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Beta (phase constant) compensation for flat voltage profile in a transmission line

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ABSTRACT

The reliability of the bulk electric power systems is not only dependent on the generation but also on the transmission system. In long lines, the voltage along the transmission line varies with the loading conditions. Proper reactive power compensation helps to maintain the voltage along the line within acceptable limits. This paper proposes a compensation method to adjust the phase constant of the transmission line in order to obtain a flat voltage profile along the line. Case studies conducted on a long transmission line are presented.

1. Introduction

The economic growth of any country depends on the availability of a quality electric power, one criterion of which is to maintain a constant voltage throughout the length of the line and making it flat. The major generating plants may not be near to the load centers and this condition dictates the power transmission over long distances. The bulk power transmission over longer distances is possible and economical if transmitted at higher voltages (EHV and UHV) [1]. The fundamental requirement for bulk transmission of electrical power is to keep the voltages near to its rated value and voltage variation along the line within acceptable limits. This is more important while transmitting at higher voltages in particular in order to keep the stress on the insulators below the rated value. One of the major concerns while transmitting power at high voltages is inadmissible increase of voltage along line on no load or low load conditions. Reactive power compensation provides voltage support and reduces the voltage fluctuation at a given terminal of a transmission line. It also improves the system stability by increasing the maximum active power that can be transmitted. It also maintains almost a flat voltage profile along the transmission line [2]. If a lossless line is operated at its surge impedance loading, it exhibits a flat voltage profile. This has been discussed in detail in [3] controlling the surge impedance of the line by shunt compensation to make the corresponding surge impedance loading always equal to the actual loading. In this paper, a beta compensation (BC) method is proposed to implement the similar concept in a different way with a procedure to determine the required compensator values. This method requires a dynamic compensation and it can be achieved by using proper FACTS devices.

2. Long transmission line equivalent circuit

In a long transmission line model, sending-end voltage and receiving-end voltage can be related by

$$V_s = V_r \cosh \gamma l + I_r Z_c \sinh \gamma l \quad (1)$$

where V_s is the sending-end voltage; V_r is the receiving-end voltage; I_r is the receiving-end current; Z_c is the characteristic impedance, $\sqrt{z/y}$; γ is the propagation constant, $\sqrt{yz} = \alpha + j\beta$; α is an attenuation constant, nepers/unit length; β is the phase constant, radians/unit length; l is the length of the line; z is the series impedance/unit length and y is the shunt admittance/unit length.

The voltage at any distance ‘x’ from the receiving-end can be expressed as

$$V_x = V_r \cosh \gamma x + I_r Z_c \sinh \gamma x \quad (2)$$

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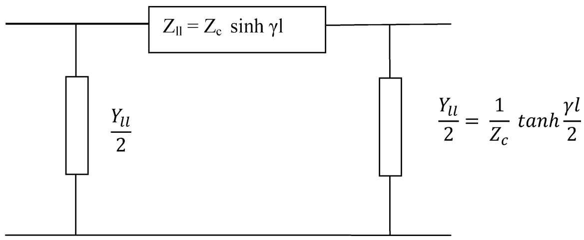

Fig. 1. Long line equivalent circuit.

By re-arranging the terms from Eqs. (1) and (2), the following equation can be derived

$$V_x = \frac{V_r \sinh\gamma(l-x) + V_s \sinh\gamma x}{\sinh\gamma l} \tag{3}$$

The lumped parameter equivalent circuit of a long line can be expressed as in Fig. 1.where Z_{ll} is an equivalent long line series impedance and Y_{ll} is an equivalent long line shunt admittance.

3. Voltage profiles

It is assumed in the work the line is lossless. This is of course a crude model far from reality. However, it is well known all initial generalizations are always made in the literature with respect to stability, economic operation of power systems and several optimization problems assuming the system is lossless. In this paper a new algorithm for providing a required compensation using the newly proposed Flatness Index is tested for implementation and hence to make possible approximate generalizations the line is assumed lossless. To make the line lossless, the series resistance and shunt conductance are neglected. Hence the attenuation constant (α) becomes zero.

$$\gamma = \sqrt{yz} = \alpha + j\beta = j\beta$$

$$j\beta = \sqrt{j\omega C j\omega L}$$

$$\beta = \omega\sqrt{LC}$$

The quantity βl is the electrical length of the line expressed in radians [4].

