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ILP-Based Optimal PMU Placement with the Inclusion of the Effect of a Group of Zero-Injection Buses

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Abstract

Zero-injection buses (ZIBs) can reduce the number of phasor measurement units (PMUs) required to be installed for complete observability. It has been demonstrated that a group of neighboring ZIBs can further reduce this required number of PMUs. In contrast to a single ZIB, the effect of a group of ZIBs has not been incorporated into the model of optimal PMU placement (OPP) using mathematical approaches, except for some heuristics methods. In this paper, a novel methodology to incorporate the effect of a group of ZIBs into the OPP model is proposed, using the integer linear programming approach. Two common contingencies—line outage and PMU loss—are also considered. Moreover, two visualization techniques are used for illustrating the locations of PMUs obtained from OPP: (1) using a graph-based force-directed method and (2) on a map using the geographic information system. The proposed method is verified using several small- and large-scale test systems and compared with other related studies.

Keywords Zero-injection bus (ZIB) · Integer linear programming (ILP) · Optimal PMU placement (OPP) · Phasor measurement unit (PMU)

1 Introduction

Phasor measurement units (PMUs) are intelligent electronic devices that provide synchronized phasor measurements of voltages and currents (Mazhari et al. 2013). PMU synchronization is achieved by time stamping the voltage and current waveforms using a reference time signal provided by the global positioning system (GPS). PMUs are the most suitable measurement devices for recent technological developments of wide-area measurement systems (Azizi et al. 2013). The most challenging barriers limiting the number of PMUs that can be installed are the installation cost and availability of communication facilities (Hooshmand et al. 2016). It is neither necessary nor economical to have a PMU installed at each bus in a system (Nuqui and Phadke 2005). A PMU installed at a bus can measure the voltage phasors (magnitude and angle) of buses and all the branch current phasors incident into the bus, assuming that the PMU has sufficient number of channels (Manousakis et al. 2012). In other words, in contrast to traditional meters, a PMU can render the bus where it is installed and all the neighboring buses observable. The objective of optimal PMU placement (OPP) is to determine the minimal number of PMUs and their locations required to render the entire system observable.

Buses with no generator or load are called zero-injection buses (ZIBs). The sum of all the currents flowing to a ZIB is zero. The cluster of a bus is a set of buses that includes the bus itself and all the neighboring buses. A zero-injection cluster (ZIC) is the combination of the ZIB and all the incident buses. ZIBs have the potential to reduce the required number of PMUs to be installed for complete observability. However, modeling of zero-injection constraints is one of the main challenges of OPP owing to the intrinsic nonlinearity associated with it (Dua et al. 2008).

A ZIB can be categorized into two types: (1) a ZIB connected only to load and/or generator buses—in other words, all its neighboring buses are non-ZIBs; or (2) a ZIB connected to at least one other ZIB and can form a group of ZIBs. OPP by considering the first type of ZIBs is widely addressed in the literature (Aminifar et al. 2010; Gou 2008a, b; Abbasy and Ismail 2009; Aminifar et al. 2011; Chakrabarti and Kyr-

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iakides 2008; Manousakis and Korres 2016; Huang et al. 2014; Wen et al. 2013; Abd Rahman and Zobaa 2017; Aghaei et al. 2015). However, there are only a few studies that examine the effect of a group of ZIBs on the OPP model. All the techniques used in these studies are based on intelligent search techniques, and there are no mathematical approaches that incorporate the effect of a group of ZIBs. The concept of a group of ZIBs was first introduced by the authors in Hajian et al. (2007) using particle swarm optimization. Subsequently, in Aminifar et al. (2009), the authors used an immunity genetic algorithm to handle a group of ZIBs. The results of both studies show that considering a group of ZIBs can further reduce the number of PMUs required for complete observability.

The solutions used in the literature for OPP can be classified into two techniques: deterministic and heuristic.

Deterministic techniques are based on the mathematical representation of the model. The most widely used deterministic technique is integer linear programming (ILP), which can solve large-scale power systems without getting stuck in local minima (Gou 2008b; Huang et al. 2014). Other deterministic techniques used in the literature are integer nonlinear programming (Gou 2008a), integer quadratic programming (Chakrabarti et al. 2009), weighted least square algorithm (Manousakis and Korres 2013), and probability-based constraint method (Aminifar et al. 2011).

Heuristic solutions are intelligent search-based methods to solve the OPP without a mathematical representation of the model. Several heuristic methods are described and used in the literature including the genetic algorithm (Marin et al. 2003), Tabu search (Koutsoukis et al. 2013), simulated annealing (Nuqui and Phadke 2005), particle swarm optimization (Saleh et al. 2017), immune algorithm (Aminifar et al. 2009), decision tree (Mahmoodianfard et al. 2009), and cuckoo algorithm (Dalali and Kazemi Karegar 2016). Despite some advantages, the major drawback of heuristic methods is that these techniques are approximate methods and global optimum may not be achieved owing to the possible trapping in local minima (Monti and Muscas 2016).

Another important consideration is visualization of the PMU locations obtained from the numerical results of OPP. In the literature, only tables are used to illustrate these locations. For large power systems such as the Polish test systems, the optimal locations of PMUs have not been reported in the previous studies. Instead, only the number of PMUs required for complete observability is mentioned. The operator or planner may want to view the placement results visualized on a map or as a graph to know the details of the problem and obtain some insights into the solution. This motivates us to determine some techniques to visualize the optimal PMU locations graphically and geographically.

The main contributions in this paper are: (1) developing a mathematical methodology based on the ILP technique to incorporate the effect of a group of ZIBs into the OPP model to further reduce the number of PMUs required for observability; (2) presenting different visualization techniques for illustrating the numerical results of OPP graphically and on a map using graph theory and geographic information system (GIS), respectively; (3) employing large-scale and practical test systems such as the 13,659-bus European system and the Texas and Tennessee synthetic test systems. In order to deal with multiple solutions, the measurement redundancy is incorporated into the OPP with a maximization objective. Moreover, the impact of different contingencies such as PMU losses or line outages is also included in the model by considering the influence of a group of ZIBs. The obtained results are compared with seven other related studies.

The rest of this paper is organized as follows: The following section describes the observability rules. In Sect. 3, the mathematical equations for the basic ILP are formulated. Subsequently, the effect of a single ZIB is incorporated into the equations. The proposed method is described and expressed as a general formula with an example. The numerical results are presented in Sect. 4. The visualization techniques are introduced in Sect. 5. Finally, the conclusions are presented in Sect. 6.

2 PMU-Based Observability Analysis

A system is considered observable if all its states are known, i.e., the bus voltage phasors (magnitudes and angles) are measured directly or by calculation using circuit rules. The observability of power systems can be classified into two categories: numerical and topological (Huang et al. 2014). Numerical observability can be carried out based on the gain matrix of state estimation. If the gain matrix has a full column rank, the system is said to be fully observable. In spite of some advantages, the high computation burden is the main disadvantage of the numerical approach. Further, a large condition number of the measurement matrix may lead to an inaccurate solution. The system is considered topologically observable if there is a spanning tree of the full rank of the network (Huang et al. 2014).

