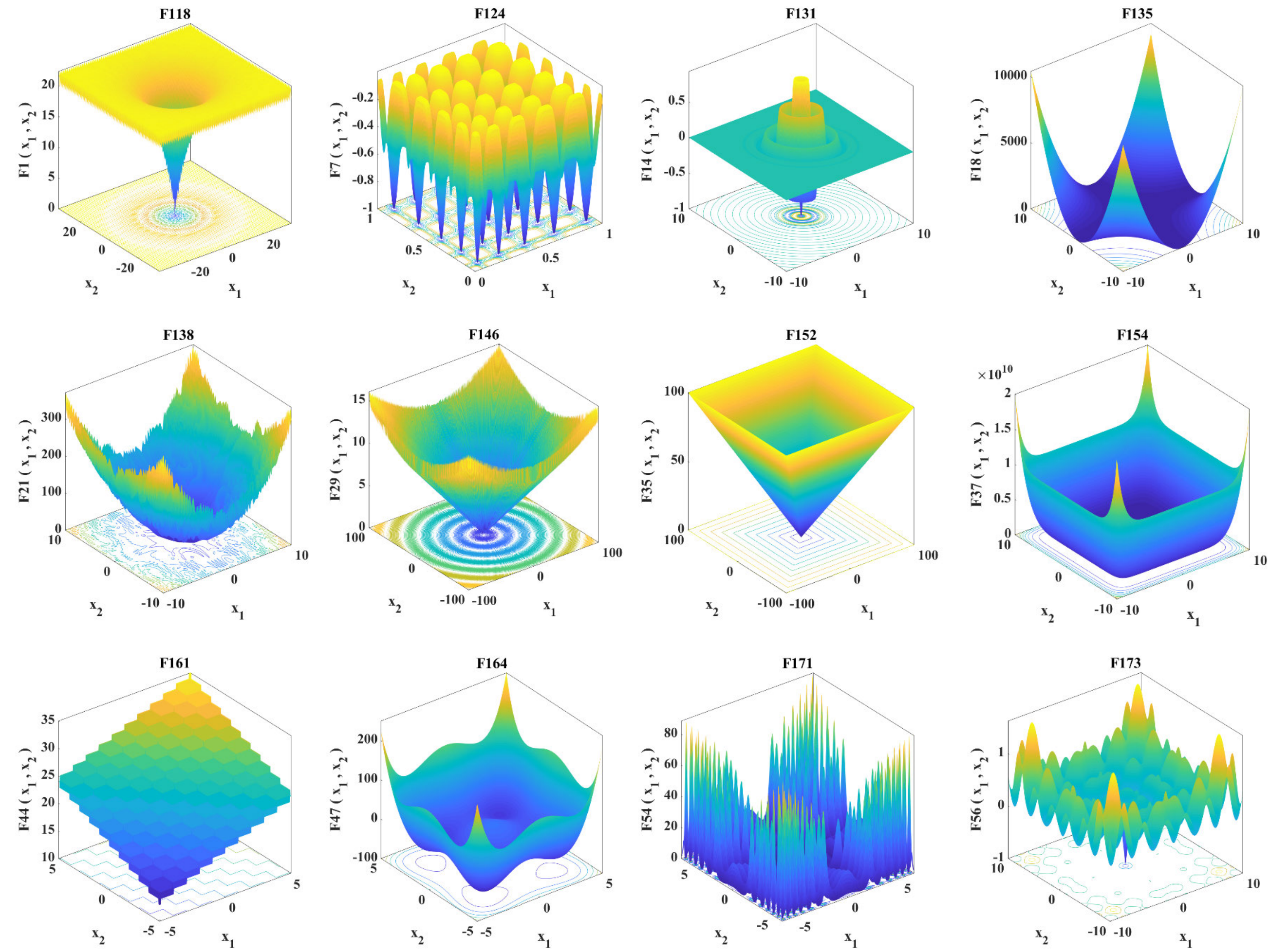


**FIGURE 4. The 3D plots of the 50D mathematical functions.**

each algorithm is adjusted based on the selected maximum number of FEs. As collecting quantitative results are of great importance in dealing with different optimization problems, CryStAl and the other algorithms are utilized 100 times with different initializations and the mean and standard deviation (std) of the best approximated solutions in the last iteration are reported. A tolerance of $1 \times 10^{-12}$ is also considered for the convergence results of the algorithms in which the optimization runs are stopped at this tolerance of the Global Best (GB). It is assumed that the GB results are achieved by these optimization runs within this tolerance and the results of the GB are utilized instead of the final results of the optimization runs. The number of FEs are also calculated based on the selected tolerance. It should be noted that the above-mentioned is utilized as the stopping criterion in order to save time from a computational complexity perspective. In other words, if the algorithm reaches to this tolerance of the global best for the considered problem, the global best is reported as the final solution of the algorithm which requires less computational time. Therefore, the computational time for the considered 100 optimization runs will be reasonable. Besides, the initial random state of each optimization run for each alternative algorithm has been selected equally in order to form a fair judgment about the performance of the proposed and alternative algorithms.

The detailed results of CryStAl and the other selected methods are presented in the Supplementary Materials which includes the convergence history of the proposed algorithm. It turned out that CryStAl can find the exact global results of 156 functions (65%); moreover, its result is very close to the global best result for 83 problems. Further investigations into the results of CryStAl compared to those of the other methods are performed in the next sections using some advanced statistical approaches. Moreover, the convergence curves of the proposed algorithm in dealing with some of the considered mathematical test functions are provided in the Supplementary Materials.

## VI. STATISTICAL ANALYSIS

In this section, the maximum error values of the optimization convergence data have been calculated and utilized for statistical analysis. To this end, the difference between the Global

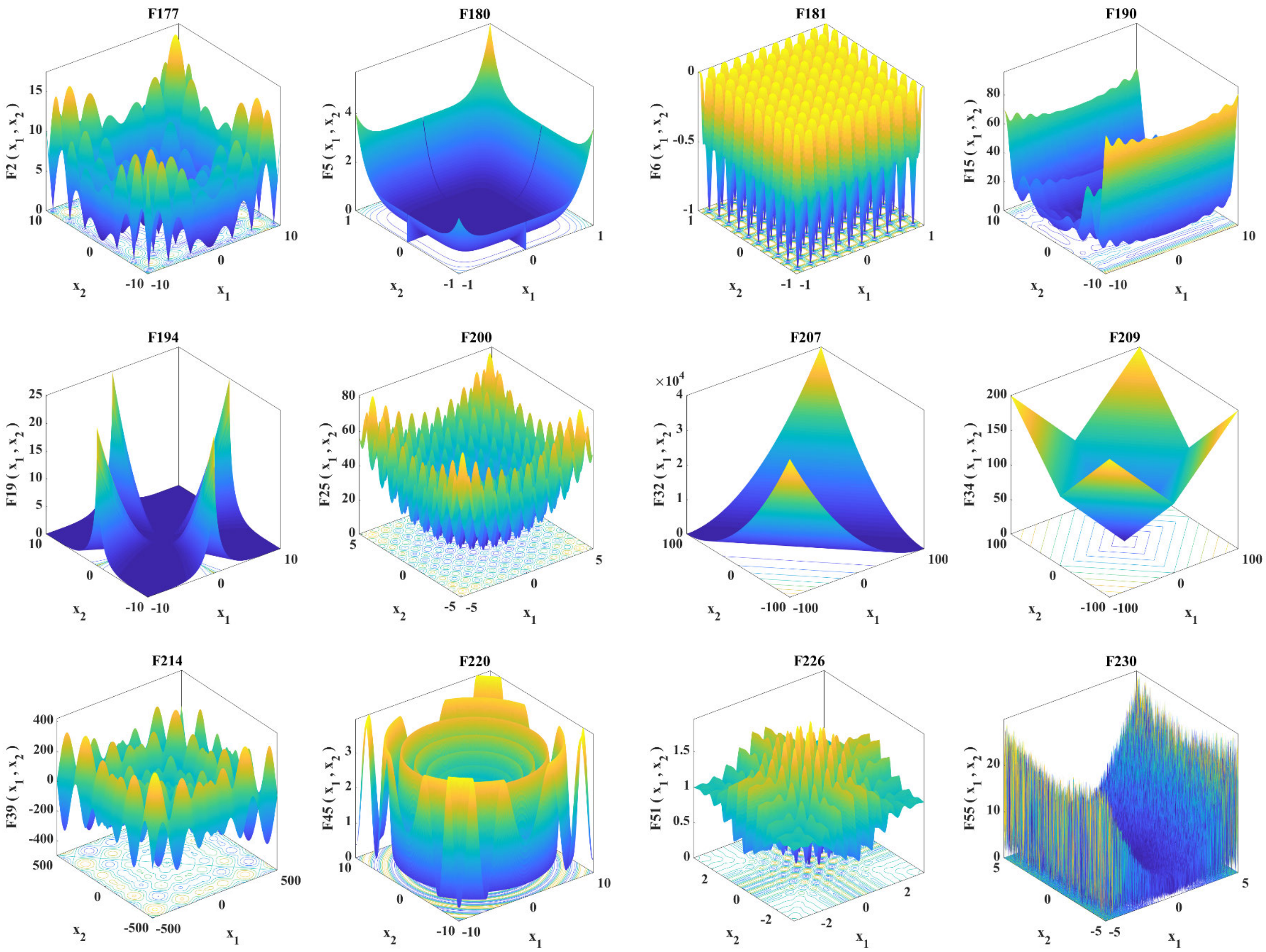


**FIGURE 5.** The 3D plots of the 100D mathematical functions.

Best (GB) of the functions and the obtained optimal values resulted from the optimization runs are considered as the error values. For statistical analysis purposes, four statistical tests have been conducted in which the Kolmogorov-Smirnov (K-S) test is utilized for normality issues, the Mann-Whitney U (M-W) test is implemented for comparing the summation of the ranks of different metaheuristics in a two-by-two comparing manner, the Kruskal-Wallis (K-W) test is conducted for comparing the overall rankings of the metaheuristics by considering the mean of their rankings, and the Post-Hoc (P-H) analysis is conducted based on the results of the K-W tests for further investigations.

### *A. KOLMOGOROV-SMIRNOV TEST*

There are two kinds of statistical tests which are applicable to all of the obtained statistical data from multiple applications, known as the parametric and non-parametric statistical tests. One of the most important criteria which demonstrates the possibility of utilizing each method in a specific situation is the Kolmogorov-Smirnov test. This test shows that the distribution of data is either normal or non-normal in which the distribution of each sample among the statistical data are considered and checked accordingly. If the K-S test is rejected, the data are normally distributed, and there is the possibility of using parametric statistical tests for the research. Conversely, if the K-S test is confirmed, the data do not have a normal distribution, so the nonparametric tests should be used in the study.