Very high voltages along the line may occur when $\beta l = n\pi$, where $n = 1, 2, 3\ldots$

For a loss-less line Eq. (3) becomes

$$V_x = \frac{V_r \sinh j\beta(l-x) + V_s \sinh j\beta x}{\sinh j\beta l} = \frac{V_r \sin\beta(l-x) + V_s \sin\beta x}{\sin\beta l} \tag{4}$$

With some arbitrary value of $\beta = 0.00205558$ rad/mile [5], $V_s = 1\angle 90^\circ$ pu, and $V_r = 1\angle 0$ pu the voltage profile along the long transmission line has been shown in Fig. 2.

From Eq. (4), the mid-point voltage, V_m can be expressed as

$$V_m = \frac{V_r + V_s}{2\cos(\beta l/2)}$$

If $V_s = 1\angle\delta$ pu, $V_r = 1\angle 0$ pu, where δ is the power angle in radians.

$$V_m = \frac{\cos(\delta/2)}{\cos(\beta l/2)}\angle\frac{\delta}{2} \tag{5}$$

$$|V_m| = \frac{\cos(\delta/2)}{\cos(\beta l/2)} \tag{6}$$

Conditions for voltage sag, rise, and flat voltage profile:

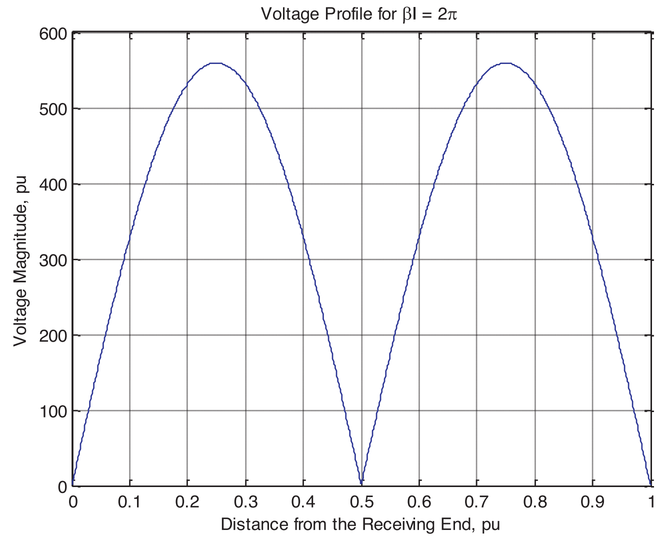

Fig. 2. Voltage profile with $n = 2$.

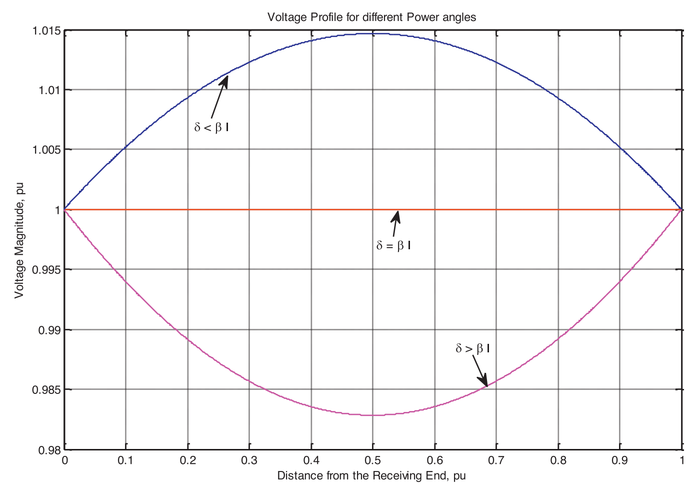

Fig. 3. Voltage profile for different power angles.

$|V_m| < 1$, when $\delta > \beta l$, i.e. voltage sag
$|V_m| > 1$, when $\delta < \beta l$, i.e. voltage rise
$|V_m| = 1$, when $\delta = \beta l$, i.e. flat voltage

With the assumption that both δ and βl vary between 0 to π radians ($0 \le \delta \le \pi$).

The following are the voltage profiles for these three different conditions.

4. Beta compensation (BC) method

From Eq. (6) and Fig. 3, it is evident that a flat voltage profile along the line can be obtained if the power angle (δ) and the electrical length (βl) of the line are maintained at the same value. As explained earlier in Section 3, by neglecting the line series resistance and shunt conductance, the variation of β with line series reactance and shunt susceptance can be shown as in Fig. 4.