Topological observability can be accomplished by applying the following common rules (Hajian et al. 2007):

(a) A bus is considered directly observable if it has a PMU installed that measures the voltage phasor of that bus. If this PMU also has a channel to measure the current phasor of the line connected to the bus, the line current is considered to be directly measurable.
(b) If the voltage and current phasors of one end of a line are known, the voltage phasor of the other end of the line can be calculated using Ohm's law. This bus with the calculated voltage phasor is considered indirectly observable.

(c) If the voltage phasors of both ends of a line are directly or indirectly known, the current phasor of this line can be computed, assuming that the line impedance is known. This line current is considered indirectly measurable.
(d) For a single observable ZIB, if the voltage phasors of this bus and all adjacent buses are known except one, the unknown voltage phasor can be calculated using Kirchhoff's current law at the ZIB.
(e) For a single unobservable ZIB, if the voltage phasors of all of its neighboring buses are known, the ZIB is also considered observable as its voltage phasor can be calculated using the node equation.
(f) For a group of unobservable ZIBs, if the voltage phasors of all the neighboring buses are known, all buses in the group are considered observable as their voltage phasors can be calculated using the nodal equation.

The first three rules are general rules and can be applied to any bus in the system. The effect of a single ZIB (type 1 of ZIBs) is indicated by rules (d) and (e). These two rules can be combined with the following statement: if all the buses in a ZIC are observable except one, the exception can also be observed using the node equation at the ZIB. This indicated that observing only (n − 1) buses in a ZIC renders the entire cluster observable including the unseen bus, where n is the total number of buses in the cluster. Therefore, ZIBs can reduce the number of PMUs required for complete observability. The final rule considers the effect of a group(s) of ZIBs, which can further lead to have an observable system with fewer number of PMUs. This rule is attempted to be incorporated into the OPP formulation using ILP.

3 ILP Formulation

ILP is a mathematical technique widely used in the literature for solving OPP with or without considering ZIBs. In this section, the mathematical equations of the OPP are first formulated for the basic ILP. Subsequently, the effect of a single ZIB is incorporated into the OPP using ILP. Finally, the proposed method for incorporating the effect of a group of ZIBs into the OPP model is presented.

3.1 Basic ILP for Solving OPP

The OPP can be formulated with ILP using the first three rules of observability as follows (Huang et al. 2014):

$$\text{Minimize} \sum\nolimits_{i=1}^{N} c_i X_i \tag{1}$$

$$\text{Subject to } f_i = \sum\nolimits_{j=1}^{N} a_{i,j} X_j \geq 1 \ \forall i, \tag{2}$$

where

- $X_i = \begin{cases} 1 \text{ if a PMU is installed at Bus-i} \\ 0 \text{ otherwise} \end{cases}$
- c_i is the total installation cost of PMU-i
- $a_{i,j}$ is the ij-th entry of the connectivity matrix
- $a_{i,j} = \begin{cases} 1 \text{ if } i = j \\ 1 \text{ if } i \text{ and } j \text{ are connected} \\ 0 \text{ otherwise} \end{cases}$
- N is number of buses in the system

The objective function (1) is the total required number of PMUs that should be minimized to obtain a completely observable system. The inequality constraint (2) guarantees that all the buses in the system are observable by at least one PMU. In order to maximize the measurement redundancy, i.e., the number of times a bus can be reached by PMUs (Huang et al. 2014), another term can be added to the objective function as follows:

$$\text{Minimize} \sum\nolimits_{i=1}^{N} c_i X_i - \sum\nolimits_{i=1}^{N} R_i \tag{3}$$

where R_i is the redundancy of Bus-i and the negative sign is added to convert the redundancy maximization problem to a minimization objective function.

3.2 ILP with the Effect of a Single ZIB

The impact of a single ZIB indicated by rules (d) and (e) can be mathematically incorporated into the model by introducing auxiliary variables added as additional constraints. This can be achieved for N buses and N_z ZIBs as follows:

$$\text{Minimize} \sum\nolimits_{i=1}^{N} c_i X_i - \sum\nolimits_{i=1}^{N} R_i \tag{4}$$

$$\text{S.t.} f_i = \sum\nolimits_{j=1}^{N} a_{i,j} X_j + \sum\nolimits_{z=1}^{N_z} a_{i,z} y_z \geq 1 \ \forall i, \tag{5}$$

$$\sum\nolimits_{i=1}^{N} a_{i,z} y_z \leq 1 \ \forall z \tag{6}$$

where

- $a_{i,z} = \begin{cases} 1 \text{ if } i \text{ and } z \text{ are connected} \\ 0 \text{ otherwise} \end{cases}$
- z is the index of ZIB
- y_z is the auxiliary variable added to each bus $\in ZIC_i$
- N_z is the number of ZIBs in the system

There is no change in the objective function. However, the second term of (2) is added to include the effect of the ZIBs. Moreover, another inequality constraint is added to ensure that only one unseen bus is considered in each ZIC. This unseen bus becomes observable through the effect of the ZIB according to rules (d)–(e).

An alternative mathematical formula reported in the literature to incorporate the effect of a ZIB is as follows (Manousakis and Korres 2013):

$$\text{Minimize} \sum_{i=1}^{N} c_i X_i - \sum_{i=1}^{N} R_i \tag{7}$$

$$\text{S.t.}\, f_i = \sum_{j=1}^{N} a_{i,j} X_j - \sum_{z=1}^{N_z} a_{i,z} y_z \geq \text{b}\ \forall i \tag{8}$$

$$\sum_{i=1}^{N} a_{i,z} y_z \geq n - 1\ \forall z \tag{9}$$

where:

$$b = \begin{cases} 0 \text{ if bus} - i \text{ is a ZIB} \\ 0 \text{ if bus} - i \text{ is connected to a ZIB} \\ 1 \text{ otherwise} \end{cases}$$

and $n$is the total number of buses in the cluster. Constraint equations (8)–(9) are similar to (5)–(6). The only difference is that rather than searching for at most one unobservable bus in a cluster to satisfy rules (d)–(e), the search is now intended for at least $(n - 1)$ observable buses. This method will be discussed with an example in the following section.

3.3 ILP with the Effect of a Group of ZIBs

As mentioned earlier, the effect of a group of ZIBs indicated by rule (f) has not been considered for solving the OPP using a mathematical approach. Further, it was demonstrated in Hajian et al. (2007) and Aminifar et al. (2009) that incorporating this rule into the model can reduce the required number of installed PMUs for complete observability.

In order to incorporate the effect of a group of ZIBs into the ILP formulation, suppose that a ZIB that belongs to this group, such as ZIB-i, is connected to n_1 ZIBs and n_2 non-ZIBs, and together they form ZIC-i as shown in Fig. 1a. It is possible to modify this ZIC-i in order to include only one ZIB in this cluster (in this example, ZIB-i) connected to n_2 non-ZIBs neighboring to the group. In other words, all the neighboring ZIBs-1…n_1 are removed from the corresponding constraint equations of ZIB-i. If all the buses in the new modified cluster (Fig. 1b) are observable except one, the exception is also observable through the effect of ZIB-$i$according to rules (d) and (e). The same procedure is applied to each ZIB and cluster in the group. Thus, all the buses in the group are observable if the neighboring buses are observable. Notably, by applying this approach, the lines joining the ZIBs of the group are disconnected. The connection can be remade easily by introducing an additional inequality constraint related to each ZIB in the group with equal to or greater than zero. These constraints are added to ensure that, if a PMU is installed at a ZIB in the group, all the neighboring buses can be reached by this PMU.