The results of the K-S test for the error values of the minimum, mean, standard deviation, and maximum function evaluations of the optimization runs for the 2D, 50D, and 100D functions are presented in Table 8. This test is conducted as a two-sample test in which the distributions of the CryStAl data are compared with the data obtained from other metaheuristics. It should be noted that if the Asymptotic Significance (Asymp. Sig.) value is less than 0.05, the presented data are not distributed normally, so the non-parametric statistical tests should be conducted for further investigations. The obtained results of the K-S test demonstrate that the Asymp. Sig. values in most of the investigated cases are less than 0.05, so the non-parametric statistical tests should be utilized for further considerations.

**TABLE 3.** Details of the 2D to 10D mathematical functions (First group).

| No. | Name | Type | R | D | Min. |
|---|---|---|---|---|---|
| $F_{1}$ | Ackley 2 | C, D, NS, Sc,M | [-35, 35] | 2 | -200 |
| $F_{2}$ | Ackley 3 | C, D, NS, NSc,U | [-32, 32] | 2 | -195.629 |
| $F_{3}$ | Adjiman | C, D, NS, NSc, M | [-1, 2] & [-1, 1] | 2 | -2.02181 |
| $F_{4}$ | Bartels Conn | C, ND, NS, NSc, M | [-500, 500] | 2 | 1 |
| $F_{5}$ | Beale | C, D, NS, NSc, U | [-4.5, 4.5] | 2 | 0 |
| $F_{6}$ | Becker-Lago | S | [-10, 10] | 2 | 0 |
| $F_{7}$ | Biggs EXP2 | C, D, NS, NSc, M | [0, 20] | 2 | 0 |
| $F_{8}$ | Bird | C, D, NS, NSc, M | [-2π, π] | 2 | -106.765 |
| $F_{9}$ | Bohachevsky 1 | C, D, S, NSc, M | [-100, 100] | 2 | 0 |
| $F_{10}$ | Bohachevsky 2 | C, D, NS, NSc, M | [-100, 100] | 2 | 0 |
| $F_{11}$ | Bohachevsky 3 | C, D, NS, NSc, M | [-100, 100] | 2 | 0 |
| $F_{12}$ | Booth | C, D, NS, NSc, U | [-10, 10] | 2 | 0 |
| $F_{13}$ | Branin RCOS | C, D, NS, NSc M | [-5, 10] & [0, 15] | 2 | 0.397887 |
| $F_{14}$ | Branin RCOS 2 | C, D, NS, NSc, M | [-5, 15] | 2 | 5.559037 |
| $F_{15}$ | Brent | C, D, NS, NSc, U | [-10, 10] | 2 | 0 |
| $F_{16}$ | Bukin 4 | C, ND, S, NSc, M | [-15, -5] & [-3, 3] | 2 | 0 |
| $F_{17}$ | Bukin 6 | C, ND, NS, NSc, M | [-15, -5] & [-3, 3] | 2 | 0 |
| $F_{18}$ | Camel - 3 Hump | C, D, NS, NSc, M | [-5, 5] | 2 | 0 |
| $F_{19}$ | Camel - 6 Hump | C, D, NS, NSc, M | [-5, 5] | 2 | -1.0316 |
| $F_{20}$ | Carrom table | NS | [-10, 10] | 2 | -24.1568 |
| $F_{21}$ | Chen Bird | C, D, NS, NSc, M | [-500, 500] | 2 | -2000 |
| $F_{22}$ | Chen V | C, D, NS, NSc, M | [-500, 500] | 2 | -2000 |
| $F_{23}$ | Chichinadze | C, D, S, NSc, M | [-30, 30] | 2 | -42.9444 |
| $F_{24}$ | Cross-in-Tray | C, NS, NSc, M | [-10, 10] | 2 | -2.06261 |
| $F_{25}$ | Cube | C, D, NS, NSc, U | [-10, 10] | 2 | 0 |
| $F_{26}$ | Damavandi | C, D, NS, NSc, M | [0, 14] | 2 | 0 |
| $F_{27}$ | Deckkers-Aarts | C, D, NS, NSc, M | [-20, 20] | 2 | -24771.1 |
| $F_{28}$ | Easom | C, D, S, NSc, M | [-100, 100] | 2 | -1 |
| $F_{29}$ | El-Attar-Vidyasagar-Dutta | C, D, NS, NSc, M | [-500, 500] | 2 | 1.7128 |
| $F_{30}$ | Egg Crate | C, D, NS, Sc, M | [-5, 5] | 2 | 0 |
| $F_{31}$ | Exp 2 | S | [0, 20] | 2 | 0 |
| $F_{32}$ | Freudenstein Roth | C, D, NS, NSc, M | [-10, 10] | 2 | 0 |
| $F_{33}$ | Giunta | C, D, S, Sc, M | [-1, 1] | 2 | 0.060447 |
| $F_{34}$ | Goldstein Price | C, D, NS, NSc, M | [-2, 2] | 2 | 3 |
| $F_{35}$ | Hansen | C, D, S, NSc, M | [-10, 10] | 2 | -165.953 |
| $F_{36}$ | Himmelblau | C, D, NS, NSc, M | [-5, 5] | 2 | 0 |
| $F_{37}$ | Hosaki | C, D, NS, NSc, M | [0, 5] & [0, 6] | 2 | -2.3458 |
| $F_{38}$ | Jennrich-Sampson | C, D, NS, NSc, M | [-1, 1] | 2 | 124.3612 |
| $F_{39}$ | Keane | C, D, NS, NSc, M | [0, 10] | 2 | -0.67367 |
| $F_{40}$ | Leon | C, D, NS, NSc, U | [-1.2, 1.2] | 2 | 0 |
| $F_{41}$ | Levy 3 | S | [-10, 10] | 2 | -176.542 |
| $F_{42}$ | Levy 5 | NS | [-10, 10] | 2 | -176.138 |
| $F_{43}$ | Matyas | C, D, NS, NSc, U | [-10, 10] | 2 | 0 |
| $F_{44}$ | McCormick | C, D, NS, NSc, M | [-1.5, 4] & [-3, 3] | 2 | -1.9133 |
| $F_{45}$ | Mexican hat | NS | [-10, 10] | 2 | -19.6683 |
| $F_{46}$ | Michaelewicz 2 | S | [0, π] | 2 | -1.8013 |
| $F_{47}$ | Mishra 3 | C, D, NS, NSc, M | [-10, 10] | 2 | -0.18465 |
| $F_{48}$ | Mishra 4 | C, D, NS, NSc, M | [-10, 10] | 2 | -0.19941 |
| $F_{49}$ | Mishra 5 | C, D, NS, NSc, M | [-10, 10] | 2 | -1.01983 |
| $F_{50}$ | Mishra 6 | C, D, NS, NSc, M | [-10, 10] | 2 | -2.28395 |
| $F_{51}$ | Mishra 8 | C, D, NS, NSc, M | [-10, 10] | 2 | 0 |
| $F_{52}$ | Mishra 10 | C, D, NS, NSc, M | [-10, 10] | 2 | 0 |
| $F_{53}$ | Parsopoulos | C, D, S, Sc, M | [-5, 5] | 2 | 0 |
| $F_{54}$ | Pen Holder | C, D, NS, NSc, M | [-11, 11] | 2 | -0.96354 |
| $F_{55}$ | Periodic | S | [-10, 10] | 2 | 0.9 |
| $F_{56}$ | Price 1 | C, ND, S, NSc, M | [-500, 500] | 2 | 0 |
| $F_{57}$ | Price 2 | C, D, NS, NSc, M | [-10, 10] | 2 | 0.9 |
| $F_{58}$ | Price 3 | C, D, NS, NSc, M | [-500, 500] | 2 | 0 |
| $F_{59}$ | Price 4 | C, D, NS, NSc, M | [-500, 500] | 2 | 0 |
| $F_{60}$ | Quadratic | C, D, NS, NSc | [-10, 10] | 2 | -3873.72 |
| $F_{61}$ | Ripple 1 | NS | [0, 1] | 2 | -2.2 |
| $F_{62}$ | Ripple 25 | NS | [0, 1] | 2 | -2 |
| $F_{63}$ | Rosenbrock Modified | C, D, NS, NSc, M | [-2, 2] | 2 | 34.3712 |
| $F_{64}$ | Rotated Ellipse | C, D, NS, NSc, U | [-500, 500] | 2 | 0 |
| $F_{65}$ | Rotated Ellipse 2 | C, D, NS, NSc, U | [-500, 500] | 2 | 0 |
| $F_{66}$ | Rump | C, D, NS, NSc, U | [-500, 500] | 2 | 0 |
| $F_{67}$ | Scahffer 1 | C, D, NS, NSc, U | [-100, 100] | 2 | 0 |
| $F_{68}$ | Scahffer 2 | C, D, NS, NSc, U | [-100, 100] | 2 | 0 |
| $F_{69}$ | Scahffer 3 | C, D, NS, NSc, U | [-100, 100] | 2 | 0.001567 |
| $F_{70}$ | Scahffer 4 | C, D, NS, NSc, U | [-100, 100] | 2 | 0.292579 |
| $F_{71}$ | Schwefel 2.6 | C, D, NS, NSc, U | [-100, 100] | 2 | 0 |
| $F_{72}$ | Schwefel 2.36 | C, D, S, Sc, M | [0, 500] | 2 | -3456 |
| $F_{73}$ | Table 1 / Holder Table 1 | C, D, S, NSc, M | [-10, 10] | 2 | -26.9203 |
| $F_{74}$ | Table 2 / Holder Table 2 | C, D, S, NSc, M | [-10, 10] | 2 | -19.2085 |
| $F_{75}$ | Table 3 / Carrom Table | C, D, NS, NSc, M | [-10, 10] | 2 | -24.1568 |
| $F_{76}$ | Testtube Holder | C, D, S, NSc, M | [-10, 10] | 2 | -10.8723 |
| $F_{77}$ | Trecanni | C, D, S, NSc, U | [-5, 5] | 2 | 0 |
| $F_{78}$ | Trefethen | C, D, NS, NSc, M | [-10, 10] | 2 | -3.30687 |
| $F_{79}$ | Tripod | C, D, NS, NSc, M | [-100, 100] | 2 | 0 |
| $F_{80}$ | Ursem 1 | S | [-2.5, 3] & [-2, 2] | 2 | -4.81681 |
| $F_{81}$ | Ursem 3 | NS | [-2, 2] & [-1.5, 1.5] | 2 | -2.5 |
| $F_{82}$ | Ursem 4 | NS | [-2, 2] | 2 | -1.5 |
| $F_{83}$ | Ursem Waves | NS | [-0.9, 1.2] & [-1.2, 1.2] | 2 | -7.307 |
| $F_{84}$ | Venter Sobiezcczanski-Sobieski | C, D, S, NSc | [-50, 50] | 2 | -400 |
| $F_{85}$ | Wayburn Seader 1 | C, D, NS, Sc, U | [-500, 500] | 2 | 0 |
| $F_{86}$ | Wayburn Seader 2 | C, D, NS, Sc, U | [-500, 500] | 2 | 0 |
| $F_{87}$ | Wayburn Seader 3 | C, D, NS, Sc, U | [-500, 500] | 2 | 21.35 |
| $F_{88}$ | Zettl | C, D, NS, NSc, U | [-5, 10] | 2 | -0.00379 |
| $F_{89}$ | Zirilli or Aluffi-Pentini | C, D, S, NSc, U | [-10, 10] | 2 | -0.3523 |
| $F_{90}$ | Zirilli 2 | C, D, S, S, M | [-500, 500] | 2 | 0 |
| $F_{91}$ | Biggs EXP3 | C, D, NS, NSc, M | [0, 20] | 3 | 0 |
| $F_{92}$ | Gulf Research Problem | C, D, NS, NSc, M | [0.1, 100] & [0, 25.6] & [0, 6.5] | 3 | 0 |
| $F_{93}$ | Hartman 3 | C, D, NS, NSc, M | [0, 1] | 3 | -3.86278 |
| $F_{94}$ | Helical Valley | C, D, NS, Sc, M | [-10, 10] | 3 | 0 |
| $F_{95}$ | Meyer-Roth | NS | [0, 1] | 3 | 4.00E-05 |
| $F_{96}$ | Mishra 9 | C, D, NS, NSc, M | [-10, 10] | 3 | 0 |
| $F_{97}$ | Wolfe | C, D, S, Sc, M | [0, 2] | 3 | 0 |
| $F_{98}$ | Biggs EXP4 | C, D, NS, NSc, M | [0, 20] | 4 | 0 |
| $F_{99}$ | Colville | C, D, NS, NSc, M | [-10, 10] | 4 | 0 |
| $F_{100}$ | Corana | DC, ND, S, Sc, M | [-500, 500] | 4 | 0 |
| $F_{101}$ | DeVilliers Glasser 1 | C, D, NS, NSc, M | [1, 100] | 4 | 0 |
| $F_{102}$ | Gear | NS | [12, 60] | 4 | 2.70E-12 |
| $F_{103}$ | Kowalik | NS | [-5, 5] | 4 | 0.000308 |
| $F_{104}$ | Miele Cantrell | C, D, NS, NSc, M | [-1, 1] | 4 | 0 |
| $F_{105}$ | Shekel 5 | C, D, NS, Sc, M | [0, 10] | 4 | -10.1532 |
| $F_{106}$ | Shekel 7 | C, D, NS, Sc, M | [0, 10] | 4 | -10.4029 |
| $F_{107}$ | Shekel 10 | C, D, NS, Sc, M | [0, 10] | 4 | -10.5364 |
| $F_{108}$ | Biggs EXP5 | C, D, NS, NSc, M | [0, 20] | 5 | 0 |
| $F_{109}$ | DeVilliers Glasser 2 | C, D, NS, NSc, M | [1, 60] | 5 | 0 |
| $F_{110}$ | Dolan | C, D, NS, NSc, M | [-100, 100] | 5 | -529.871 |
| $F_{111}$ | Langerman-5 | C, D, NS, Sc, M | [0, 10] | 5 | -0.965 |
| $F_{112}$ | Biggs EXP6 | C, D, NS, NSc, M | [-20, 20] | 6 | 0 |
| $F_{113}$ | Hartman 6 | C, D, NS, NSc, M | [0, 1] | 6 | -3.32236 |
| $F_{114}$ | Trid 6 | C, D, NS, NSc, M | [-36, 36] | 6 | -50 |
| $F_{115}$ | Ann-XOR | NS | [-1, 1] | 9 | 0.95979 |
| $F_{116}$ | Paviani | C, D, NS, Sc, M | [2.0001, 10] | 10 | -45.778 |
| $F_{117}$ | Trid 10 | C, D, NS, NSc, M | [-100,100] | 10 | -210 |