With this keeping in mind, the following method has been proposed to maintain a flat voltage along the lossless long transmission line.

a. Load flow study is performed considering both ends of the line, one as slack bus and the other as PV bus (voltage controlled bus).
b. Power angle, the δ_{new} is compared with βl.

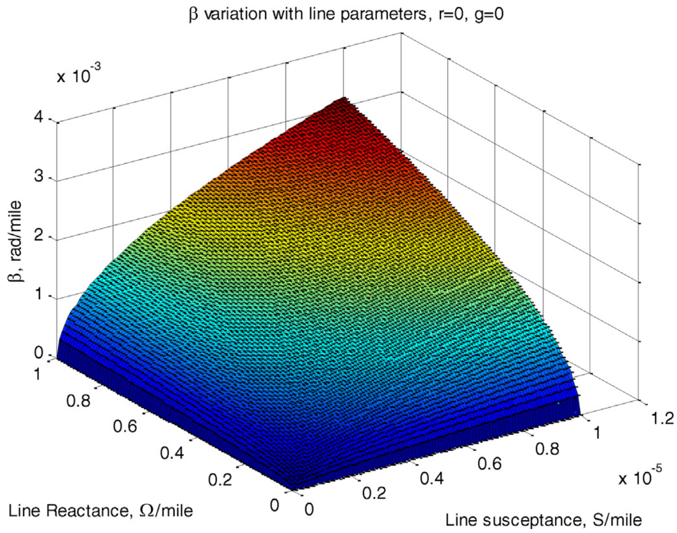

Fig. 4. β variation with line series reactance and shunt susceptance.

c. If $\delta_{new} \neq \beta l$, line parameters are adjusted by adding external compensation using the long line equivalent circuit and $\beta_{new} l = \delta_{new}$ so that

$$Z_{new} = jZ_c \sin\delta_{new} \quad \text{and}$$
$$Y_{new} = j\frac{2}{Z_c}\tan\frac{\delta_{new}}{2} \tag{7}$$

d. The modified Y_{bus} is framed.
e. Process is repeated until $\delta_{new} = \beta_{new} l$.

The same is shown as the flow chart in Fig. 5.

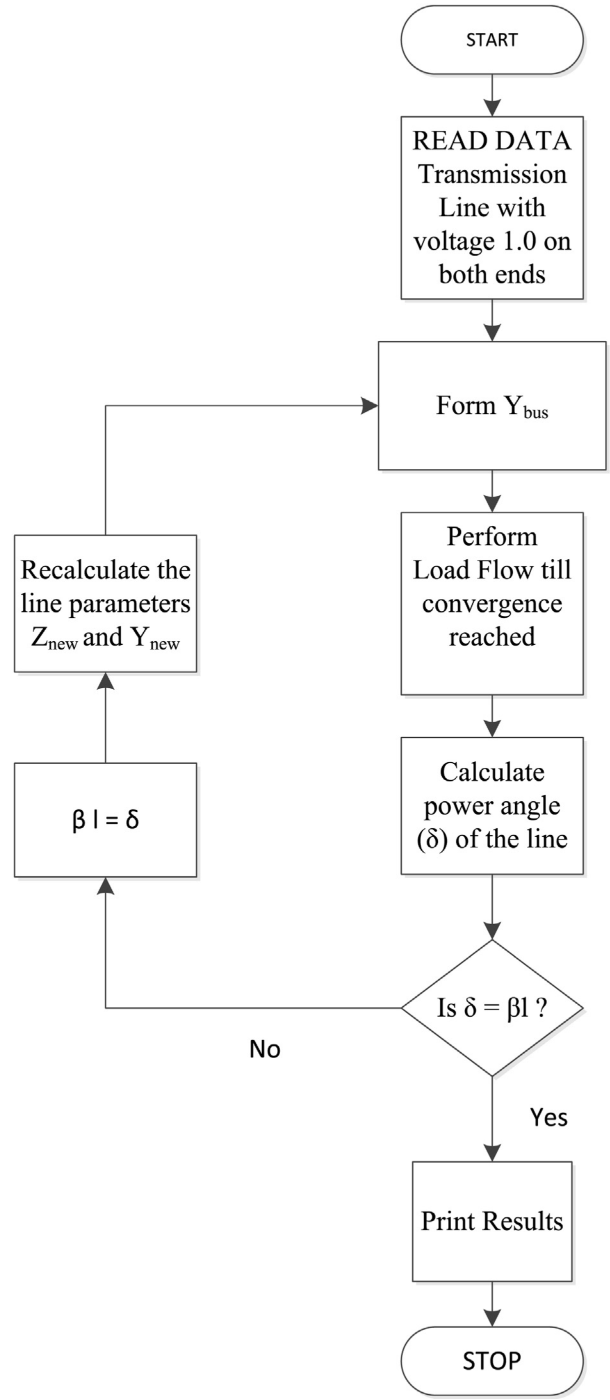

Fig. 5. Flow chart of the beta compensation method.