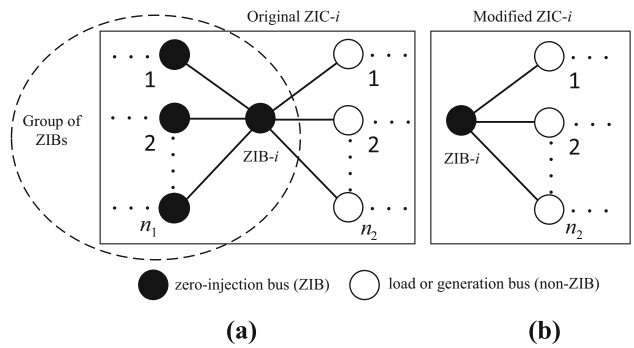

Fig. 1 The proposed ILP approach

For the sake of illustration, and before expressing the generalized formulas for the proposed method, a small test system with a group of ZIBs is considered as an example. The IEEE 14-bus test system has only one ZIB and cannot be used for verifying the proposed method. The smallest test system with a group of ZIBs is the IEEE 30-bus system (Fig. 2) with two separate groups of ZIBs: (1) {6, 9} and (2) {25, 27, 28}. Bus-22 is also a ZIB but is not connected to any neighboring ZIBs. The second group is used herein to clarify the proposed method. In this group, each ZIB has two neighboring ZIBs; both are eliminated from the corresponding cluster of this ZIB, according to the proposed method. For instance, Bus-27 has four neighboring buses—25, 28, 29, and 30—as shown in Fig. 3. The modified constraint equation of this bus includes only the bus itself in addition to the non-ZIBs, which are buses 29 and 30.

The original constraints for the cluster set of Bus-27 with their auxiliary variable constraints can be written using (8)–(9), as follows:

$$\text{Bus} - 25: \quad x_{24} + x_{25} + x_{26} + x_{27} - y_{25} \geq 0 \tag{10}$$

$$\text{Bus} - 27: \quad x_{25} + x_{27} + x_{28} + x_{29} + x_{30} - y_{27} \geq 0 \tag{11}$$

$$\text{Bus} - 28: \quad x_6 + x_8 + x_{27} + x_{28} - y_{28} \geq 0 \tag{12}$$

$$\text{Bus} - 29: \quad x_{27} + x_{29} + x_{30} - y_{29} \geq 0 \tag{13}$$

$$\text{Bus} - 30: \quad x_{27} + x_{29} + x_{30} - y_{30} \geq 0 \tag{14}$$

$$y_{25} + y_{27} + y_{28} + y_{29} + y_{30} \geq 4 \tag{15}$$

Equations (10)–(14) are the inequality constraints of the cluster set of Bus-27. Equation (15) is the auxiliary variable constraint that retains at least $(n - 1)$ buses, which is four in this case, as observables.

Therefore, based on the proposed approach, (10)–(12) and (15) can be modified as follows:

$$\text{Bus} - 25: \quad x_{24} + x_{25} + x_{26} - y_{25} \geq 0 \tag{16}$$

$$\text{Bus} - 27: \quad x_{27} + x_{29} + x_{30} - y_{27} \geq 0 \tag{17}$$

$$\text{Bus} - 28: \quad x_8 + x_{28} - y_{28} \geq 0 \tag{18}$$

$$y_{27} + y_{29} + y_{30} \geq 2 \tag{19}$$

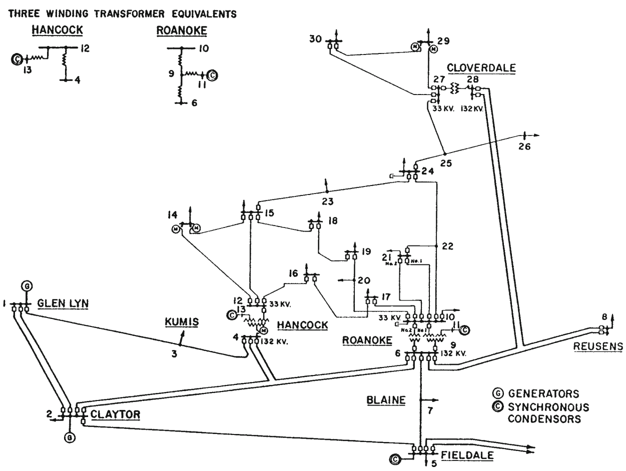

Fig. 2 IEEE 30-bus test system (Wen et al. 2013)

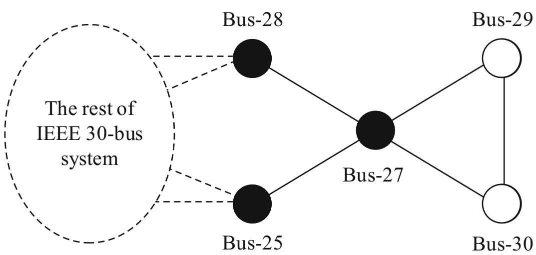

Fig. 3 Bus-27 and its adjacent buses in IEEE 30-bus system

Each of the above constraint equations now has only one ZIB, which is the corresponding bus of the constraint. Note that auxiliary variable constraint (19) has to observe two buses to render the entire cluster observable. If any two buses among 27, 29, and 30 are observable, the third bus is also observable through the effect of ZIB. Moreover, the eliminated buses from the constraint of Bus-27, i.e., buses 25 and 28, are observable through their modified equations if $(n-1)$ buses of their cluster sets are observable. Thus, all the buses in the group of ZIBs are observable. For the inequality constraints of buses neighboring to the group such as (13) and (14), all their equations remain unchanged. In order to retain the connections between the ZIBs in the group, the following constraints are added without auxiliary variables:

$$\text{Bus}-25: \quad x_{24}+x_{25}+x_{26}+x_{27} \geq 0 \tag{20}$$
$$\text{Bus}-27: \quad x_{25}+x_{27}+x_{28}+x_{29}+x_{30} \geq 0 \tag{21}$$
$$\text{Bus}-28: \quad x_{6}+x_{8}+x_{27}+x_{28} \geq 0 \tag{22}$$

It can be observed that all the ZIBs in the group are now connected through their variables on the left-hand side of the inequality constraints. Note that, in contrast to (10)–(12), there are no auxiliary variables in these constraints. Moreover, as the right-hand sides of (20)–(22) are zero, these constraints do not affect the placement except if there is a PMU installed at one of the ZIBs of the group. In this case, the neighboring buses are also observable.

The same procedure can be applied to the rest of ZIBs in all the groups of ZIBs. Notably, as ZIB-22 is not connected to any neighboring ZIB, the basic ILP is applied to this bus. Moreover, there is no additional inequality that should be added for Bus-22 in contrast to (20)–(22), which are added for buses 25, 27, and 28, respectively.