In Table 9, the maximum difference between the statistical data of CryStAl and the other metaheuristics are also presented in order to have an initial judgment about the obtained results of the new algorithm. The maximum and minimum differences of CryStAl with the alternative algorithms are represented by bold font-weight and underlined font, respectively. The bolded values designate those algorithms which have the maximum difference with CryStAl among other metaheuristics, while the underlined values show the algorithms which have the minimum difference with CryStAl among other metaheuristics.

## B. MANN-WHITNEY U TEST

The Mann-Whitney U (M-W) test is a non-parametric test that allows two groups of data to be compared in which the null hypothesis denotes that it is equally likely that a randomly-selected value from one sample will be less than or greater than a randomly-selected value from a second sample. This test can be used to investigate whether two independent samples were selected from populations having the same distribution. This test provides the summation of the ranks for two sets of statistical data considered for comparative analysis. As an essential criterion, if the summation of the

**TABLE 4. Details of the 50D mathematical functions (Second group).**

| No. | Name | Type | R | D | Min. |
|---|---|---|---|---|---|
| $F_{118}$ | Ackley 1 | C, D, NS, Sc,M | [-35, 35] | 50 | 0 |
| $F_{119}$ | Alpine 1 | C, ND, S, NSc,U | [-10, 10] | 50 | 0 |
| $F_{120}$ | Brown | C, D, NS, Sc, U | [-1, 4] | 50 | 0 |
| $F_{121}$ | Chung Reynolds | C, D, PS, Sc, U | [-100, 100] | 50 | 0 |
| $F_{122}$ | Csendes | C, D, S, Sc, M | [-1, 1] | 50 | 0 |
| $F_{123}$ | Deb 1 | C, D, S, Sc, M | [-1, 1] | 50 | -1 |
| $F_{124}$ | Deb 3 | C, D, S, Sc, M | [0, 1] | 50 | -1 |
| $F_{125}$ | Dixon & Price | C, D, NS, Sc, U | [-10, 10] | 50 | 0 |
| $F_{126}$ | Extended Easom | C, D, S, NSc, M | [-2π, 2π] | 50 | -1 |
| $F_{127}$ | Exponential | C, D, NS, Sc, M | [-1, 1] | 50 | -1 |
| $F_{128}$ | Griewank | C, D, NS, Sc, M | [-100,100] | 50 | 0 |
| $F_{129}$ | Holzman 2 | S | [-10, 10] | 50 | 0 |
| $F_{130}$ | Hyper-ellipsoid | C, U | [-500, 500] | 50 | 0 |
| $F_{131}$ | Inverted cosine wave | NS | [-10, 10] | 50 | -49 |
| $F_{132}$ | Levy 8 | NS | [-10, 10] | 50 | 0 |
| $F_{133}$ | Mishra 1 | C, D, NS, Sc, M | [0, 1] | 50 | 2 |
| $F_{134}$ | Mishra 2 | C, D, NS, Sc, M | [0, 1] | 50 | 2 |
| $F_{135}$ | Mishra 7 | C, D, NS, NSc, M | [-10, 10] | 50 | 0 |
| $F_{136}$ | Mishra 11 | C, D, NS, NSc, M | [-10, 10] | 50 | 0 |
| $F_{137}$ | Pathological | C, D, NS, NSc, M | [-100, 100] | 50 | 0 |
| $F_{138}$ | Pint´er | C, D, NS, Sc, M | [-10, 10] | 50 | 0 |
| $F_{139}$ | Powell Singular | C, D, NS, Sc, U | [-4, 5] | 50 | 0 |
| $F_{140}$ | Powell Singular 2 | C, D, NS, Sc, U | [-4, 5] | 50 | 0 |
| $F_{141}$ | Powell Sum | C, D, S, Sc, U | [-1, 1] | 50 | 0 |
| $F_{142}$ | Rastrigin | C, D, S, M | [-5.12, 5.12] | 50 | 0 |
| $F_{143}$ | Qing | C, D, S, Sc, M | [-500, 500] | 50 | 0 |
| $F_{144}$ | Quintic | C, D, S, NSc, M | [-10, 10] | 50 | 0 |
| $F_{145}$ | Rosenbrock | C, D, NS, Sc, U | [-30, 30] | 50 | 0 |
| $F_{146}$ | Salomon | C, D, NS, Sc, M | [-100, 100] | 50 | 0 |
| $F_{147}$ | Schumer Steiglitz | C, D, S, Sc, U | [-100, 100] | 50 | 0 |
| $F_{148}$ | Schwefel | C, D, PS, Sc, U | [-100, 100] | 50 | 0 |
| $F_{149}$ | Schwefel 1.2 | C, D, NS, Sc, U | [-100, 100] | 50 | 0 |
| $F_{150}$ | Schwefel 2.4 | C, D, S, NSc, M | [0, 10] | 50 | 0 |
| $F_{151}$ | Schwefel 2.20 | C, ND, S, Sc, U | [-100, 100] | 50 | 0 |
| $F_{152}$ | Schwefel 2.21 | C, ND, S, Sc, U | [-100, 100] | 50 | 0 |
| $F_{153}$ | Schwefel 2.22 | C, D, NS, Sc, U | [-100, 100] | 50 | 0 |
| $F_{154}$ | Schwefel 2.23 | C, D, NS, Sc, U | [-10, 10] | 50 | 0 |
| $F_{155}$ | Schwefel 2.25 | C, D, S, NSc, M | [0, 10] | 50 | 0 |
| $F_{156}$ | Schwefel 2.26 | C, D, S, Sc, M | [-500, 500] | 50 | -418.98 |
| $F_{157}$ | Sphere | C, D, S, Sc, M | [0, 10] | 50 | 0 |
| $F_{158}$ | Step | DC, ND, S, Sc, U | [-100, 100] | 50 | 0 |
| $F_{159}$ | Step 2 | DC, ND, S, Sc, U | [-100, 100] | 50 | 0 |
| $F_{160}$ | Step 3 | DC, ND, S, Sc, U | [-100, 100] | 50 | 0 |
| $F_{161}$ | Stepint | DC, ND, S, Sc, U | [-5.12, 5.12] | 50 | -275 |
| $F_{162}$ | Stretched V Sine Wave | C, D, NS, Sc, U | [-10, 10] | 50 | 0 |
| $F_{163}$ | Sum Squares | C, D, S, Sc, U | [-10, 10] | 50 | 0 |
| $F_{164}$ | Styblinski-Tang | C, D, NS, NSc, M | [-5, 5] | 50 | -1958.3 |
| $F_{165}$ | Trid | C, D, NS, NSc, U | [-D^2, D^2] | 50 | -22050 |
| $F_{166}$ | Trigonometric 1 | C, D, NS, Sc, M | [0, π] | 50 | 0 |
| $F_{167}$ | Trigonometric 2 | C, D, NS, Sc, M | [-500, 500] | 50 | 1 |
| $F_{168}$ | W / Wavy | C, D, S, Sc, M | [-π, π] | 50 | 0 |
| $F_{169}$ | Xin-She Yang (1) | DC, ND, NS, Sc, M | [-20, 20] | 50 | -1 |
| $F_{170}$ | Xin-She Yang (2) | DC, ND, NS, Sc, M | [-10, 10] | 50 | 0 |
| $F_{171}$ | Xin-She Yang (3) | DC, ND, NS, Sc, M | [-2π, 2π] | 50 | 0 |
| $F_{172}$ | Xin-She Yang (4) | DC, ND, NS, Sc, M | [-5, 5] | 50 | 0 |
| $F_{173}$ | Xin-She Yang (5) | DC, ND, NS, Sc, M | [-10, 10] | 50 | -1 |
| $F_{174}$ | Xin-She Yang (6) | DC, ND, NS, Sc, M | [-5, 5] | 50 | 0 |
| $F_{175}$ | Zakharov | C, D, NS, Sc, M | [-5, 10] | 50 | 0 |