5. Value and location of the compensators

In this proposed method, the line is modeled as two equal cascaded π networks. Hence, 1/4 of the total line susceptance is considered at each end and 1/2 of the line susceptance is taken at the middle of the line. So the shunt compensation is also divided in the same ratio. Line series reactance is divided into two equal parts, one in each section. So the series compensator is also divided equally into the two sections.

The diagram in Fig. 6 describes what has been proposed.where X_L is the total line series reactance; Y_L is the total line shunt susceptance; X_{comp} is the total series compensation, and Y_{comp} is the total shunt compensation.

6. Flatness Index (FI) of the transmission line

Flatness Index (FI) of a transmission line is defined by authors [6,7] as the difference between the maximum voltage-sag/hump of an uncompensated line and a compensated line expressed as a fraction of the maximum voltage-sag/hump of the uncompensated line.

$$\text{Flatness Index, FI} \triangleq \frac{(\text{Max. voltage sag of the uncompensated line} - \text{Max. voltage sag of the compensated line})}{\text{Max. voltage sag of the uncompensated line}}$$

Voltage hump (swell) can be considered as negative sag.

$$\text{FI} = \frac{(V_{ref} - V_u) - (V_{ref} - V_c)}{(V_{ref} - V_u)} = 1 - \frac{(V_{ref} - V_c)}{(V_{ref} - V_u)} \tag{8}$$

where V_{ref} is the reference voltage; V_u is the min/max voltage along the line when there is no compensation; V_c is the min/max voltage along line with compensation.

7. Variation of FI and voltage profile along the transmission line with respect to compensation

The shunt and series compensations are adjusted to give rise to following cases

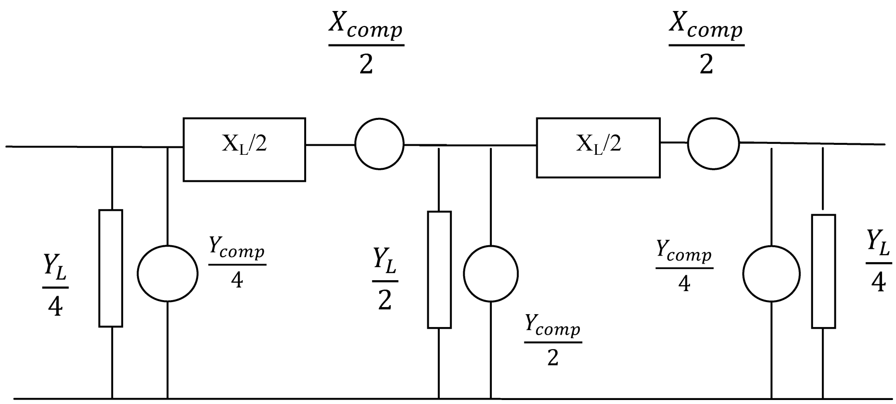

Fig. 6. Equivalent circuit of transmission line with beta compensation.

a. $V_u < V_{ref}$, $V_c \le V_{ref}$, $V_c > V_u$
This is a voltage sag case and from Eq. (8), it is evident that the FI varies between 0 and 1 ($0 \le FI \le 1$). The voltage profile can be shown as in Fig. 7(a). V_c varies anywhere in the shaded area.

b. $V_u < V_{ref}$, $V_c \le V_{ref}$, $V_c < V_u$
In this case the voltage along the line is further decreased after compensation. Hence FI becomes zero or negative ($FI \le 0$). The voltage profile can be shown as in Fig. 7(b).

c. $V_u < V_{ref}$, $V_c \ge V_{ref}$, $V_c > V_u$
In this case the voltage profile is changed from sag to hump after compensation. The FI will be 1 or greater than 1 ($FI \ge 1$). The voltage profile can be shown as in Fig. 7(c).