Thus, the modified ILP constraints for the IEEE 30-bus test system considering a group of ZIBs can be written as follows:

$$\text{Minimize} \sum_{i=1}^{N} c_i X_i - \sum_{i=1}^{N} R_i \tag{23}$$
$$\text{Bus}-1 \quad x_1+x_2+x_3 \geq 1 \tag{24}$$
$$\text{Bus}-2 \quad x_1+x_2+x_4+x_5+x_6-y_2 \geq 0 \tag{25}$$
$$\text{Bus}-3 \quad x_1+x_3+x_4 \geq 1 \tag{26}$$
$$\text{Bus}-4 \quad x_2+x_3+x_4+x_6+x_{12}-y_4 \geq 0 \tag{27}$$
$$\text{Bus}-5 \quad x_2+x_5+x_7 \geq 1 \tag{28}$$
$$\text{Bus}-6 \quad x_2+x_4+x_6+x_7+x_8+x_{10}-y_6 \geq 0 \tag{29}$$
$$\text{Bus}-7 \quad x_5+x_6+x_7-y_7 \geq 0 \tag{30}$$
$$\text{Bus}-8 \quad x_6+x_8+x_{28}-y_8 \geq 0 \tag{31}$$
$$\text{Bus}-9 \quad x_9+x_{10}+x_{11}-y_9 \geq 0 \tag{32}$$
$$\begin{aligned}\text{Bus}-10 \quad & x_6+x_9+x_{10}+x_{17}+x_{20}+x_{21} \\ & +x_{22}-y_{10} \geq 0\end{aligned} \tag{33}$$
$$\text{Bus}-11 \quad x_9+x_{11}-y_{11} \geq 0 \tag{34}$$
$$\text{Bus}-12 \quad x_4+x_{12}+x_{13}+x_{14}+x_{15}+x_{16} \geq 1 \tag{35}$$
$$\text{Bus}-13 \quad x_{12}+x_{13} \geq 1 \tag{36}$$
$$\text{Bus}-14 \quad x_{12}+x_{14}+x_{15} \geq 1 \tag{37}$$
$$\text{Bus}-15 \quad x_{12}+x_{14}+x_{15}+x_{18}+x_{23} \geq 1 \tag{38}$$
$$\text{Bus}-16 \quad x_{12}+x_{16}+x_{17} \geq 1 \tag{39}$$
$$\text{Bus}-17 \quad x_{10}+x_{16}+x_{17} \geq 1 \tag{40}$$
$$\text{Bus}-18 \quad x_{15}+x_{18}+x_{19} \geq 1 \tag{41}$$
$$\text{Bus}-19 \quad x_{18}+x_{19}+x_{20} \geq 1 \tag{42}$$
$$\text{Bus}-20 \quad x_{10}+x_{19}+x_{20} \geq 1 \tag{43}$$
$$\text{Bus}-21 \quad x_{10}+x_{21}+x_{22}-y_{21} \geq 0 \tag{44}$$
$$\text{Bus}-22 \quad x_{10}+x_{21}+x_{22}+x_{24}-y_{22} \geq 0 \tag{45}$$
$$\text{Bus}-23 \quad x_{15}+x_{23}+x_{24} \geq 1 \tag{46}$$
$$\text{Bus}-24 \quad x_{22}+x_{23}+x_{24}+x_{25}-y_{24} \geq 0 \tag{47}$$
$$\text{Bus}-25 \quad x_{24}+x_{25}+x_{26}-y_{25} \geq 0 \tag{48}$$
$$\text{Bus}-26 \quad x_{25}+x_{26}-y_{26} \geq 0y \tag{49}$$
$$\text{Bus}-27 \quad x_{27}+x_{29}+x_{30}-y_{27} \geq 0 \tag{50}$$
$$\text{Bus}-28 \quad x_8+x_{28}-y_{28} \geq 0 \tag{51}$$
$$\text{Bus}-29 \quad x_{27}+x_{29}+x_{30}-y_{29} \geq 0 \tag{52}$$
$$\text{Bus}-30 \quad x_{27}+x_{29}+x_{30}-y_{30} \geq 0 \tag{53}$$
$$y_2+y_4+y_6+y_7+y_8+y_{10} \geq 5 \tag{54}$$
$$y_9+y_{10}+y_{11} \geq 2 \tag{55}$$
$$y_{10}+y_{21}+y_{22}+y_{24} \geq 3 \tag{56}$$
$$y_{24}+y_{25}+y_{26} \geq 2 \tag{57}$$
$$y_{27}+y_{29}+y_{30} \geq 2 \tag{58}$$
$$y_8+y_{28} \geq 1 \tag{59}$$

$$x_2 + x_4 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{28} \geq 0 \quad (60)$$
$$x_6 + x_9 + x_{10} + x_{11} \geq 0 \quad (61)$$
$$x_{24} + x_{25} + x_{26} + x_{27} \geq 0 \quad (62)$$
$$x_{25} + x_{27} + x_{28} + x_{29} + x_{30} \geq 0 \quad (63)$$
$$x_6 + x_8 + x_{27} + x_{28} \geq 0 \quad (64)$$

Solving (23)–(64) yields the OPP for the IEEE 30-bus test system. Notably, for this small system, the number of PMUs required for complete observability is the same as those mentioned in the literature. However, for practical larger systems with multiple groups of ZIBs, complete observability can be achieved with fewer PMUs as explained in the following section.

To generalize, the mathematic equations for the proposed method can be written as:

$$\text{Minimize} \sum_{i=1}^{N} c_i X_i - \sum_{i=1}^{N} R_i \quad (65)$$
$$\text{S. t:} \sum_{j=1}^{N} a_{i,j} X_j - \sum_{g=1}^{N_z} a_{i,g} X_g - \sum_{z=1}^{N_z} a_{i,z} y_z \geq b \ \forall i \quad (66)$$
$$\sum_{i=1}^{n} a_{i,z} y_z \geq n - n_z - 1 \ \forall z \quad (67)$$

where

- $a_{i,g} = \begin{cases} 1 \text{ if } i \text{ and } g \text{ are connected and both are ZIB} \\ 0 \text{ if } i = g \\ 0 \text{ otherwise} \end{cases}$
- g is the index of the ZIB neighboring to ZIB-i
- X_g is the neighboring ZIB to ZIB-i
- n is the total number of buses in ZIC$_i$
- n_z is the number of neighboring ZIB in ZIC$_i$

Equation (65) is the objective function that should be minimized. Equation (66) is the inequality constraint for Bus-i obtained by subtracting the neighboring ZIBs from the corresponding ZIB. Equation (67) is the auxiliary variable constraint.

From (65)–(67), the following two special cases can be observed:

(1) For a single ZIB (rules (d) and (e)), the second terms of (66) and n_z in (67) are zeros. Thus, (65)–(67) are reduced to (7)–(9).