**TABLE 5. Details of the 100D mathematical functions (Third group).**

| No. | Name | Type | R | D | Min. |
|---|---|---|---|---|---|
| $F_{176}$ | Ackley 1 | C, D, NS, Sc,M | [-35, 35] | 100 | 0 |
| $F_{177}$ | Alpine 1 | C, ND, S, NSc,U | [-10, 10] | 100 | 0 |
| $F_{178}$ | Brown | C, D, NS, Sc, U | [-1, 4] | 100 | 0 |
| $F_{179}$ | Chung Reynolds | C, D, PS, Sc, U | [-100, 100] | 100 | 0 |
| $F_{180}$ | Csendes | C, D, S, Sc, M | [-1, 1] | 100 | 0 |
| $F_{181}$ | Deb 1 | C, D, S, Sc, M | [-1, 1] | 100 | -1 |
| $F_{182}$ | Deb 3 | C, D, S, Sc, M | [0, 1] | 100 | -1 |
| $F_{183}$ | Dixon & Price | C, D, NS, Sc, U | [-10, 10] | 100 | 0 |
| $F_{184}$ | Extended Easom | C, D, S, NSc, M | [-2π, 2π] | 100 | -1 |
| $F_{185}$ | Exponential | C, D, NS, Sc, M | [-1, 1] | 100 | -1 |
| $F_{186}$ | Griewank | C, D, NS, Sc, M | [-100,100] | 100 | 0 |
| $F_{187}$ | Holzman 2 | S | [-10, 10] | 100 | 0 |
| $F_{188}$ | Hyper-ellipsoid | C, U | [-500, 500] | 100 | 0 |
| $F_{189}$ | Inverted cosine wave | NS | [-10, 10] | 100 | -99 |
| $F_{190}$ | Levy 8 | NS | [-10, 10] | 100 | 0 |
| $F_{191}$ | Mishra 1 | C, D, NS, Sc, M | [0, 1] | 100 | 2 |
| $F_{192}$ | Mishra 2 | C, D, NS, Sc, M | [0, 1] | 100 | 2 |
| $F_{193}$ | Mishra 7 | C, D, NS, NSc, M | [-10, 10] | 100 | 0 |
| $F_{194}$ | Mishra 11 | C, D, NS, NSc, M | [-10, 10] | 100 | 0 |
| $F_{195}$ | Pathological | C, D, NS, NSc, M | [-100, 100] | 100 | 0 |
| $F_{196}$ | Pint´er | C, D, NS, Sc, M | [-10, 10] | 100 | 0 |
| $F_{197}$ | Powell Singular | C, D, NS, Sc, U | [-4, 5] | 100 | 0 |
| $F_{198}$ | Powell Singular 2 | C, D, NS, Sc, U | [-4, 5] | 100 | 0 |
| $F_{199}$ | Powell Sum | C, D, S, Sc, U | [-1, 1] | 100 | 0 |
| $F_{200}$ | Rastrigin | C, D, S, M | [-5.12, 5.12] | 100 | 0 |
| $F_{201}$ | Qing | C, D, S, Sc, M | [-500, 500] | 100 | 0 |
| $F_{202}$ | Quintic | C, D, S, NSc, M | [-10, 10] | 100 | 0 |
| $F_{203}$ | Rosenbrock | C, D, NS, Sc, U | [-30, 30] | 100 | 0 |
| $F_{204}$ | Salomon | C, D, NS, Sc, M | [-100, 100] | 100 | 0 |
| $F_{205}$ | Schumer Steiglitz | C, D, S, Sc, U | [-100, 100] | 100 | 0 |
| $F_{206}$ | Schwefel | C, D, PS, Sc, U | [-100, 100] | 100 | 0 |
| $F_{207}$ | Schwefel 1.2 | C, D, NS, Sc, U | [-100, 100] | 100 | 0 |
| $F_{208}$ | Schwefel 2.4 | C, D, S, NSc, M | [0, 10] | 100 | 0 |
| $F_{209}$ | Schwefel 2.20 | C, ND, S, Sc, U | [-100, 100] | 100 | 0 |
| $F_{210}$ | Schwefel 2.21 | C, ND, S, Sc, U | [-100, 100] | 100 | 0 |
| $F_{211}$ | Schwefel 2.22 | C, D, NS, Sc, U | [-100, 100] | 100 | 0 |
| $F_{212}$ | Schwefel 2.23 | C, D, NS, Sc, U | [-10, 10] | 100 | 0 |
| $F_{213}$ | Schwefel 2.25 | C, D, S, NSc, M | [0, 10] | 100 | 0 |
| $F_{214}$ | Schwefel 2.26 | C, D, S, Sc, M | [-500, 500] | 100 | -418.98 |
| $F_{215}$ | Sphere | C, D, S, Sc, M | [0, 10] | 100 | 0 |
| $F_{216}$ | Step | DC, ND, S, Sc, U | [-100, 100] | 100 | 0 |
| $F_{217}$ | Step 2 | DC, ND, S, Sc, U | [-100, 100] | 100 | 0 |
| $F_{218}$ | Step 3 | DC, ND, S, Sc, U | [-100, 100] | 100 | 0 |
| $F_{219}$ | Stepint | DC, ND, S, Sc, U | [-5.12, 5.12] | 100 | -575 |
| $F_{220}$ | Stretched V Sine Wave | C, D, NS, Sc, U | [-10, 10] | 100 | 0 |
| $F_{221}$ | Sum Squares | C, D, S, Sc, U | [-10, 10] | 100 | 0 |
| $F_{222}$ | Styblinski-Tang | C, D, NS, NSc, M | [-5, 5] | 100 | -3916.6 |
| $F_{223}$ | Trid | C, D, NS, NSc, U | [-D^2, D^2] | 100 | -171600 |
| $F_{224}$ | Trigonometric 1 | C, D, NS, Sc, M | [0, π] | 100 | 0 |
| $F_{225}$ | Trigonometric 2 | C, D, NS, Sc, M | [-500, 500] | 100 | 1 |
| $F_{226}$ | W / Wavy | C, D, S, Sc, M | [-π, π] | 100 | 0 |
| $F_{227}$ | Xin-She Yang (1) | DC, ND, NS, Sc, M | [-20, 20] | 100 | -1 |
| $F_{228}$ | Xin-She Yang (2) | DC, ND, NS, Sc, M | [-10, 10] | 100 | 0 |
| $F_{229}$ | Xin-She Yang (3) | DC, ND, NS, Sc, M | [-2π, 2π] | 100 | 0 |
| $F_{230}$ | Xin-She Yang (4) | DC, ND, NS, Sc, M | [-5, 5] | 100 | 0 |
| $F_{231}$ | Xin-She Yang (5) | DC, ND, NS, Sc, M | [-10, 10] | 100 | -1 |
| $F_{232}$ | Xin-She Yang (6) | DC, ND, NS, Sc, M | [-5, 5] | 100 | 0 |
| $F_{233}$ | Zakharov | C, D, NS, Sc, M | [-5, 10] | 100 | 0 |

ranks for one sample has lower values than the other one, the one with a smaller sum of ranks has better statistical results and the utilized metaheuristic is superior to the other one. The results of the M-W test for different mathematical functions based on the obtained results of the optimization runs are presented in Tables 10 to 12. In these tables, the upper and lower values are the summation of the ranks related to the alternative metaheuristics and CryStAl, respectively. Based on the statistical results, the related values of CryStAl for the summation of the ranks in most cases are lower than those of the other metaheuristics (bolded values in the table) which demonstrates the superiority of CryStAl to its competitors in dealing with optimization functions.