d. $V_u > V_{ref}$, $V_c \ge V_{ref}$, $V_c < V_u$
In this case the voltage hump is reduced to V_{ref} or anywhere between V_u and V_{ref}, the shaded area in Fig. 7(d). FI varies between 0 and 1 ($0 \le FI \le 1$).

e. $V_u > V_{ref}$, $V_c \ge V_{ref}$, $V_c > V_u$
The voltage hump is further increased after compensation. Hence FI becomes zero or negative ($FI \le 0$). The voltage profile can be shown as in Fig. 7(e).

f. $V_u > V_{ref}$, $V_c \le V_{ref}$, $V_c < V_u$
In this case the voltage profile is changed from hump to sag after compensation. The FI will be 1 or greater than 1 ($FI \ge 1$). The voltage profile can be shown as in Fig. 7(f).

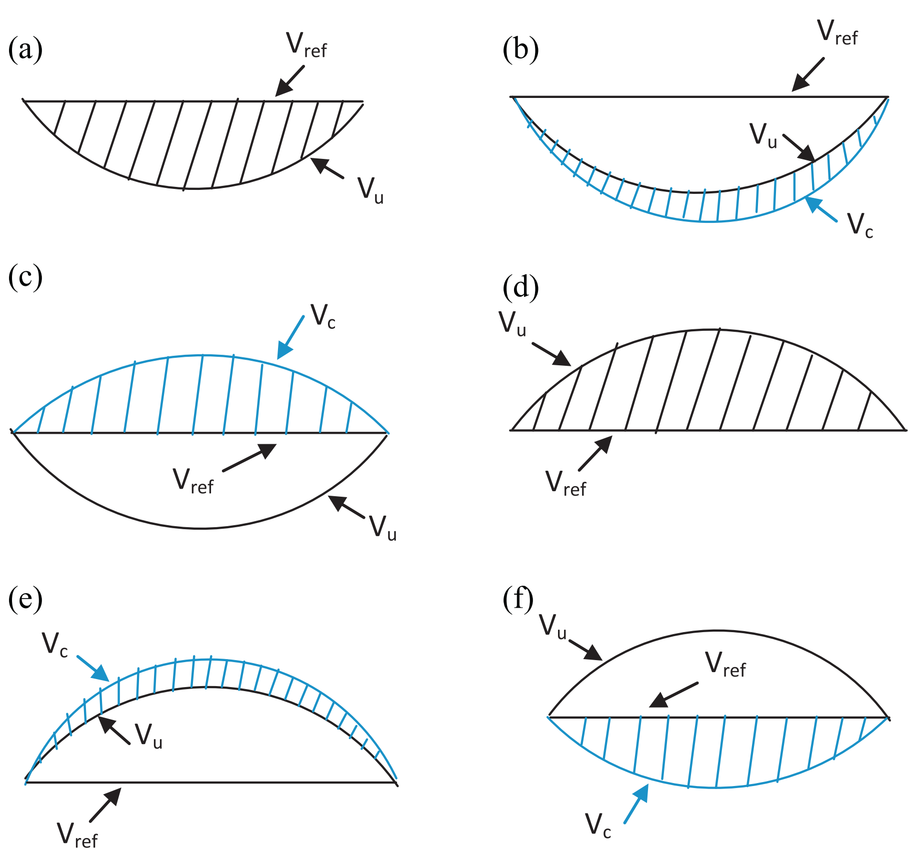

Fig. 7. (a) Voltage profile of sag case with proper compensation. (b) Voltage profile of sag case with improper compensation. (c) Voltage profile of sag case with over compensation. (d) Voltage profile of hump case with proper compensation. (e) Voltage profile of hump case with improper compensation. (f) Voltage profile of hump case with over compensation.

Table 1
Results of beta compensation under different conditions.