(2) If all the neighboring buses of a ZIB are also ZIBs, i.e., $n_z = n - 1$ in (67), the right-hand side of auxiliary constraint (67) equals zero. In other words, a ZIB that has only ZIB neighbors is observable if the neighboring buses to the group of the ZIBs are observable. As auxiliary inequality constraint (67) must be equal to or greater than zero in this case, this constraint has no effect on the solution except if a PMU is installed at this ZIB, in which case, it can be used for observing the neighboring ZIBs.

4 Numerical Result

The proposed method is applied to three different test systems, to demonstrate the ability of the proposed method to solve standard, large, and practical power systems: (1) IEEE test systems including 14-bus, 30-bus, 39-bus, and 118-bus systems, (2) large test systems such as the Polish 2383-bus, Polish 3375-bus, European 13,659-bus systems, and (3) the synthetic power systems of Tennessee and Texas. The impact of different contingencies such as PMU loss or line outage is also incorporated into the model of OPP. The number and locations of ZIBs for all the IEEE and large power systems are obtained from MATPOWER package using simple commands in MATLAB. The total time for solving the problem is provided based on a PC with 8 MB memory and 2.3 GHz CPU. All the operations are carried out using CPLEX.

4.1 Normal Operation

4.1.1 IEEE Test Systems

It is important to compare the result obtained by the proposed method with those in the literature based on the same technique, i.e., ILP, or those based on other techniques but incorporate the effect of a group of ZIBs in their OPP model. The obtained results are compared with several ILP-based studies reported in Dua et al. (2008), Aminifar et al. (2010), Gou (2008a), Abbasy and Ismail (2009), and Huang et al. (2014). These studies include the effect of a single ZIB, but the effect of a group of ZIBs is not considered. Furthermore, the difference between the results obtained by the proposed approach and the two heuristics-based techniques (Hajian et al. 2007; Aminifar et al. 2009) are also introduced.

Table 1 presents a comparison of the results obtained by the proposed approach and other ILP studies. For IEEE 14-bus, 30-bus, and 39-bus test systems, the results are similar. However, for larger systems with several groups of ZIBs such as the IEEE 118-bus system, the number of PMUs required to render the system observable using the proposed method is less by one compared with the results in Dua et al. (2008), Gou (2008a), and Abbasy and Ismail (2009), and equal to those in Aminifar et al. (2010) and Huang et al. (2014). Notably, although the results obtained by the proposed methods and Aminifar et al. (2010) and Gou (2008a) are the same, the proposed method requires fewer PMUs when there are contingency conditions as demonstrated in the following section in Table 7.

For methods not based on the ILP technique but that incorporate the effect of a group of ZIBs, the obtained results are compared with the related results presented in Table 2. It can be observed that, although the number of PMUs required by

Table 1 Comparison between the proposed ILP method and previous ILP methods

IEEE sys.	No. of ZIBs	ZIB location	Number of PMUs						Location of PMU (proposed)	Time (s)
			ILP (Dua et al. 2008)	ILP (Aminifar et al. 2010)	ILP (Gou 2008a)	ILP (Abbasy and Ismail 2009)	ILP (Huang et al. 2014)	Proposed ILP		
14	1	7	3	3	3	–	3	3	2, 6, 9	0.1101
30	6	6, 9, 22, 25, 27, 28	7	7	-	7	7	7	2, 4, 10, 12, 19, 24, 27	0.1547
39	11	1, 2, 5, 6, 9, 11, 13, 14, 17, 19, 22	–	8	–	–	8	8	3, 8, 10, 16, 20, 23, 25, 29	0.2084
118	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81	29	28	29	29	28	28	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 62, 65, 72, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 28	0.1984

Table 2 Comparison between the proposed and previous work considering a group of ZIB

IEEE sys.	Number of															
	PMU	Times the bus are observable by PMU			Buses observable/unobservable by PMU						Lines with a PMU installed at one end			Lines with PMUs installed at both ends		
		Hajian et al. (2007)	Aminifar et al. (2009)	Proposed	Hajian et al. (2007)		Aminifar et al. (2009)		Proposed		Hajian et al. (2007)	Aminifar et al. (2009)	Proposed	Hajian et al. (2007)	Aminifar et al. (2009)	Proposed
14	3	15	15	15	13	1	13	1	13	1	16	16	16	0	0	0
30	7	29	33	35	26	4	26	4	26	4	22	26	24	0	0	2
39	8	–	32	33	–	–	28	11	29	10	–	24	25	–	0	0
118	28	138	137	145	108	10	108	10	110	8	105	104	106	6	6	9

Table 3 PMU placement for large power systems

System	Number of			Time (s)
	Buses	ZIBs	PMUs	
Polish 2383-bus	2383	553	559	2.5888
Polish 3375-bus	3375	896	764	5.8925
European 13,659	13,659	4068	2,582	129.7238

the proposed method and other relevant studies are similar, the locations of these PMUs are different. The OPP is used to obtain the optimal number and locations of PMUs. These locations can affect the OPP through different observability views or indices such as the number of times the buses are observable, and the number of buses and lines that can be reached by a PMU. For a small system such as the IEEE 14-bus system with only one ZIB, the results are the same for all the studies including the proposed method. However, for other cases such as the IEEE 30-bus, 39-bus, and 118-bus systems, there is a noticeable difference between the proposed method and the previous work. The proposed method demonstrates a higher number of buses and lines with a PMU installed, although they all require the same number of PMUs. For instance, for the IEEE 118-bus, the number of times the buses are observable in the proposed method is 145, whereas it is 137 and 138 in Hajian et al. (2007) and Aminifar et al. (2009), respectively. Note that, for the IEEE 30-bus system, the proposed method demonstrates a lower number of lines with a PMU installed at one end; however, there are two lines whose both ends are connected to PMUs. Consequently, there are more PMUs connected to the ends of the lines in the proposed method compared with the ones in Hajian et al. (2007) and Aminifar et al. (2009). Hence, the redundancy of the lines and buses is higher in the proposed approach. The full comparison and details are listed in Table 2.

4.1.2 The Polish and European Systems

The proposed method is also applied to large power systems such as the Polish 2383-bus, 3375-bus, and the European 13,659-bus systems, as presented in Table 3. The results demonstrate the ability of the proposed method to solve large power system in a short time. For instance, it takes only 129 s to solve the 13,659-bus European test system. Notably, in the literature, the largest test system reported and used for the OPP, to the best of the authors' knowledge, was the Polish 3375-bus (Esmaili et al. 2013). In this study, the European system, which is the largest test system available and provided by MATPOWER, is employed for verifying the proposed method, and the result is presented in Table 3.