### C. KRUSKAL-WALLIS TEST

The Kruskal-Wallis (K-W) test is a non-parametric method for testing whether or not different statistical samples are originated from the same distribution. It is used for comparing two or more independent samples of equal or different sample sizes. It extends the Mann-Whitney U test, which is used for comparing only two groups. A significant K-W test indicates that at least one sample stochastically dominates another sample. This test provides the mean of the ranks for multiple sets of statistical data which are considered for comparative analysis. As an important criterion, if the mean of the ranks for one sample has lower values than the other ones, the one

**TABLE 6.** Details of the composite and hybrid mathematical functions (Fourth group).

| No. | Descriptions | R | D | Min. |
|---|---|---|---|---|
| $F_{234}$ | Basic Functions: Sphere Function<br>$f_1, f_2, f_3, \ldots, f_{10}$ = Sphere Function<br>$[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$<br>$[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [5/100, 5/100, 5/100, \ldots, 5/100]$ | [-5, 5] | 10 | 0 |
| $F_{235}$ | Basic Functions: Griewank Function<br>$f_1, f_2, f_3, \ldots, f_{10}$ = Griewank Function<br>$[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$<br>$[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [5/100, 5/100, 5/100, \ldots, 5/100]$ | [-5, 5] | 10 | 0 |
| $F_{236}$ | Basic Functions: Griewank Function<br>$f_1, f_2, f_3, \ldots, f_{10}$ = Griewank Function<br>$[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$<br>$[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [1, 1, 1, \ldots, 1]$ | [-5, 5] | 10 | 0 |
| $F_{237}$ | Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions<br>$f_1, f_2$ = Ackley Function<br>$f_3, f_4$ = Rastrigin Function<br>$f_5, f_6$ = Weierstrass Function<br>$f_7, f_8$ = Griewank Function<br>$f_9, f_{10}$ = Sphere Function<br>$[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$<br>$[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [5/32, 5.32, 1, 1, 5/0.5, 5/0.5, 5/100, 5/100, 5/100, 5/100]$ | [-5, 5] | 10 | 0 |
| $F_{238}$ | Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions<br>$f_1, f_2$ = Rastrigin Function<br>$f_3, f_4$ = Weierstrass Function<br>$f_5, f_6$ = Griewank Function<br>$f_7, f_8$ = Ackley Function<br>$f_9, f_{10}$ = Sphere Function<br>$[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$<br>$[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [1/5, 1/5, 5/0.5, 5/0.5, 5/100, 5/100, 5/32, 5/32, 5/100, 5/100]$ | [-5, 5] | 10 | 0 |
| $F_{239}$ | Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions<br>$f_1, f_2$ = Rastrigin Function<br>$f_3, f_4$ = Weierstrass Function<br>$f_5, f_6$ = Griewank Function<br>$f_7, f_8$ = Ackley Function<br>$f_9, f_{10}$ = Sphere Function<br>$[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]$<br>$[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [0.1\times1/5, 0.2\times1/5, 0.3\times5/0.5, 0.4\times5/0.5, 0.5\times5/100, 0.6\times5/100, 0.7\times5/32, 0.8\times5/32, 0.9\times5/100, 1\times5/100]$ | [-5, 5] | 10 | 0 |

**TABLE 7.** Parameter summary of the alternative metaheuristic algorithms.

| Metaheuristic | Parameter | Description | Value |
|---|---|---|---|
| ABC | $N_{pop}$ | Colony Size | 50 |
| | $N_o$ | Number of Onlooker Bees | 50 |
| | $L$ | Abandonment Limit Parameter | 60 |
| | $a$ | Acceleration Coefficient Upper Bound | 1 |
| ACO | $N_{pop}$ | Archive Size | 50 |
| | $N_s$ | Sample Size | 50 |
| | $q$ | Intensification Factor | 0.5 |
| | $\zeta$ | Deviation-Distance Ratio | 1 |
| BA | $N_{pop}$ | Number of Scout Bees | 50 |
| | $N_{ss}$ | Number of Selected Sites | 25 |
| | $N_{se}$ | Number of Selected Elite Sites | 10 |
| | $N_{rs}$ | Number of Recruited Bees for Selected Sites | 25 |
| | $N_{re}$ | Number of Recruited Bees for Elite Sites | 50 |
| | $r$ | Neighborhood Radius | 0.1 |
| | $r_{damp}$ | Neighborhood Radius Damp Rate | 0.95 |
| FA | $N_{pop}$ | Number of Fireflies (Swarm Size) | 50 |
| | $\gamma$ | Light Absorption Coefficient | 1 |
| | $\beta$ | Attraction Coefficient Base Value | 2 |
| | $\alpha$ | Mutation Coefficient | 0.2 |
| | $\alpha_{damp}$ | Mutation Coefficient Damping Ratio | 0.98 |
| | $\delta$ | Uniform Mutation Range | ±0.05 |
| GA | $p_c$ | Crossover Percentage | 0.8 |
| | $p_m$ | Mutation Percentage | 0.3 |
| | $\mu$ | Mutation Rate | 0.02 |
| | $\beta$ | Roulette wheel selection pressure | 1 |
| HS | $HMS$ | Harmony Memory Size | 50 |
| | $N_{new}$ | Number of New Harmonies | 20 |
| | $HMCR$ | Harmony Memory Consideration Rate | 0.9 |
| | $PAR$ | Pitch Adjustment Rate | 0.1 |
| | $FW$ | Fret Width (Bandwidth) | ±0.02 |
| | $FW_{damp}$ | Fret Width Damp Ratio | 0.995 |
| PSO | $N_{pop}$ | Swarm Size | 50 |
| | $w$ | Inertia Weight | 1 |
| | $w_d$ | Inertia Weight Damping Ratio | 0.99 |
| | $c_1$ | Personal Learning Coefficient | 2 |
| | $c_2$ | Global Learning Coefficient | 2 |
| SA | $N_{pop}$ | Population Size | 50 |
| | $M_{subit}$ | Maximum Number of Sub-iterations | 15 |
| | $T_0$ | Initial Temperature | 0.025 |
| | $\alpha$ | Temperature Reduction Rate | 0.99 |
| | $N_m$ | Number of Neighbors per Individual | 5 |
| | $\mu$ | Mutation Rate | 0.5 |
| | $\sigma$ | Mutation Range (Standard Deviation) | 0.1 |

with a smaller mean of ranks has better statistical results and the utilized metaheuristic is superior to the other one. The results of the K-W test for different studied functions based on the obtained results of the optimization runs have been presented in Tables 13 to 15. Based on the results, the CryStAl related values for the mean of the ranks in most of the cases are lower than the related values for the other metaheuristics which represents the superiority of CryStAl. In these tables, the bolded values are related to the metaheuristic which is superior to the other ones while the values related to CryStAl are all underlined.

### D. POST-HOC ANALYSIS

Post-hoc is a Latin phrase, meaning ''after this'' or ''after the event''. In a scientific study, a Post-Hoc (P-H) analysis consists of statistical analyses that were not specified before the data was seen. A P-H analysis involves looking at the data after a study has been concluded, and trying to find patterns that were not the primary objectives of the study.

In this section, the P-H analysis is conducted in order to derive the overall rankings of the metaheuristic algorithms for all of the 2D, 50D, and 100D functions based on the achieved results of the K-W test. The overall rankings of the metaheuristics obtained by the P-H analysis are presented in Table 16. It should be noted that CryStAl provides a success estimation of 100 percent in outranking the other metaheuristics, which demonstrates the superiority of this proposed novel optimization algorithm.

**TABLE 8.** The K-S test results (Asymp. Sig.) for different algorithms.

| *Main Algorithm* | *Function Type* | *Data Type* | *Alternative Metaheuristic Algorithms* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ***ABC*** | ***ACO*** | ***BA*** | ***FA*** | ***GA*** | ***HS*** | ***MFO*** | ***MVO*** | ***PSO*** | ***SA*** | ***SCA*** | ***SSA*** |
| **CryStAl** | **2D** | **Min.** | 6.92E-07 | 8.66E-01 | 8.09E-11 | 1.79E-21 | 2.07E-01 | 3.51E-01 | 9.97E-01 | 2.35E-13 | 1E+00 | 3.61E-16 | 4.92E-10 | 2.72E-01 |
| | | **Mean** | 1.30E-04 | 8.66E-01 | 1.11E-15 | 1.15E-16 | 2.91E-14 | 5.04E-03 | 8.19E-02 | 4.73E-12 | 4.04E-02 | 8.35E-14 | 1.56E-07 | 7.92E-03 |
| | | **Std.** | 2.01E-10 | 2.07E-01 | 6.59E-21 | 5.04E-25 | 4.80E-22 | 6.95E-04 | 1.14E-01 | 8.43E-20 | 4.04E-02 | 1.26E-22 | 8.35E-14 | 2.09E-05 |
| | | **Fun. Evl.** | 6.47E-09 | 1.14E-01 | 9.99E-15 | 4.80E-22 | 4.73E-12 | 8.35E-14 | 2.07E-01 | 1.79E-21 | 2.76E-02 | 1.00E-18 | 3.20E-11 | 1.79E-21 |
| | **50D** | **Min.** | 2.48E-14 | 1.09E-15 | 4.17E-17 | 3.23E-11 | 1.40E-03 | 1.12E-13 | 6.67E-04 | 4.69E-07 | 1.05E-02 | 2.17E-16 | 1.40E-03 | 3.06E-04 |
| | | **Mean** | 9.37E-06 | 4.69E-07 | 1.69E-08 | 6.67E-04 | 6.07E-01 | 3.06E-04 | 5.76E-05 | 3.24E-01 | 7.65E-01 | 9.37E-06 | 2.22E-01 | 3.24E-01 |
| | | **Std.** | 3.06E-04 | 3.06E-04 | 1.32E-06 | 1.46E-01 | 9.99E-01 | 9.30E-02 | 6.67E-04 | 7.65E-01 | 7.65E-01 | 1.05E-02 | 4.54E-01 | 6.07E-01 |
| | | **Fun. Evl.** | 1.40E-03 | 6.67E-04 | 1.35E-04 | 1.35E-04 | 2.22E-01 | 1.35E-04 | 8.99E-01 | 6.67E-04 | 7.65E-01 | 5.76E-05 | 1.46E-01 | 3.37E-02 |
| | **100D** | **Min.** | 2.04E-12 | 4.87E-13 | 2.48E-14 | 1.21E-10 | 5.76E-05 | 8.27E-12 | 5.31E-08 | 1.61E-07 | 1.40E-03 | 1.12E-13 | 4.69E-07 | 3.06E-04 |
| | | **Mean** | 3.58E-06 | 3.58E-06 | 1.61E-07 | 6.67E-04 | 6.07E-01 | 1.35E-04 | 2.37E-05 | 2.22E-01 | 3.24E-01 | 5.76E-05 | 5.58E-03 | 8.99E-01 |
| | | **Std.** | 3.06E-04 | 3.06E-04 | 5.76E-05 | 3.37E-02 | 9.99E-01 | 5.58E-03 | 1.35E-04 | 8.99E-01 | 6.07E-01 | 2.85E-03 | 1.05E-02 | 6.07E-01 |
| | | **Fun. Evl.** | 1.92E-02 | 1.05E-02 | 6.67E-04 | 6.67E-04 | 5.70E-02 | 6.67E-04 | 9.30E-02 | 6.67E-04 | 3.24E-01 | 6.67E-04 | 5.58E-03 | 5.70E-02 |