Scenario		Voltage				Compensation in line between buses 1 and 2		Power angle δ rad	βl rad	$1/Z_c$	Real power transfer pu
		Bus 2		Bus 15		Series X_{comp} pu	Shunt Y_{comp} pu				
		$\|V\|$ pu	Angle deg	$\|V\|$ pu	Angle deg						
Base case	NLF	1.0	−24.801	1.067	−12.401	–	–	0.4329	0.8221	1.148	0.586
	BCLF	1.0	−13.386	1.0	−6.693	−0.5144	−0.6745	0.233	0.233	1.148	1.148
$P_{Gbus2} = 15$	NLF	1.0	−28.705	1.058	−14.353	–	–	0.501	0.8221	1.148	0.671
	BCLF	1.0	−18.666	1.0	−9.333	−0.4373	−0.5666	0.3258	0.3258	1.148	1.148
$P_{Gbus2} = 0$	NLF	1.0	−31.140	1.052	−15.570	–	–	0.543	0.8221	1.148	0.722
	BCLF	1.0	−21.918	1.0	−10.959	−0.3909	−0.4993	0.383	0.383	1.148	1.148
$Z_{se} = 0.358$	NLF	1.0	−18.672	1.008	−9.336	–	–	0.326	0.411	1.148	0.894
$Y_{sh} = 0.472$	BCLF	1.0	−13.519	1.0	−6.760	−0.1544	−0.1998	0.235	0.235	1.148	1.148
$Z_{se} = 1.074$	NLF	1.0	−27.496	1.199	−13.748	–	–	0.4799	1.233	1.148	0.4299
$Y_{sh} = 1.416$	BCLF	1.0	−13.386	1.0	−6.693	−0.8724	−1.1465	0.233	0.233	1.148	1.148
$Z_{se} = 0.358$	NLF	1.0	−18.723	1.003	−9.362	–	–	0.327	1.0	1.0	0.897
$Y_{sh} = 0.358$	BCLF	1.0	−16.619	1.0	−8.309	−0.072	−0.066	0.290	0.290	1.0	1.0
$Z_{se} = 0.716$	NLF	1.0	−25.583	0.982	−12.792	–	–	0.4465	0.2369	0.3309	0.6031
$Y_{sh} = 0.0784$	BCLF	1.0	−32.737	1.0	−16.368	0.9182	0.116	0.571	0.571	0.3309	0.3309
Line 5–6 removed	NLF	1.0	−25.716	1.065	−12.858	–	–	0.449	0.8221	1.148	0.606
	BCLF	1.0	−14.880	1.0	−7.440	−0.4924	−0.6441	0.2597	0.2597	1.148	1.148

NLF, normal load flow; BCLF, beta compensation load flow.

8. Case studies

To illustrate the proposed BC method, the IEEE 14 bus system is slightly modified as follows and the modified line is shown in Fig. 8.

a. Line between buses 1 and 2 is replaced with a lossless line with
   Line series reactance = 0.716 pu.
   Line shunt susceptance = 0.944 pu.
b. A new PQ bus 15 is introduced at the mid-point between buses 1 and 2 with $P = 0$ and $Q = 0$.
c. The bus 2 voltage is specified as 1.0 and reactive power limits are increased in order to keep the voltage at 1.0.

At each iteration in the algorithm, the series reactance and the shunt admittance values are varied in such a way that the mid-point voltage reduced gradually and settled at 1.0 pu, Fig. 9(a) and (b) obviates this. The difference between δ and βl is reduced with each iteration and finally both reached to the same value, Fig. 9(d). The Flatness Index (FI) is shown as zero at uncompensated and 1.0 at fully compensated where $\delta = \beta l$, Fig. 9(f) obviates this. The real power transfer is increased with each iteration and reached to maximum and equal to the power at surge impedance loading, i.e. at $\delta = \beta l$ as shown in Fig. 9(e).

Different scenarios are created on the system and the corresponding values of various parameters with and without beta compensation are noted. The following table shows various values of all those scenarios (Table 1).

For the case study conducted, the beta compensation method has been applied for different scenarios and observed how the compensation has to be varied in order to get the flat voltage profile.

In scenario-1, all the bus data and line data have been kept as given in IEEE 14 bus except for the line replacement between buses 1 and 2. A hump (swell) in the voltage profile along the line is observed. After proper beta compensation, new βl and new δ become equal and the required flat voltage profile is got.

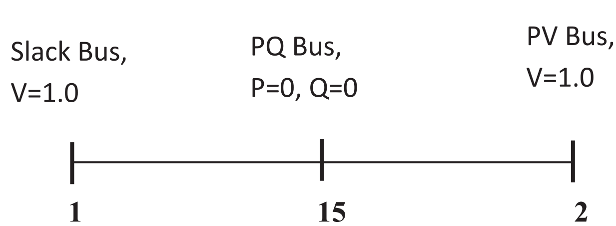

Fig. 8. Line under study in IEEE 14 bus system with modifications.