Table 4 Specifications of the synthetic systems

Number	kV level			Total
	345	115	13.8	
Buses	225	1500	282	2007
Lines	338	2233	–	2571
Transformers	562			
Generators	282			
Loads	1417			

Table 5 PMU placement for synthetic power systems

System	Number of			Time (s)
	Buses	*ZIBs*	*PMUs*	
Tennessee S.S.	150	33	32	0.2828
Texas S.S.	2,007	312	491	2.2286

4.1.3 Synthetic Systems of Tennessee and Texas

The previous test systems lack geographic information such as the longitude and latitude data of the substations, and hence, they cannot be used for visualizing the obtained results of OPP on their maps. Recently, a group of researchers (ICSEG 2016a, b) built some synthetic test systems by providing the geographic locations of the substations in the system. These systems are created based on public data and statistical analysis, and they are not related to the actual grid. However, they can be used to illustrate the results of OPP on maps using a geographic information system (GIS). Two synthetic test systems of Texas and Tennessee are employed in this paper for visualization purposes. The layers of the maps for these systems are built completely by the authors using ArcGIS Desktop software. The geographic data are converted from excel sheet to geodatabase files with different layers of buses and lines. These layers are classified according to the network voltage level and bus types such as generator and load buses. Table 4 provides some information about these synthetic systems including the voltage level, number of substations, buses, lines, transformers, loads, and generators.

The proposed ILP method is applied to these systems, and the results are given in Table 5. Visualizing and mapping the obtained results will be discussed in Sect. 5.

4.2 Contingency Cases (Line Outage or PMU Loss)

The previous OPP assumed a fixed network topology and absolute reliable measurement devices. However, in practical cases, PMUs may fail owing to several reasons, such as the loss of communication channels, loss of GPS signal, or failure in measurement instruments (Huang et al. 2014). Power systems are also subjected to line outages. Therefore,

Table 6 PMU locations without considering ZIB

System	Number of PMUs	
	Normal operation	PMU loss
IEEE 14-bus	4	9
IEEE 30-bus	10	21
IEEE 39-bus	13	28
IEEE 118-bus	32	68
Polish 2383-bus	746	1681
Polish 3375-bus	1083	2405
European 13,659	3369	10,467
Tennessee S.S.	37	98
Texas S.S.	578	1319

the operators may plan to install additional PMUs in the network for reliable monitoring of the system. In this section, the effect of a group of ZIBs is incorporated to the formulations of ILP against the line outage or PMU loss. The objective function of OPP with PMU or line outage remains the same. However, the right-hand side of inequality constraints (2), (5), (8), and (66) should be multiplied by 2. Therefore, the OPP formulation can be written as follows:

$$\text{Minimize} \sum_{i=1}^{N} c_i X_i - \sum_{i=1}^{N} R_i \tag{68}$$

$$\text{S. t:} \sum_{j=1}^{N} a_{i,j} X_j - \sum_{g=1}^{N_z} a_{i,g} X_g - \sum_{z=1}^{N_z} a_{i,z} y_z \geq 2b \; \forall i \tag{69}$$

$$\sum_{i=1}^{n} a_{i,z} y_z \geq n - n_z - 1 \; \forall z \tag{70}$$

The same test systems used for normal operation are employed here for the contingency cases. Table 6 lists the obtained results without considering ZIBs in the network. These results are similar to those in the literature.

Table 7 lists the results obtained by the proposed method and other related studies with considering ZIBs in the network. In general, the proposed method requires less or equal number of PMUs with respect to those in the literature. For instance, IEEE 14-bus requires 11 PMUs by Huang et al. (2014), 8 by Aminifar et al. (2010), 7 by Dua et al. (2008), and 7 PMUs by the proposed methods. For the IEEE 30-bus system, the observability of the system can be achieved with 3, 2, and 6 PMUs less than those in Aminifar et al. (2010), Abbasy and Ismail (2009), and Huang et al. (2014), respectively. Similarly, for the IEEE 39-bus system, the proposed method requires three PMUs less than those obtained by Aminifar et al. (2010) and Huang et al. (2014). For the IEEE 118-bus, the obtained result is similar, less, or more compared to those in Dua et al. (2008), {Aminifar et al. (2010), Huang et al. (2014)}, and Abbasy and Ismail (2009), respectively. The rest of the results are lsited in Table 7. Table 8

Table 7 PMU placement for the contingency case using ILP

System	Number of PMUs						Time (s)
	ILP (Dua et al. 2008)	ILP (Aminifar et al. 2010)	ILP (Gou 2008a)	ILP (Abbasy and Ismail 2009)	ILP (Huang et al. 2014)	Prop. ILP	
IEEE 14	7	8	–	–	11	7	0.1
IEEE 30	–	17	–	16	20	14	0.2
IEEE 39	–	22	–	–	22	19	0.2
IEEE 118	64	65	–	61	65	64	0.2
P-2,383	–		–	–	1217	1271	3.1
P-3,375	–		–	–	–	1755	12
E-13,659	–		–	–	–	7338	143
TN S.S.	–		–	–	–	90	0.2
TX S.S.	–		–	–	–	1145	2

Table 8 PMU Locations for PMU loss using the proposed ILP

IEEE system	Location of PMU (proposed)
14	2, 4, 5, 6, 9,11,13
30	1, 3, 5, 7, 10, 12, 13, 15, 17, 19, 20, 24, 25, 27, 29
39	3, 4, 8, 9, 10, 12, 16, 17, 20, 22, 23, 26, 29, 32, 34, 36, 37, 38
118	1, 3, 5, 6, 8, 9, 11, 12, 15, 17, 19, 21, 22, 23, 24, 27, 28, 31, 32, 34, 35, 37, 40, 42, 44, 45, 46, 49, 51, 53, 54, 56, 57, 59, 62, 66, 68, 69, 70, 71, 75, 77, 78, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 115, 117, 118

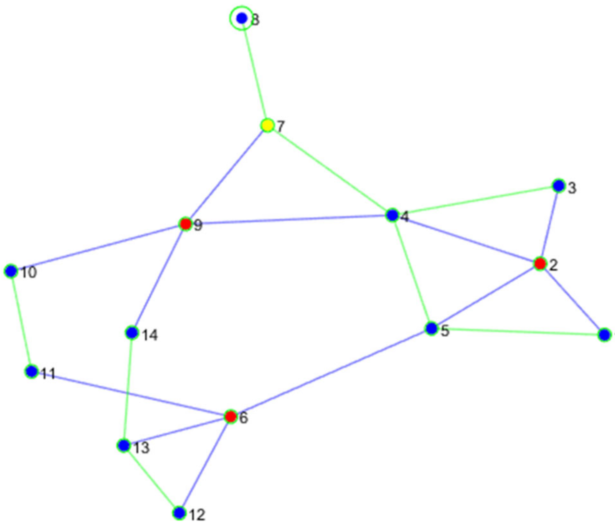

Fig. 4 IEEE 14-bus with PMU locations (red), ZIB (yellow), lines with PMU (blue), lines with no PMU (green) (Color figure online)