**TABLE 9.** The K-S test results (the overall difference between data) for different algorithms.

| *Main Algorithm* | *Function Type* | *Data Type* | *Alternative Metaheuristic Algorithms* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ***ABC*** | ***ACO*** | ***BA*** | ***FA*** | ***GA*** | ***HS*** | ***MFO*** | ***MVO*** | ***PSO*** | ***SA*** | ***SCA*** | ***SSA*** |
| **CryStAl** | **2D** | **Min.** | 0.3504 | 0.0769 | 0.4444 | **0.6325** | 0.1368 | 0.1197 | 0.0513 | 0.4957 | 0.0427 | 0.5470 | 0.4274 | 0.1282 |
| | | **Mean** | 0.2821 | 0.0769 | 0.5385 | **0.5556** | 0.5128 | 0.2222 | 0.1624 | 0.4701 | 0.1795 | 0.5043 | 0.3675 | 0.2137 |
| | | **Std.** | 0.4359 | 0.1368 | 0.6239 | **0.6838** | 0.6410 | 0.2564 | 0.1538 | 0.6068 | 0.1795 | 0.6496 | 0.5043 | 0.3077 |
| | | **Fun. Evl.** | 0.4017 | 0.1538 | 0.5214 | **0.6410** | 0.4701 | 0.5043 | 0.1368 | 0.6325 | 0.1880 | 0.5897 | 0.4530 | 0.6325 |
| | **50D** | **Min.** | 0.7241 | 0.7586 | **0.7931** | 0.6379 | 0.3448 | 0.7069 | 0.3621 | 0.5000 | 0.2931 | 0.7759 | 0.3448 | 0.3793 |
| | | **Mean** | 0.4483 | 0.5000 | **0.5517** | 0.3621 | 0.1379 | 0.3793 | 0.4138 | 0.1724 | 0.1207 | 0.4483 | 0.1897 | 0.1724 |
| | | **Std.** | 0.3793 | 0.3793 | **0.4828** | 0.2069 | 0.0690 | 0.2241 | 0.3621 | 0.1207 | 0.1207 | 0.2931 | 0.1552 | 0.1379 |
| | | **Fun. Evl.** | 0.3448 | 0.3621 | 0.3966 | 0.3966 | 0.1897 | 0.3966 | 0.1034 | 0.3621 | 0.1207 | **0.4138** | 0.2069 | 0.2586 |
| | **100D** | **Min.** | 0.6724 | 0.6897 | **0.7241** | 0.6207 | 0.4138 | 0.6552 | 0.5345 | 0.5172 | 0.3448 | 0.7069 | 0.5000 | 0.3793 |
| | | **Mean** | 0.4655 | 0.4655 | **0.5172** | 0.3621 | 0.1379 | 0.3966 | 0.4310 | 0.1897 | 0.1724 | 0.4138 | 0.3103 | 0.1034 |
| | | **Std.** | 0.3793 | 0.3793 | **0.4138** | 0.2586 | 0.0690 | 0.3103 | 0.3966 | 0.1034 | 0.1379 | 0.3276 | 0.2931 | 0.1379 |
| | | **Fun. Evl.** | 0.2759 | 0.2931 | 0.3621 | 0.3621 | 0.2414 | 0.3621 | 0.2241 | **0.3621** | 0.1724 | 0.3621 | 0.3103 | 0.2414 |

**TABLE 10.** The M-W test results (summation of the ranks) for 2D mathematical functions.

| *Main Algorithm* | *Function Type* | *Data Type* | *Alternative Metaheuristic Algorithms* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ***ABC*** | ***ACO*** | ***BA*** | ***FA*** | ***GA*** | ***HS*** | ***MFO*** | ***MVO*** | ***PSO*** | ***SA*** | ***SCA*** | ***SSA*** |
| **CryStAl** | **2D** | **Min.** | 16014.00 | 14047.50 | 16845.50 | 18294.50 | 14779.00 | 14616.50 | 14146.50 | 16896.50 | 14064.00 | 17406.50 | 16912.50 | 14595.00 |
| | | | **11481.00** | **13447.50** | **10649.50** | **9200.50** | **12716.00** | **12878.50** | **13348.50** | **10598.50** | **13431.00** | **10088.50** | **10582.50** | **12900.00** |
| | | **Mean** | 15181.00 | 14235.50 | 17494.00 | 17703.00 | 17137.00 | 15344.00 | 14945.50 | 16680.00 | 14752.00 | 16954.00 | 16323.00 | 14893.00 |
| | | | **12314.00** | **13259.50** | **10001.00** | **9792.00** | **10358.00** | **12151.00** | **12549.50** | **10815.00** | **12743.00** | **10541.00** | **11172.00** | **12602.00** |
| | | **Std.** | 16016.50 | 14528.00 | 17779.00 | 18090.50 | 17774.00 | 15501.00 | 14778.50 | 17393.00 | 14751.00 | 17583.50 | 16729.00 | 15552.50 |
| | | | **11478.50** | **12967.00** | **9716.00** | **9404.50** | **9721.00** | **11994.00** | **12716.50** | **10102.00** | **12744.00** | **9911.50** | **10766.00** | **11942.50** |
| | | **Fun. Evl.** | 15657.00 | **13306.00** | 17180.00 | 18550.00 | 16372.00 | 16576.00 | 14609.00 | 18323.00 | 14689.00 | 18335.00 | 17293.00 | 17633.00 |
| | | | **11838.00** | 14189.00 | **10315.00** | **8945.00** | **11123.00** | **10919.00** | **12886.00** | **9172.00** | **12806.00** | **9160.00** | **10202.00** | **9862.00** |

**TABLE 11.** The M-W test results (summation of the ranks) for 50D mathematical functions.

| *Main Algorithm* | *Function Type* | *Data Type* | *Alternative Metaheuristic Algorithms* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ***ABC*** | ***ACO*** | ***BA*** | ***FA*** | ***GA*** | ***HS*** | ***MFO*** | ***MVO*** | ***PSO*** | ***SA*** | ***SCA*** | ***SSA*** |
| **CryStAl** | **50D** | **Min.** | 4570.00 | 4632.00 | 4753.00 | 4563.00 | 3995.00 | 4610.00 | 3765.00 | 4308.00 | 3625.00 | 4702.00 | 3870.00 | 4003.00 |
| | | | **2216.00** | **2154.00** | **2033.00** | **2223.00** | **2791.00** | **2176.00** | **3021.00** | **2478.00** | **3161.00** | **2084.00** | **2916.00** | **2783.00** |
| | | **Mean** | 4210.00 | 4292.50 | 4468.50 | 4089.50 | 3545.00 | 4060.00 | 4080.50 | 3681.00 | **3390.50** | 4268.00 | 3710.00 | 3539.00 |
| | | | **2576.00** | **2493.50** | **2317.50** | **2696.50** | **3241.00** | **2726.00** | **2705.50** | **3105.00** | 3395.50 | **2518.00** | **3076.00** | **3247.00** |
| | | **Std.** | 3943.00 | 4002.00 | 4209.00 | 3795.00 | **3384.50** | 3760.50 | 3953.00 | 3530.50 | **3289.00** | 3938.00 | 3562.50 | **3387.50** |
| | | | **2843.00** | **2784.00** | **2577.00** | **2991.00** | 3401.50 | **3025.50** | **2833.00** | **3255.50** | 3497.00 | **2848.00** | **3223.50** | 3398.50 |
| | | **Fun. Evl.** | 3937.00 | 3968.00 | 4069.00 | 4063.00 | 3697.00 | 4048.00 | 3536.00 | 3983.00 | **3355.00** | 4079.00 | 3748.00 | 3816.00 |
| | | | **2849.00** | **2818.00** | **2717.00** | **2723.00** | **3089.00** | **2738.00** | **3250.00** | **2803.00** | 3431.00 | **2707.00** | **3038.00** | **2970.00** |

## VII. CEC 2017 COMPETITION RESULTS

In order to evaluate the overall performance of the proposed algorithm, CryStAl, it is necessary to consider state-of-the-art mathematical test functions alongside state-of-the-art algorithms. To this end, a recent competition on single-objective real-parameter numerical optimization named "CEC 2017" [86] is considered in this section. In this regard, a list of 30 mathematical functions are studied and

**TABLE 12.** The M-W test results (summation of the ranks) for 100D mathematical functions.

| *Main Algorithm* | *Function Type* | *Data Type* | *Alternative Metaheuristic Algorithms* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | *ABC* | *ACO* | *BA* | *FA* | *GA* | *HS* | *MFO* | *MVO* | *PSO* | *SA* | *SCA* | *SSA* |
| CryStAl | 100D | Min. | 4516.00 | 4555.00 | 4712.00 | 4571.00 | 4113.00 | 4610.00 | 4251.00 | 4381.00 | 3943.00 | 4650.00 | 4303.00 | 4047.00 |
| | | | **2270.00** | **2231.00** | **2074.00** | **2215.00** | **2673.00** | **2176.00** | **2535.00** | **2405.00** | **2843.00** | **2136.00** | **2483.00** | **2739.00** |
| | | Mean | 4163.00 | 4193.00 | 4360.00 | 4069.00 | 3517.00 | 4145.00 | 4171.00 | 3688.00 | 3587.00 | 4208.00 | 3987.00 | 3469.00 |
| | | | **2623.00** | **2593.00** | **2426.00** | **2717.00** | **3269.00** | **2641.00** | **2615.00** | **3098.00** | **3199.00** | **2578.00** | **2799.00** | **3317.00** |
| | | Std. | 3889.00 | 3918.00 | 4071.00 | 3756.00 | **3347.00** | 3865.00 | 4042.00 | 3485.00 | 3461.00 | 3916.00 | 3824.00 | **3325.00** |
| | | | **2897.00** | **2868.00** | **2715.00** | **3030.00** | 3439.00 | **2921.00** | **2744.00** | **3301.00** | **3325.00** | **2870.00** | **2962.00** | 3461.00 |
| | | Fun. Evl. | 3825.50 | 3858.00 | 3995.00 | 3995.00 | 3766.50 | 3989.00 | 3739.00 | 3989.00 | 3655.50 | 3997.00 | 3888.50 | 3775.50 |
| | | | **2960.50** | **2928.00** | **2791.00** | **2791.00** | **3019.50** | **2797.00** | **3047.00** | **2797.00** | **3130.50** | **2789.00** | **2897.50** | **3010.50** |