In the next two scenarios (2 and 3), the real power generation at bus 2 is varied. The voltage profile is observed without compensation and with compensation. As real power generation is reduced at bus 2, more real power has to flow from bus 1. So the voltage angle at bus 2 is increased compare to scenario-1 as the real power generation is decreased at bus 2. Hence, the βl value is changed to lower value with compensation.

In scenario-4 and 5, the line length is varied keeping the characteristic impedance at the same value. The line length is made half in the case of scenario-4 and it is increased to 1.5 times to the base case (scenario-1) in case of scenario-5. The compensation value is varied with the line length. It is decreased with the decrease in line length (scenario-4) and increased as the line length increased (scenario-5).

In scenario-6, the characteristic impedance value is changed in order to observe the change in real power transfer along the line. In all the scenarios, the real power transfer with compensation is equal to the value of the reciprocal of the line characteristic impedance. In this case the characteristic impedance is changed and so the real power transfer accordingly.

In scenario-7, the voltage sag is observed. The voltage profile becomes flat with the compensation but the real power transfer has been reduced and is equal to the reciprocal of the characteristic impedance. In this case, the series compensation is inductive and shunt compensation is capacitive as expected for voltage sag. The chosen values may not be realistic but considered to observe the case of voltage sag.

In scenario-8, the line outage between buses 5 and 6 is considered and the changes in values with compensation are observed.

In all the scenarios, it is suggested that both series and shunt compensation need to be provided in order to get the flat voltage profile. Flatness Index is zero at uncompensated and becomes 1.0 with beta compensation for all the scenarios.

The voltage profiles for different scenarios are shown in Fig. 10. The straight line with 1.0 pu is the profile for all the scenarios with compensation.

Surge impedance loading (SIL) is the power delivered by a line to purely resistive load equal to its surge impedance. [5]