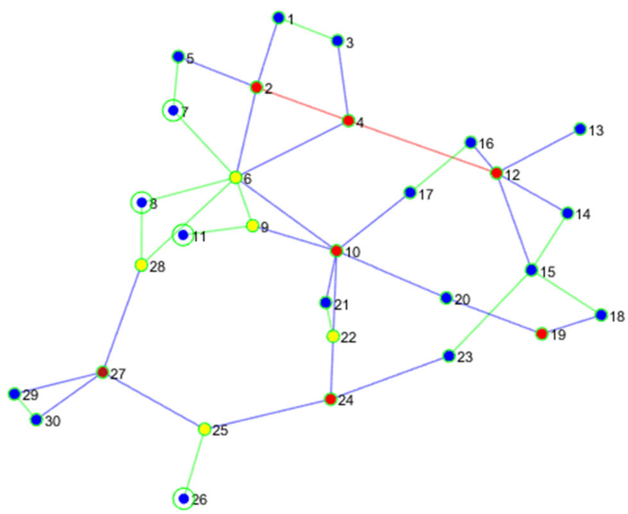

Fig. 5 IEEE 30-bus with PMU locations (red), ZIBs (yellow), PMU and ZIB (dark red), lines with two PMUs (red), lines with one PMU (blue), line with no PMU (green) (Color figure online)

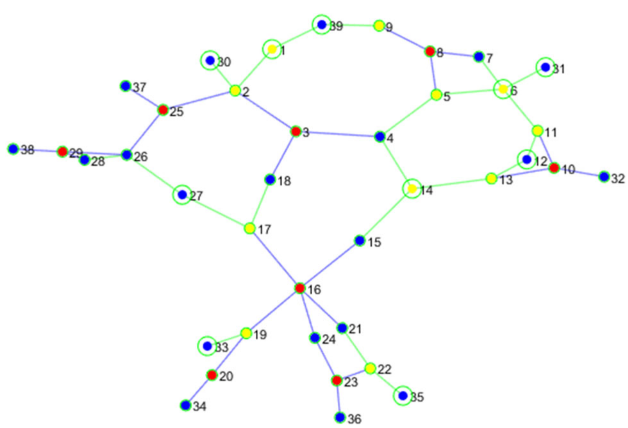

Fig. 6 IEEE 39-bus with PMU locations (red) and ZIBs (yellow) (Color figure online)

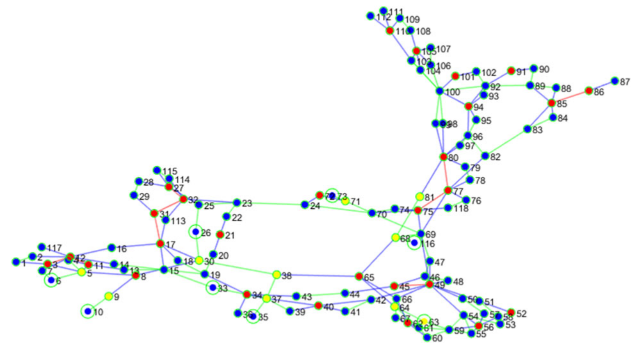

Fig. 7 IEEE 118-bus with PMU locations (red) and ZIBs (yellow) (Color figure online)

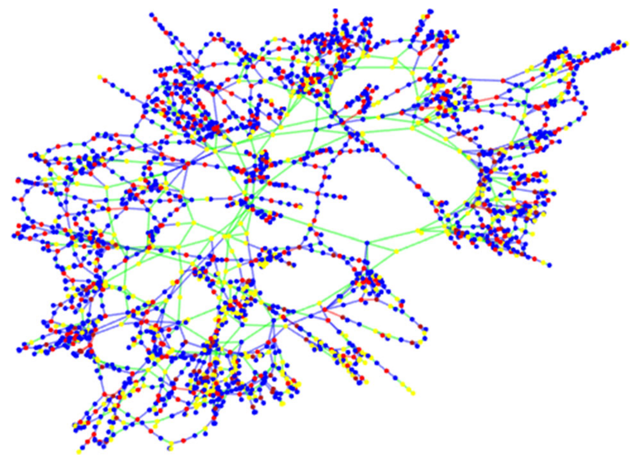

Fig. 8 Polish 2,383-bus system with PMU location (red) and ZIBs (yellow) (Color figure online)

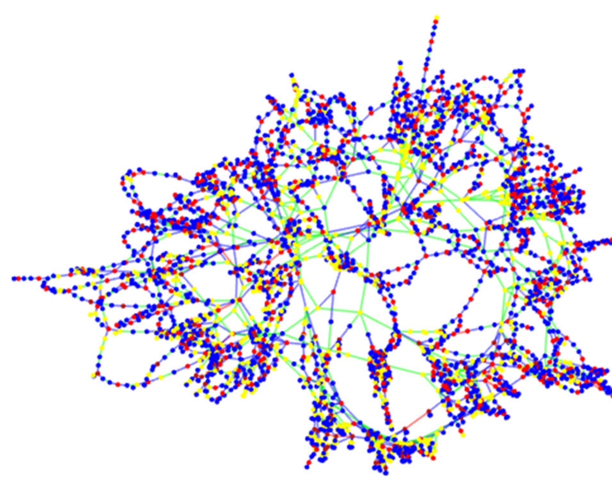

Fig. 9 Polish 3,375-bus system with PMU locations (red) and ZIBs (yellow) (Color figure online)

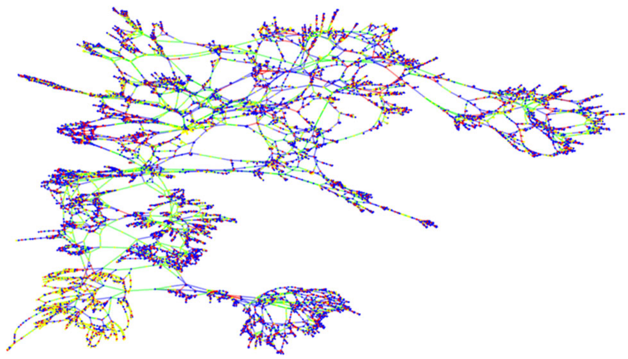

Fig. 10 European 13,659-bus system with PMU locations (red) and ZIBs (yellow) (Color figure online)

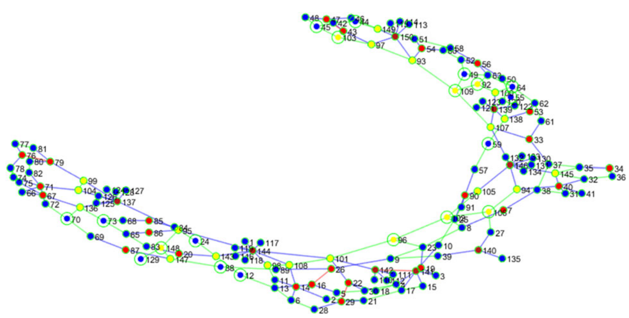

Fig. 11 Tennessee 150-bus synthetic system with PMU locations (red) and ZIBs (yellow) (Color figure online)

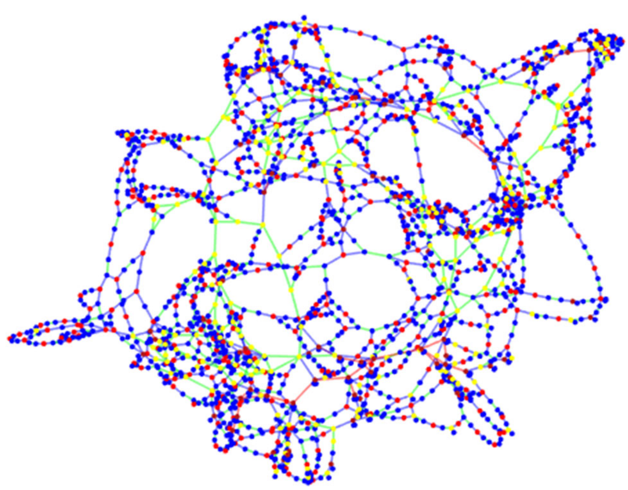

Fig. 12 Texas 2007-bus synthetic system with PMU locations (red) and ZIBs (yellow) (Color figure online)

presents the locations of PMUs obtained using the proposed method.