**TABLE 13.** The K-W test results (mean of the ranks) for 2D mathematical functions.

| *Ranking* | 2D | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | *Min.* | | *Mean* | | *Std.* | | *Fun. Evl.* | |
| | *Algorithms* | *Mean of Ranks* | *Algorithms* | *Mean of Ranks* | *Algorithms* | *Mean of Ranks* | *Algorithms* | *Mean of Ranks* |
| 1 | ***CryStAl*** | **558.8803** | ***CryStAl*** | **533.0342** | ***CryStAl*** | **491.7137** | ***CryStAl*** | **474.2222** |
| 2 | *PSO* | 595.5940 | *ACO* | 592.4744 | *ACO* | 580.1197 | *ACO* | 484.2607 |
| 3 | *ACO* | 601.7350 | *SSA* | 631.2607 | *SSA* | 622.9487 | *PSO* | 545.1838 |
| 4 | *MFO* | 608.3205 | *ABC* | 649.4615 | *MFO* | 629.0043 | *MFO* | 555.2906 |
| 5 | *SSA* | 648.7650 | *PSO* | 668.5684 | *PSO* | 630.4573 | *HS* | 698.1624 |
| 6 | *HS* | 659.7521 | *MFO* | 687.2735 | *ABC* | 685.3034 | *GA* | 700.3162 |
| 7 | *GA* | 682.7180 | *HS* | 727.0556 | *HS* | 701.8889 | *ABC* | 768.0342 |
| 8 | *ABC* | 793.7863 | *SCA* | 829.2863 | *SCA* | 843.2094 | *SSA* | 817.4274 |
| 9 | *MVO* | 875.8120 | *MVO* | 830.9573 | *MVO* | 860.1026 | *SCA* | 884.8205 |
| 10 | *BA* | 905.4060 | *SA* | 870.3333 | *SA* | 899.4744 | *BA* | 903.0769 |
| 11 | *SCA* | 927.8162 | *GA* | 897.7094 | *GA* | 931.0171 | *MVO* | 1000.7521 |
| 12 | *SA* | 950.4103 | *FA* | 983.9444 | *FA* | 992.2222 | *SA* | 1004.3590 |
| 13 | *FA* | 1084.0043 | *BA* | 991.6410 | *BA* | 1025.5385 | *FA* | 1057.0940 |
| ***Chi-sq.*** | 253.8161 | | 168.4093 | | 225.9612 | | 332.7130 | |
| ***Prob>Chi-sq.*** | 2.1883E-47 | | 1.0096E-29 | | 1.3755E-41 | | 6.1870E-64 | |

**TABLE 14.** The K-W test results (mean of the ranks) for 50D mathematical functions.

| *Ranking* | 50D | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | *Min.* | | *Mean* | | *Std.* | | *Fun. Evl.* | |
| | *Algorithms* | *Mean of Ranks* | *Algorithms* | *Mean of Ranks* | *Algorithms* | *Mean of Ranks* | *Algorithms* | *Mean of Ranks* |
| 1 | ***CryStAl*** | **193.3621** | ***CryStAl*** | **263.3879** | ***PSO*** | **290.0776** | ***PSO*** | **280.4397** |
| 2 | *PSO* | 244.8793 | *PSO* | 269.4224 | *CryStAl* | 307.8707 | *CryStAl* | 281.2414 |
| 3 | *MFO* | 297.6034 | *GA* | 296.2586 | *GA* | 308.6638 | *MFO* | 313.6552 |
| 4 | *SCA* | 299.5000 | *SSA* | 298.9310 | *SSA* | 312.3190 | *GA* | 351.0603 |
| 5 | *GA* | 300.1638 | *MVO* | 321.7931 | *MVO* | 337.1121 | *SCA* | 359.1983 |
| 6 | *SSA* | 309.4655 | *SCA* | 332.7845 | *SCA* | 348.0776 | *SSA* | 377.9138 |
| 7 | *MVO* | 348.3276 | *HS* | 389.2241 | *HS* | 380.3190 | *ABC* | 404.4828 |
| 8 | *HS* | 429.8621 | *FA* | 413.8190 | *FA* | 396.2500 | *MVO* | 406.4828 |
| 9 | *FA* | 449.7500 | *MFO* | 423.7672 | *SA* | 420.7500 | *ACO* | 411.5000 |
| 10 | *SA* | 489.8362 | *SA* | 450.6207 | *MFO* | 433.0259 | *HS* | 427.7328 |
| 11 | *ABC* | 490.7586 | *ABC* | 456.6638 | *ABC* | 434.9483 | *FA* | 429.3707 |
| 12 | *ACO* | 508.1121 | *ACO* | 473.9224 | *ACO* | 447.7500 | *BA* | 429.9569 |
| 13 | *BA* | 545.8793 | *BA* | 516.9052 | *BA* | 490.3362 | *SA* | 434.4655 |
| ***Chi-sq.*** | 189.8568 | | 105.8629 | | 61.5589 | | 99.1510 | |
| ***Prob>Chi-sq.*** | 4.0193E-34 | | 3.9241E-17 | | 1.1716E-08 | | 8.1644E-16 | |

presented in Table 17; the mathematical details of these functions have been presented by the CEC 2017 competition committee [86].

The statistical results of the CryStAl algorithm in dealing with these test functions (CEC 2017) with 10 dimensions are presented in the Supplementary Materials where the results of three other successful algorithms are also presented. It should be noted that the error values, rather than the global best values, of each run are considered in this competition and the statistical results are based on the best error values of 51 independent runs. The results show that the proposed CryStAl algorithm is capable of providing eminently acceptable results in dealing with these test functions of different dimensions.

## VIII. COMPUTATIONAL COST AND COMPLEXITY ANALYSIS

In this section, the computational cost and complexity of the proposed CryStAl method are examined and analyzed where three different approaches are considered to acquire a better understanding of these properties. In the first approach, the computational cost procedure of the CEC 2017 benchmark suite is determined while the results of three other state-of-the-art algorithms are also considered to form a fair

TABLE 15. The K-W test results (mean of the ranks) for 100D mathematical functions.

| Ranking | 100D | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Min. | | Mean | | Std. | | Fun. Evl. | |
| | Algorithms | Mean of Ranks | Algorithms | Mean of Ranks | Algorithms | Mean of Ranks | Algorithms | Mean of Ranks |
| 1 | CryStAl | 171.7069 | CryStAl | 259.5517 | SSA | 298.1638 | CryStAl | 278.2328 |
| 2 | PSO | 274.6810 | SSA | 283.4741 | GA | 299.2241 | PSO | 340.7586 |
| 3 | SSA | 291.4397 | GA | 285.7414 | CryStAl | 305.3793 | MFO | 358.7500 |
| 4 | GA | 307.5172 | PSO | 304.1983 | PSO | 323.3362 | GA | 364.9741 |
| 5 | MVO | 342.7672 | MVO | 314.6724 | MVO | 327.7759 | SSA | 366.7328 |
| 6 | SCA | 349.0948 | SCA | 389.4483 | FA | 387.4310 | ABC | 375.9569 |
| 7 | MFO | 350.9914 | FA | 405.6379 | SCA | 400.7069 | ACO | 382.9741 |
| 8 | FA | 438.8103 | HS | 417.7241 | HS | 403.2586 | SCA | 389.6810 |
| 9 | HS | 451.3190 | MFO | 427.6121 | SA | 413.5862 | HS | 409.3276 |
| 10 | SA | 466.2414 | SA | 435.2241 | ABC | 420.7759 | MVO | 409.4569 |
| 11 | ABC | 472.2845 | ABC | 445.9655 | ACO | 426.6810 | BA | 410.1379 |
| 12 | ACO | 479.4483 | ACO | 452.9741 | MFO | 441.4052 | FA | 410.1897 |
| 13 | BA | 511.1983 | BA | 485.2759 | BA | 459.7759 | SA | 410.3276 |
| Chi-sq. | 149.9211 | | 86.7540 | | 50.0012 | | 65.2723 | |
| Prob>Chi-sq. | 5.8829E-26 | | 2.0918E-13 | | 1.3965E-06 | | 2.4278E-09 | |

TABLE 16. The P-H analysis results for all of the mathematical functions.

| Ranking | 2D & 50D & 100D | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Min. | | Mean | | Std. | | Fun. Evl. | |
| | Algorithms | Mean of Ranks | Algorithms | Mean of Ranks | Algorithms | Mean of Ranks | Algorithms | Mean of Ranks |
| 1 | CryStAl | 1011.2554 | CryStAl | 1167.9185 | CryStAl | 1213.4700 | CryStAl | 1077.0343 |
| 2 | PSO | 1181.8820 | SSA | 1262.1524 | SSA | 1271.1373 | PSO | 1198.1524 |
| 3 | MFO | 1301.1180 | PSO | 1296.0794 | PSO | 1304.7704 | MFO | 1253.2403 |
| 4 | SSA | 1322.7361 | GA | 1463.3519 | ACO | 1467.9335 | ACO | 1306.1953 |
| 5 | GA | 1358.6159 | MVO | 1492.2876 | ABC | 1507.1803 | GA | 1403.3948 |
| 6 | SCA | 1552.6202 | ACO | 1520.1352 | GA | 1510.8391 | SSA | 1506.7189 |
| 7 | HS | 1575.1717 | ABC | 1523.9678 | HS | 1516.0665 | HS | 1507.9700 |
| 8 | ACO | 1577.6159 | SCA | 1542.6180 | MFO | 1517.6009 | ABC | 1555.5579 |
| 9 | MVO | 1591.6459 | MFO | 1543.5536 | MVO | 1526.1524 | SCA | 1632.6674 |
| 10 | ABC | 1679.0107 | HS | 1553.2382 | SCA | 1566.9571 | BA | 1739.8498 |
| 11 | SA | 1818.0408 | SA | 1697.1931 | SA | 1679.9893 | MVO | 1792.3734 |
| 12 | BA | 1847.7639 | FA | 1759.9850 | FA | 1738.1974 | SA | 1834.5622 |
| 13 | FA | 1877.5236 | BA | 1872.5193 | BA | 1874.7060 | FA | 1887.2833 |
| Chi-sq. | 270.5922 | | 141.3699 | | 125.5517 | | 334.3474 | |
| Prob>Chi-sq. | 6.8410E-51 | | 3.1677E-24 | | 4.8084E-21 | | 2.7999E-64 | |

judgment. In the CEC 2017 computational scenario, four different computational times, namely $T_0$, $T_1$, $T_2$ and $\widehat{T}_2$, are considered based on four specific mathematical procedures. $T_0$ refers to the running time of a predefined mathematical procedure [46], $T_1$ denotes the computational time for evaluation of the $G_{18}$ test function considering 200000 function evaluations, $T_2$ represents the computational time of the considered metaheuristic algorithm (CryStAl in this paper) for evaluation of the $G_{18}$ test function considering 200000 function evaluations, and $\widehat{T}_2$ refers to the mean values of five different assessments of $T_2$. The results of this scenario for the proposed and alternative algorithms are presented in Table 18 which demonstrates the capability of the proposed CryStAl algorithm in producing competitive results.