$$\mathrm{SIL} = \frac{|V_L|^2}{z_c} \text{ megawatts}$$

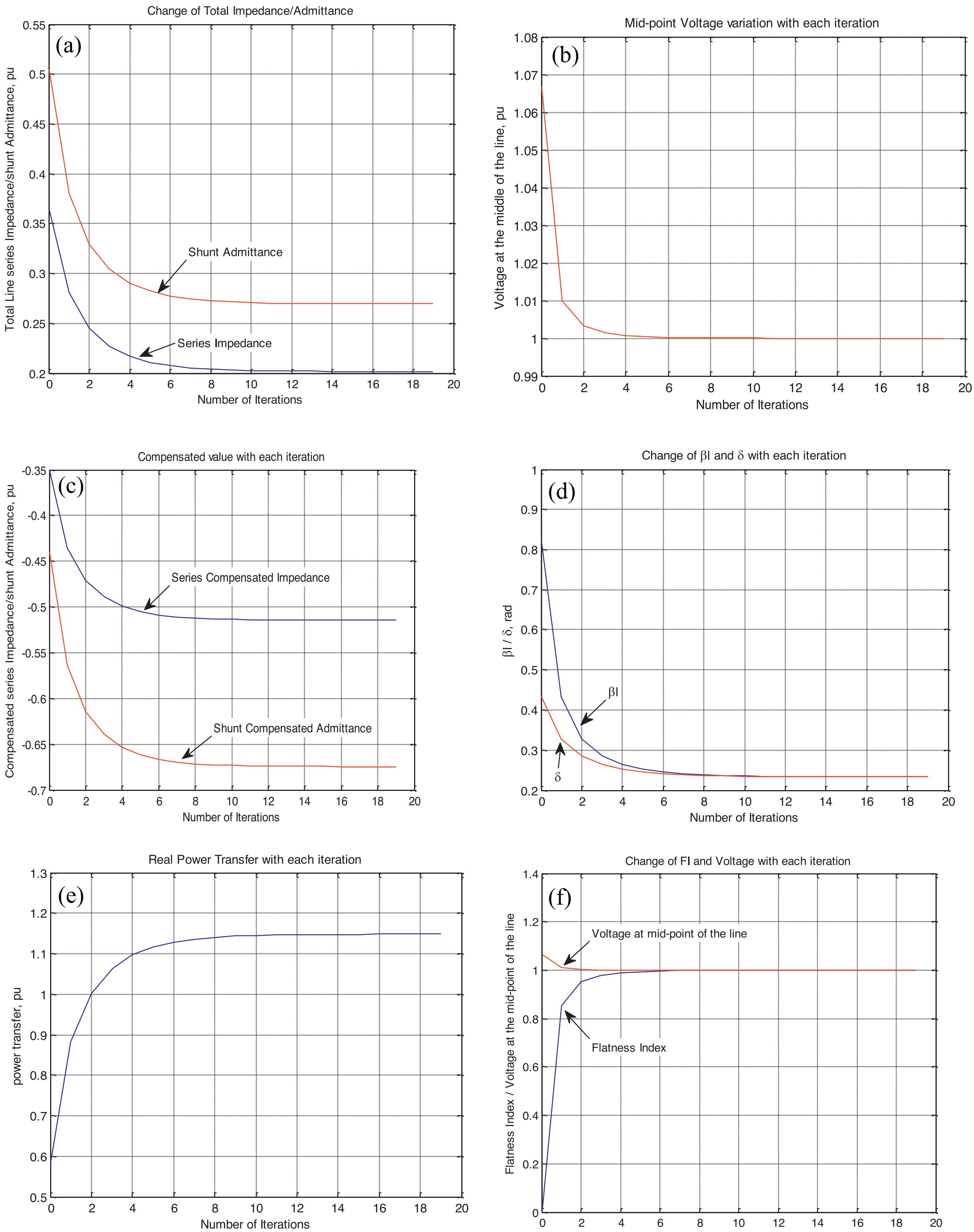

Fig. 9. (a) Total line parameters variation. (b) Mid-point voltage variation. (c) Compensator variation. (d) δ and βl variation. (e) Variation of real power transfer. (f) Flatness Index variation.

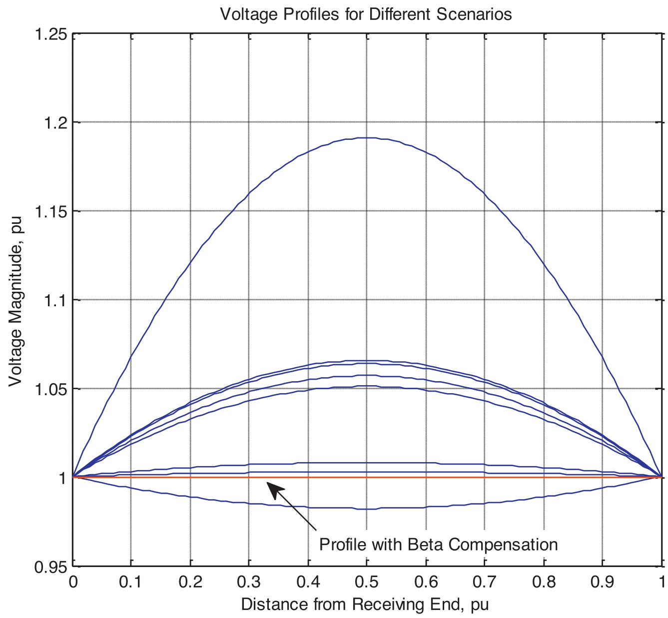

Fig. 10. Voltage profile along the line for different scenarios.

where V_L is line to line voltage at load in kV and Z_c is in ohms. If these are expressed in per unit, it will be equal to $1/Z_c$ pu, when $V_L = 1.0$ pu.

If power transfer from long line equivalent circuit (Fig. 1) for a loss less line is P, assuming the voltage on both sides is at 1.0 pu with power angle δ, then

$$P = \frac{\sin\delta}{Z_c \sinh\gamma l} = \frac{\sin\delta}{Z_c \sin\beta l}$$

With beta compensation, $\beta l = \delta$

$$\text{So } P = \frac{1}{Z_c}$$

By beta compensation it is meant that phase angle is controlled to achieve an effective active power control. In long line it is not the delta but beta is the control variable. The proposed beta compensation is nothing but controlling the surge impedance of the line. The surge impedance always operates at its SIL in order to get a flat voltage profile and the real power transfer is equal to the power delivered at its natural load (SIL).

9. Conclusions

A new method of reactive power compensation, named beta compensation (BC) method has been explained with proper case studies and the results analyzed. This method provided a mean to estimate the amount of compensation required to obtain the flat voltage profile over the transmission line. The combination of both shunt and series compensation is required to operate the line at flat voltage profile. The relation between the Flatness Index proposed and the compensation provided has also been established. The active power transfer has been controlled with beta (phase constant) of the transmission line as the control variable. The realization of the FACTS devices by proposed method and associated cost-benefit analysis is kept for future study.

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