5 Placement Visualization

In the literature, the numerical results of OPP are shown using only tables. This is not sufficient to provide us insights into the solution, especially for large power systems. As the old adage says "a picture is worth a thousand words," it is useful to show the obtained results of OPP graphically and geographically on maps. In this paper, two methods of visualization are employed using graph theory and GIS.

5.1 Visualization Using Graph Theory

One of the most flexible methods to draw a network consisting of nodes and edges with pleasing and free-crossing layouts is the force-directed method (Tamassia 2014). This method is used for visualizing the placement results obtained in the previous section, without considering the theoretical details as they are beyond the scope of the paper. Owing to space limitations, the graphs of contingency cases are skipped.

The following colors are used for the graphs shown in Figs. 4, 5, 6, 7, 8, 9, 10, 11, 12: the ZIBs are marked with yellow circles, and the PMU locations are highlighted by red circles. A dark red color is used for ZIBs that have PMU installed, as in the case of Bus-27 in the IEEE 30-bus system. The external green rings are related to those buses that are unobservable by PMU but observable through the effect of ZIB, such as bus 8 in the IEEE 14-bus system. The line currents directly measured by a PMU installed at one end of the line are colored by blue. Also, lines with indirectly measured currents are highlighted by green color. These lines are calculated using Ohm's law as both ends are observable through neighboring buses.

One observation emerging from the graphs and also the numerical results is that the unobservable bus in a ZIC is always the one with a lower number of incident lines, except if all the adjacent buses are known by the PMUs.

For instance, the unobservable bus in the IEEE 14-bus system (Bus-8) is the radial one with only one incident line. For IEEE 30-bus, all the radial buses that are connected to ZIBs are selected by the solver as unobservable buses as they have a lower number of incident lines. This is because, in contrast to the radial buses, the buses with a higher number of incident lines have higher possibility to be reached by PMUs through their adjacent buses. If all the buses in a ZIC are reachable by PMUs, there are no unobservable buses in the cluster, as can be observed for Bus-22 in IEEE 30-bus system. For the sake of simplicity in large test systems, only the buses with PMUs or ZIBs are highlighted without labeling the bus numbers or illustrating the unobservable buses.

5.2 Visualization Using GIS

If the network used for optimal PMU placement has geographic data, i.e., the longitude and latitude of each bus, it will be useful to show the placement on a real map. GIS is a combination of software, hardware, and geographic data that

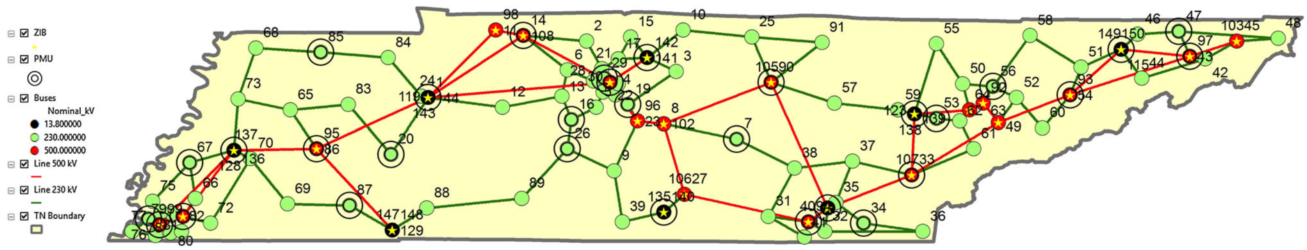

Fig. 13 Tennessee synthetic system with PMU locations (double circles), ZIBs (yellow stars), generator (black), green line (230 kV), and red line (500 kV) (Color figure online)

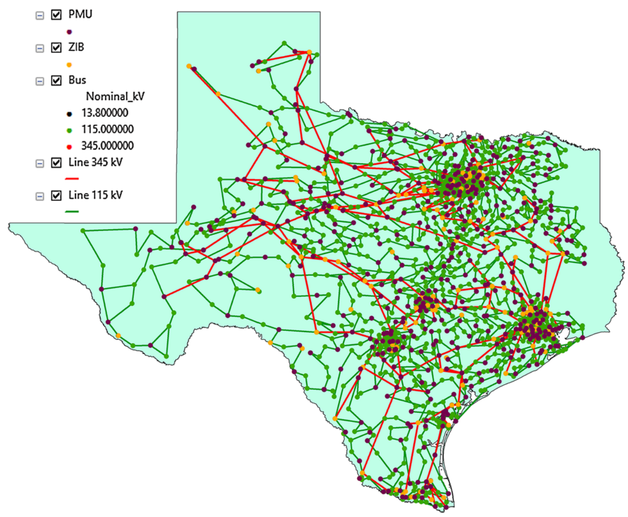

Fig. 14 Texas synthetic system with PMU locations (Violet), ZIBs (orange), green lines (115 kV), and red lines (345 kV) (Color figure online)

people interact with for visualization and analysis of data on a map (ESRI 2006). GIS has a variety of functions and tools to visualize the power system networks with different layers and to express the spatial relations between them (Shin 2004).

In this paper, two synthetic systems are used for visualizing the optimal placement of PMU on their maps using ArcGIS Desktop software. These systems are Tennessee 150-bus and Texas 2007-bus systems. For the Tennessee synthetic system (Fig. 13), the PMUs are denoted by double circles, and the ZIB buses are marked by star symbols. Moreover, the map's legend is located on the left side of the map. For the Texas synthetic power system (Fig. 14), the PMUs locations are highlighted by violet color, whereas the orange color is used for the ZIBs. The red lines are the extra-high-voltage (EHV) part of the network wheras the green color is the high-voltage (HV) network. As a substation may have more than one bus located in the same substation, some buses may not be shown on the map, but they can be observed by zooming into the map.

6 Conclusion

In this paper, a novel mathematical methodology was presented using ILP to incorporate the effect of a group of ZIBs into the OPP model. The common cases of contingency, such as PMU loss or line outage, were also studied. The proposed method was verified using several standard, large, and synthetic systems including IEEE 14, 30, 39, 118, the Polish 2383, 3375, and the European 13,659 bus test systems. The results revealed that including a group of ZIBs can enhance the OPP by reducing the number of PMUs required for complete observability. The proposed method was compared with several related studies and considering various observability views such as the number of times a bus or a line can be measured. The method demonstrated its ability to solve large power systems within a short time and provide optimal number and locations of PMUs. Furthermore, two visualization techniques were proposed for illustrating the locations of PMUs obtained from OPP: (1) graphically using a force-directed method and (2) geographically on maps using GIS.

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