In computer science, "Big O notation" is a mathematical notation that determines the required running time and memory space of an algorithm by considering its growth rate in dealing with different inputs. In the following, the computational cost of the proposed CryStAl method is presented using this notation which is the second approach for testing the complexity of the proposed algorithm. For CryStAl, the random selection process in the initialization phase of the algorithm has a computational complexity of $O(NP \times D)$ where $NP$ is the initial population size and $D$ is the dimension of the problem. The computational complexity of the objective function evaluation in the initialization phase of the algorithm is calculated as $O(NP) \times O(F(x))$ where $F(x)$ demonstrates the objective function value. After the initialization phase, the main loop of the algorithm is started based on the previously determined maximum number of iterations (MaxIter). By the consideration of the worst-case scenario, each line has a computational complexity of MaxIter in the main loop of the algorithm. In this loop, four new position vectors are created for each of the current vectors so the position updating process of the problem will have a computational complexity of $O(\text{MaxIter} \times NP \times D \times 4)$. In addition, the objective function evaluation in the main loop has a computational complexity of $O(\text{MaxIter} \times NP \times 4) \times O(F(x))$.

In general, the overall capacity of a metaheuristic algorithm depends on the balance between exploration and exploitation while the convergence speed is also an important factor in its evaluation. In order to demonstrate these properties for the proposed CryStAl algorithm, as the third complexity approach, the diversity graphs of CryStAl are plotted for functions $F_1$, $F_{61}$, and $F_{83}$ in the Supplementary Materials. As can be seen from these results, the population in the optimization process by CryStAl tends to localize the search for achieving better results.

**TABLE 17.** Summary of the CEC 2017 test functions [46].

| Function type | Func. No. | Function details | Func. Min. |
|---|---|---|---|
| Unimodal functions | $G_1$ | Shifted and Rotated Bent Cigar Function | 100 |
| | $G_2$ | Shifted and Rotated Sum of Different Power Function | 200 |
| | $G_3$ | Shifted and Rotated Zakharov Function | 300 |
| Simple multimodal functions | $G_4$ | Shifted and Rotated Rosenbrock's Function | 400 |
| | $G_5$ | Shifted and Rotated Rastrigin's Function | 500 |
| | $G_6$ | Shifted and Rotated Expanded Schaffer's F6 Function | 600 |
| | $G_7$ | Shifted and Rotated Lunacek Bi_Rastrigin Function | 700 |
| | $G_8$ | Shifted and Rotated Non-Continuous Rastrigin's Function | 800 |
| | $G_9$ | Shifted and Rotated Levy Function | 900 |
| | $G_{10}$ | Shifted and Rotated Schwefel's Function | 1000 |
| Hybrid functions | $G_{11}$ | Hybrid Function 1 ($N = 3$) | 1100 |
| | $G_{12}$ | Hybrid Function 2 ($N = 3$) | 1200 |
| | $G_{13}$ | Hybrid Function 3 ($N = 3$) | 1300 |
| | $G_{14}$ | Hybrid Function 4 ($N = 4$) | 1400 |
| | $G_{15}$ | Hybrid Function 5 ($N = 4$) | 1500 |
| | $G_{16}$ | Hybrid Function 6 ($N = 4$) | 1600 |
| | $G_{17}$ | Hybrid Function 6 ($N = 5$) | 1700 |
| | $G_{18}$ | Hybrid Function 6 ($N = 5$) | 1800 |
| | $G_{19}$ | Hybrid Function 6 ($N = 5$) | 1900 |
| | $G_{20}$ | Hybrid Function 6 ($N = 6$) | 2000 |
| Composition functions | $G_{21}$ | Composition Function 1 ($N = 3$) | 2100 |
| | $G_{22}$ | Composition Function 2 ($N = 3$) | 2200 |
| | $G_{23}$ | Composition Function 3 ($N = 4$) | 2300 |
| | $G_{24}$ | Composition Function 4 ($N = 4$) | 2400 |
| | $G_{25}$ | Composition Function 5 ($N = 5$) | 2500 |
| | $G_{26}$ | Composition Function 6 ($N = 5$) | 2600 |
| | $G_{27}$ | Composition Function 7 ($N = 6$) | 2700 |
| | $G_{28}$ | Composition Function 8 ($N = 6$) | 2800 |
| | $G_{29}$ | Composition Function 9 ($N = 3$) | 2900 |
| | $G_{30}$ | Composition Function 10 ($N = 3$) | 3000 |
| **Search range**: $[-100,100]^D$ | | | |

**TABLE 18.** Computational complexity results of CryStAl compared to other approaches.

| Metaheuristics | Properties | Results (sec) |
|---|---|---|
| EBO with CMAR [47] | $T_0$ | 0.0413 |
| | $T_1$ | 0.8218 |
| | $\hat{T}_2$ | 7.5794 |
| | $(\hat{T}_2 - T_1)/T_0$ | 163.6223 |
| LSHADE-cnEpSin [48] | $T_0$ | 0.1093 |
| | $T_1$ | 0.8391 |
| | $\hat{T}_2$ | 2.1835 |
| | $(\hat{T}_2 - T_1)/T_0$ | 12.30009 |
| MM-OED [49] | $T_0$ | 2.157784 |
| | $T_1$ | 0.146416 |
| | $\hat{T}_2$ | 6.704923 |
| | $(\hat{T}_2 - T_1)/T_0$ | 3.039417 |
| CryStAl (the present study) | $T_0$ | 0.027387 |
| | $T_1$ | 0.144345 |
| | $\hat{T}_2$ | 5.378017 |
| | $(\hat{T}_2 - T_1)/T_0$ | 191.10059 |

**TABLE 19.** Description of the investigated real-world design problems.

| No. (CEC No.) | Name | D | g | h |
|---|---|---|---|---|
| $M_1$ (RC 45) | SOPWM for 3-level Inverters | 25 | 24 | 1 |
| $M_2$ (RC 46) | SOPWM for 5-level Inverters | 25 | 24 | 1 |
| $M_3$ (RC 47) | SOPWM for 7-level Inverters | 25 | 24 | 1 |
| $M_4$ (RC 48) | SOPWM for 9-level Inverters | 30 | 29 | 1 |
| $M_5$ (RC 49) | SOPWM for 11-level Inverters | 30 | 29 | 1 |
| $M_5$ (RC 50) | SOPWM for 13-level Inverters | 30 | 29 | 1 |

## IX. REAL-WORLD OPTIMIZATION PROBLEMS

In this section, the applicability of the proposed algorithm, CryStAl, is investigated by considering some real-world optimization problems which can be a great challenge for the proposed method. In this regard, we have considered six difficult power electronics problems on synchronous optimal pulse-width modulation (SOPWM) which is used to regulate medium-voltage (MV) drives. This approach provides a significant decrease of switching frequency without raising the distortion, which leads to the reduction of switching losses that enhances the performance of the inverter. Generally, switching angles are calculated by reducing the distortion of current. In this study, this problem is considered as a constrained optimization problem which is benchmarked by CEC 2020 [90] regarding real-world constrained optimization. In this paper, six configurations of this problem are determined and solved by the proposed CryStAl with a simple penalty approach for constrained handling purposes. A brief explanation of these problems is presented in Table 19 while the comparative results are provided in the Supplementary Materials. The findings of this study demonstrated that the proposed method is capable of producing eminently acceptable and even better results in dealing with these challenging problems.

Based on the presented results in this and previous sections, it can be concluded that the proposed algorithm produces excellent results in most of the considered cases. One of the key aspects of this study is the conducted statistical analysis to evaluate the capability of this algorithm in dealing with an extensive set of test problems. The employed benchmark test problems of CEC and the competitive results of CryStAl in dealing with these problems demonstrate that this algorithm can be considered as a successful metaheuristic approach.

## X. CONCLUSION

This paper proposed a novel metaheuristic method called Crystal Structure Algorithm (CryStAl), inspired by the underlying principles of the formation of crystal structures from the addition of the basis to the lattice points. Four groups of mathematical test functions were selected in order to efficiently evaluate the performance of CryStAl with a total number of 12 different metaheuristic algorithms. A complete statistical analysis was conducted to provide a valid judgment about the performance of this method. The most important findings of this paper are as follows:

*(i)* CryStAl is superior to the other metaheuristics in converging to the global bests of the mathematical functions based on the selected tolerance.
*(ii)* The results of the K-S test demonstrated that the maximum difference between CryStAl and the other metaheuristics is about FA and BA in most of the cases.
*(iii)* The results of the M-W test showed that the summation of the ranks for CryStAl in most of the cases is lower than those of the other metaheuristics.
*(iv)* The results of the K-W test manifested that CryStAl is 100% successful in outranking the other metaheuristics for the 2D functions in all of the cases such as the minimum, mean, and standard deviation values alongside the number of function evaluations.
*(v)* The results of the K-W test showed that CryStAl has the first rank in the minimum and mean values of the 50D test functions while the PSO outranks CryStAl in the standard deviation and function evaluation.
*(vi)* The results of the K-W test showed that CryStAl has the first rank in the minimum and mean values alongside the number of function evaluations of the 100D test functions while the SSA and GA outrank CryStAl in the standard deviation values.
*(vii)* The overall comparison of CryStAl and the alternative metaheuristics considering all of the 2D, 50D, and 100D test functions demonstrated that CryStAl is 100 percent successful in outranking the other metaheuristics in all of the cases.

As future challenges, different applications of CryStAl can be explored and its capabilities in dealing with difficult test problems can be examined. Besides, new configurations of this algorithm can be considered as other researchers may have different viewpoints on the presented methodology.

## CODE AVAILABILITY

The MATLAB implementation of CryStAl is accessible at: https://www.mathworks.com/matlabcentral/fileexchange/91850-crystal-structure-algorithm-crystal

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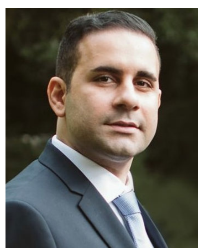

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